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Non-linear theory of poloidal correlation reflectometry

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1 Introduction

Poloidal correlation reflectometry (PCR) utilizing microwave plasma probing by several poloidaly separated antennae is used nowadays for plasma rotation diagnostics and turbulence analysis [1]. The poloidal rotation velocity (PRV) $V(x)$ is determined in this technique from the temporal shift of the maximum of the cross correlation function of scattered signals in two poloidaly separated channels Fig1. The localization of measurements is based on the assumption that the microwave scattering off long wave-length

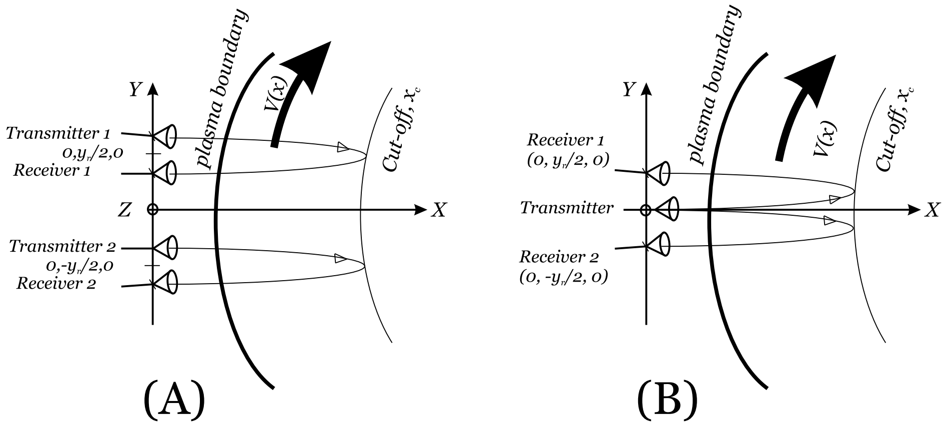

Figure 1: The poloidal correlation reflectometry experimental schemes (poloidal tokamak cross section)

fluctuations dominating in the turbulence spectra occurs in the cut-off layer $x = x_c$. In the case of low turbulence level and/or small distance between the plasma boundary and cut off, when the reflectometry scattering is linear, measurements of PRV are possible if the turbulence correlation time t_c is smaller then the temporal correlation shift caused by the rotation $t_c \ll y_r/V(x_c)$. In non-linear regime this condition is not sufficient for reliable observation of the correlation of two signals, provided by poloidaly separated channels. Even small difference of fluctuating phases in two channels due to inhomogeneity of plasma rotation and reflectometry poor locality can completely suppress correlation thus making measurements impossible. In the present paper the feasibility of PCR is treated in nonlinear regime. We perform the investigation in WKB approximation in the frame of general nonlinear approach developed in [2], [3].
As a result the explicit expression for the cross correlation function of signals in two poloidaly separated channels is obtained for arbitrary profiles of plasma density, rotation velocity, turbulence spatial distribution and wave number spectra. The conditions at which the plasma velocity measurement is possible are determined and its localiza-

tion is estimated. The derived expression is valid in nonlinear regime of fluctuation reflectometry, when the specular component is suppressed in the reflected signal. This expression is convenient for estimation of measurement accuracy and for justification of the obtained rotation velocity profiles.

