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Evolution of meso-scale plasma pressure perturbations in tokamak

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Introduction. There is vast experimental evidence of an important role of anomalous convection in the edge plasma transport in tokamaks, stellarators, and linear devices [1]. For example, tokamak experimental data clearly show that meso-scale plasma structures extended along the magnetic field lines, which often called blobs , are advected coherently from the region close to LCFS into the far SOL on a distance ~10 cm. Moreover, recent experimental observations [2] suggest that the dynamics of ELMs in the SOL plasma is very much similar to that of blobs, and result in a much larger plasma particle and energy fluxes into far SOL than it was thought of. Rather simple models, based on the effective plasma gravity caused by magnetic curvature or neutral wind [3] effects, describe many essentials of nonlinear evolution and radial advection of such meso-scale structures. Here we examine the role of drives, associated with the instabilities due to $\nabla_\perp T_e$ in the SOL [4, 5] and due to parallel shear of $\mathbf{E}\times\mathbf{B}$ drift velocity [5, 6], in nonlinear cross-field advection of plasma structures.

Plasma cross-field convection caused by the parallel shear instability. To derive the equations governing nonlinear evolution of plasma parameters associated with parallel shear of $\mathbf{E}\times\mathbf{B}$ drift velocity we use electron parallel momentum balance equation assuming that $T_e = T_0 \equiv \text{const.}$ and neglecting resistive effects

$$c^{-1}\partial_t\psi + \nabla_{||}\big((T_e/e)\ln(n) - \varphi\big) = 0, \tag{1}$$

where n is the plasma density, φ and ψ are the electrostatic and magnetic potentials, c is the speed of light. For cold ions from the condition of zero divergence of electric current we have

$$\nabla_\perp \cdot \left(cV_A^{-2} d_t \nabla_\perp \varphi\right) = (4\pi/c)\nabla_{||} j_{||} = \nabla_{||}\nabla_\perp^2\psi, \tag{2}$$

where $d_t(...) \equiv \partial_t(...) + \mathbf{V}_E\cdot\nabla(...)$, $\mathbf{V}_E = -C_s\rho_s(\nabla\phi\times\mathbf{b})$, $\mathbf{b} = \mathbf{B}/B$, B is the magnetic field strength, $C_s = \sqrt{T_0/M}$, $\rho_s = C_s/\Omega_i$, $\Omega_i = eB/Mc$, $V_A^2 = B^2/4\pi nM$, M is the ion mass, and $\phi = e\varphi/T_0$. And finally we will need the continuity equation $d_t n = 0$. Assuming that $\mathbf{b}\cdot\nabla_\perp(...) << \nabla_{||}(...)$, from the equations (1, 2) and continuity equation we find

$$\partial_t\nabla_\perp\cdot\left(V_A^{-2} d_t\nabla_\perp\phi\right) = \nabla_{||}^2\nabla_\perp^2(\phi - \Lambda), \qquad d_t\Lambda = 0, \tag{3}$$

where $\Lambda = \ln(n/n_\infty)$, $n_\infty = \text{const.}$ is the plasma density at infinity. Effectively, we neglect the nonlinearities associated with the perturbations of the magnetic field while retaining nonlinear effects due to the perturbations of the $\mathbf{E}\times\mathbf{B}$ drift velocity.

We consider the case where the plasma density profiles along the magnetic field line have a bump with parallel scale L_b. Then, neglecting the nonlinearities in the left hand side of the first equation in (3) outside the bump, we find

$$V_{A_\infty}^{-2}\partial_t^2 G = \nabla_{||}^2 G, \tag{4}$$

where $G = \phi - \Lambda$. Eq. (4) has obvious solutions $G = G_{(+)}(l_{||} - V_{A_\infty}t)$ and $G = G_{(-)}(l_{||} + V_{A_\infty}t)$, where V_{A_∞} is the Alfven velocity far from the bump. Integrating Eq. (3) over the bump in parallel coordinate, assuming weak variation of the parameters inside the bump and using solutions of (4) to evaluate derivatives $\partial(...)/\partial l_{||}$ we find