2 General approach

The analysis is performed in the slab geometry in which the background plasma parameters are dependent only on the radial co-ordinate x. The density fluctuations $\delta n\,(x, \boldsymbol{r}_\perp, \tau)$ are assumed to be 3D inhomogeneous with the correlation function $\langle \delta n\,(x_1, \boldsymbol{r}_{\perp 1}, 0)\,\delta n\,(x_2, \boldsymbol{r}_{\perp 2}, \tau)\rangle = \delta n_0^2(x) K\,(x_1 - x_2, \boldsymbol{r}_{\perp 1} - \boldsymbol{r}_{\perp 2} - \boldsymbol{e}_y V(x)\tau, \tau)$. Here $\boldsymbol{r}_\perp = (y, z)$ and $V(x)$ is the PRV. Co-ordinates y, z stand for the poloidal and toroidal directions, respectively. The density fluctuations are assumed to be small, satisfying condition $|\delta n/n_c| \ll l_{cx}/x_c \ll 1$, at which only one cut off exists in plasma at $x = x_c$, and the turbulence radial correlation length l_{cx} is thought to be large enough $l_{cx} \gg \left(c^2 x_c/\omega^2\right)^{1/3}$, so that backscattering far from the cut off is negligible. Keeping in mind ITER (or reactor scale tokamaks) conditions, we neglect the curvature effects and describe PCR's main features in the non-linear regime.

Under above assumptions the incident wave field in plasma can be represented as a superposition of WKB modes propagating at different angles. In particular, restricting ourselves to O mode reflectometry consideration, at the receiving antennae with the accuracy to the first order in density perturbation amplitude we obtain the following expression for the reflected wave:

$$E_r(\boldsymbol{r}_\perp, t) = \int_{-\infty}^{\infty} \frac{d\boldsymbol{k}_\perp}{(2\pi)^2} \tilde{E}(\boldsymbol{k}_\perp) \exp\left(i\phi_r(\boldsymbol{r}_\perp, \boldsymbol{k}_\perp, t)\right), \tag{1}$$

where $\boldsymbol{k}_\perp = (k_y, k_z)$, $\tilde{E}(\boldsymbol{k}_\perp)$ is a Fourier transformation of the spatial distribution of the unite power probing microwave beam $E_0(\boldsymbol{r}_\perp)$ at the probing antenna situated at $\boldsymbol{r}_{in} = ((\pm y_r + \delta_y)/2; 0)$. The receiving antenna is assumed to be situated at $\boldsymbol{r}_a = ((\pm y_r - \delta_y)/2; 0)$. Both the probing and the receiving antennae pattern are assumed to be Gaussian with the poloidal and the toroidal width equal ρ. Due to the condition $\rho\omega/c \gg 1$ the phase ϕ_r can be represented in the paraxial approximation as $\phi_r = \varphi_0 + \delta\varphi$, where

$$\varphi_0 = 2\int_0^{x_c} k_o\,(x, \omega)\,dx + \boldsymbol{k}_\perp \boldsymbol{r}_\perp - \frac{k_y^2 d_y^2}{2} - \frac{k_z^2 d_z^2}{2} - \frac{\pi}{2} \tag{2}$$

and

$$d_y^2(x) = 2\int_x^{x_c(\omega)} \frac{d\xi}{k_o(\xi, \omega)},\ d_z^2(x) = 2\kappa^{-2}\int_x^{x_c(\omega)} k_o(\xi, \omega) d\xi,\ \kappa = \omega/c. \tag{3}$$

The ordinary mode radial wave number is given by the expression $k_o\,(x, \omega) = \kappa\sqrt{1 - \omega_{pe}^2(x)/\omega^2}$. According to [3], the phase perturbation $\delta\varphi$ with the accuracy up to the first order terms is given by the WKB integral

$$\delta\varphi\,(\boldsymbol{r}_\perp, \boldsymbol{k}_\perp, t) = -\frac{\omega^2}{2c^2} \cdot \int_0^{x_c} \frac{\delta n\left(x, \boldsymbol{r}_\perp^-\,(x, \boldsymbol{k}_\perp)\,, t\right) + \delta n\left(x, \boldsymbol{r}_\perp^+\,(x, \boldsymbol{k}_\perp)\,, t\right)}{n_c} \frac{dx}{k_o(x)} \tag{4}$$

calculated along the unperturbed trajectory

$$\boldsymbol{r}_{\perp}^{\pm}(x, \boldsymbol{k}_{\perp}) = \boldsymbol{e}_y \left(y - \frac{k_y}{2} \left(d_y^2(0) \pm d_y^2(x) \right) \right) + \boldsymbol{e}_z \left(z - \frac{k_z}{2} \left(d_z^2(0) \pm d_z^2(x) \right) \right) \tag{5}$$

$\boldsymbol{r}_{\perp}(x, \boldsymbol{k}_{\perp})$ coming to the point $(0, \boldsymbol{r}_{\perp})$ from $(0, \boldsymbol{r}_{in})$. The upper and lower signs in Eq.(5) correspond to the parts of the ray trajectory after and before the reflection from the cut off surface. To derive the explicit expression for the cross-correlation function CCF_{12} of two signals $A_r(t_1)$ and $A_r(t_2)$

$$A_r(t_1) = \int\int d\boldsymbol{r}_{\perp} E_r(\boldsymbol{r}_{\perp}, t_1) E_0(\boldsymbol{r}_{\perp} - \boldsymbol{r}_{\perp a}),$$

registered by two antennae

$$CCF_{12} = \frac{\langle (A_r(t_1) - \langle A_r(t_1) \rangle)(A_r(t_2) - \langle A_r(t_2) \rangle)^* \rangle}{\left\langle (A_r(t_1) - \langle A_r(t_1) \rangle)^2 \right\rangle^{1/2} \left\langle (A_r(t_2) - \langle A_r(t_2) \rangle)^2 \right\rangle^{1/2}} \tag{6}$$

where $\langle ... \rangle$ stands for statistical averaging, we make several assumptions. First, it is quite natural in the case $l_{cx} \ll x_c$ to consider the random phase perturbations $\delta\varphi_{\alpha}$ as a sum of many independent random values, describing its evolution as a normal random process. Second, in the strong non-linear regime of PCR when the condition $\kappa^2 x_c l_{cx} \delta n^2 / n_c^2 ln\,(x_c/l_{cx}) \gg 1$ holds, the correlation of two reflectometer signals is not suppressed only if they are produced by the "same fluctuations" in similar propagation of probing waves, i.e. when the condition $\left(\boldsymbol{r}_{\perp}^{\pm}(x_1, \boldsymbol{k}_{\perp 1}) - \boldsymbol{r}_{\perp}^{\pm}(x_2, \boldsymbol{k}_{\perp 2}) - \boldsymbol{e}_y V\tau\right) \ll \boldsymbol{l}_{c\perp}$ is fulfilled all over the plasma. Due to the strong decorrelating effect of the turbulent phase distortions only the poloidal correlation scheme shown in Fig.1a have a chance to perform rotation measurements in the non-linear regime. In the case of second scheme (Fig.1b), where the propagation direction of probing waves is opposite, the correlation suppression makes the velocity measurements impossible. The rotation inhomogeneity due to imperfect locality of reflectometry also decrease the correlation and therefore challenge the velocity measurements in non-linear regime. We study this effect below taking into account strong anisotropy of the turbulence, usually satisfying conditions $\rho, l_{cy} \ll l_{cz}$. In this case the strong wave front distortions are important only in the poloidal direction. We consider the regime when the PRV is only weakly inhomogeneous and the turbulence poloidal radius is rather long $l_{cy} \gg \delta, d_y(0)$. In this case the incident wave front distortions are relatively small and the reflected wave amplitude (1) can be calculated using stationary phase method neglecting the fluctuating phase contribution. Introducing now an effective poloidal correlation radii and time appearing due to the incident wave propagation through the turbulent plasma as

$$\frac{1}{\rho_m^2} = \frac{\omega^4 l_{cx}}{16c^4} \int \frac{dx}{k_o^2(x)} \frac{\delta n(x)^2}{n_c^2} \frac{\partial^2}{\partial y^2} \left(\bar{K}_n(y,0)_{y=0} + \bar{K}_n(y,0)_{y=K d_y^2(x)} \right) \psi^{m-1}(x), m = 1..3,$$

$$\frac{1}{t_c^{ef2}} = \frac{\omega^4 l_{cx}}{4c^4} \int \frac{dx}{k_o^2(x)} \frac{\delta n(x)^2}{n_c^2} \frac{\partial^2}{\partial \tau^2} \left(\bar{K}_n(0,\tau) + \bar{K}_n(K d_y^2(x), \tau) \right)_{\tau=0}, \tag{7}$$

where $\bar{K}_n(y,\tau)$ means $\bar{K}_n(y,\tau) = l_{cx}^{-1}\int_0^{x_c} dx K_n(x,y,0,\tau)$ and $\psi(x) = \delta V(x)/V_c$ with $\delta V(x) = (V(x) - V_c)$,$V_c = V(x_c)$, the CCF_{12} can be represented as

$$CCF_{12} = \exp\left(-\frac{\left(y_r - V_c\left(1+\rho_1^2/\rho_2^2\right)\tau\right)^2}{\rho_1^2} - \frac{V_c^2\tau^2}{\rho_3^2}\left(1 - \frac{\rho_1^2\rho_3^2}{\rho_2^4}\right) - \frac{\tau^2}{t_c^{ef2}}\right) \tag{8}$$

The enhancing factor $1/k_o^2(x)$, as it was demonstrated in [2], is not sufficiently localized to cut-off. That results in the systematic mistakes in the case of inhomogeneous PRV. In fact, as it is shown in Eq.8 measurements rather provide information about the quantity $V_c\left(1+\rho_1^2/\rho_2^2\right)$. Suppression of the correlation stimulated by the rotation inhomogeneity is described by the term $\exp\left(-V_c^2\tau^2/\rho_3^2 \cdot (1 - \rho_1^2\rho_3^2/\rho_2^4)\right) \leq 1$ because of $(1 - \rho_1^2\rho_3^2/\rho_2^4) > 0$ according to Cauchy-Schwarz inequality. These effects are illustrated

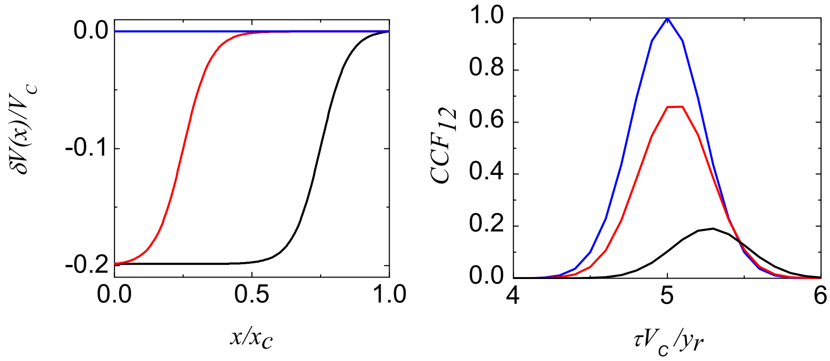

Figure 2: CCF_{12} for different rotation profiles. (a) rotation profiles, (b) CCF_{12}. Blue lines correspond to homogeneous PRV, black and red lines correspond to inhomogeneous one. The non-linearity criterion is chosen to be equal $\kappa^2\delta n^2/n_c^2 x_c l_{cx} = 7.2$, the turbulence level is assumed to be homogeneous and $y_r/l_{cy} = 4$

in Fig.2, where the CCF_{12} is shown both for homogeneous and inhomogeneous PRV.

3 Conclusions

According to our analysis only the PCR scheme (Fig.1a) have a chance of successful poloidal velocity measurements at high density level (in non-linear regime). The systematic mistake of the velocity estimation due to its non uniformity and imperfect measurements localization are shown to be significant. The correlation suppression possibility in the case of velocity inhomogeneous in the cut off is demonstrated.

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