$$L_b\partial_t\nabla_\perp\cdot(V_{A_b}^{-2} d_t\nabla_\perp\phi) = \partial_{l_{||}}\nabla_\perp^2 G \Big|_{-bump}^{+bump} = -2V_{A_\infty}^{-1}\partial_t\nabla_\perp^2 G, \tag{5}$$

which results in

$$(L_b V_{A_\infty}/2V_{A_b}^2)d_t\nabla_\perp\phi = -\nabla_\perp(\phi - \Lambda), \tag{6}$$

where V_{A_b} is the Alfven velocity in the bump region. In a long wavelength approximation, $\partial_t(...) << \mathbf{V}_E \cdot \nabla(...) << V_A / L_b$, assuming that differences in the plasma densities inside and outside the bump is not large, $V_{A_b} \approx V_{A_b} = V_A = \mathrm{const.}$, from (3) and (6) we obtain

$$\partial_t \phi = -(L_b / 2V_A)\mathbf{V}_E \cdot \{(\mathbf{V}_E \cdot \nabla_\perp)\nabla_\perp \phi\} \quad (7)$$

Using Eq. (7), one easily recovers expression for linear growth rate of the instability found in Ref. 6, $\gamma = (L_b / 2V_A)(\mathbf{V}_E \cdot \mathbf{k})^2$. Next we consider nonlinear solutions of the Eq. (7) in the form of the wedge in cross field frame (x,y)

$$\phi(x,y,t) = (x + Ut)^\alpha F\left(y/(x+Ut)^\beta\right), \quad (8)$$

where U is the wedge convection velocity, F is unknown function, and α and β are the adjustable parameters. Substituting (8) into (7) we find $\beta = \alpha - 1/2$ and

$$u\left(\alpha F - (\alpha - 1/2)\eta d_\eta F\right) = -\alpha^2 F^2 d_\eta^2 F + \left(d_\eta F\right)^2 \{\alpha(2-\alpha)F - (\alpha-1/2)(3/2-\alpha)\eta d_\eta F\}, \quad (9)$$

where $\eta = y/(x+Ut)^{\alpha - 1/2}$ and $u = 2V_A U / L_b (C_s \rho_s)^2$. The analysis of Eq. (9) shows that F approaches zero at some point $\eta = \eta_0$ as $F(\eta) \propto (\eta_0 - \eta)^\sigma$ where $\sigma = 1/2$ for $\alpha = 3/2$ and $\sigma = 1$ otherwise. The case with $\alpha = 3/2$ ($\sigma = 1/2$) could be matched through some boundary layer with unperturbed plasma ($F \propto \phi = 0$) outside the wedge if we would account for the plasma density diffusion in Eg. (3). Other cases, which give just continuous transition of potential from positive to negative values. Therefore we consider solution with $\alpha = 3/2$ as a "real" solution. We solve Eq. (9) numerically for $\alpha = 3/2$ and show the result in Fig. 1. From the value $\eta_0 \approx 1.4$, which follows from Fig. 1 we find

$$U \approx U_{||} \equiv C_s (L_b C_s / \rho_s V_A)(\phi_0(x))^2 (\rho_s / y_0(x))^3, \quad (10)$$

where $\phi_0(x)$ and $y_0(x)$ are the normalized potential at y=0 and the width of a wedge as a function of x, notice that for our solutions $\phi_0(x) \propto (y_0(x))^{3/2}$.

Plasma cross-field convection caused by $\nabla_\perp \mathbf{T_e}$ instability. The drive related to the $\nabla_\perp T_e$ instability can be important mechanism for plasma convection in the SOL region. This instability is associated with the effective plasma resistivity caused by the sheath [4, 5] and which relates plasma current through the sheath, j_{sh}, and the plasma potential φ relative to the wall, $j_{sh} = e n_{sh}\sqrt{T_e/M}\{1 - \sqrt{M/2\pi m}\exp(-e\varphi/T_e)\}$, where m is the electron mass and n_{sh} is the plasma density at the entrance to the sheath. Then, assuming that plasma density, temperature, and electrostatic potential are uniform along the field lines, we integrate Eq. (2) in the SOL region from one end plate to another (for simplicity we assume that end plate is normal to the magnetic field lines) and use j_{sh} as a boundary conditions for $j_{||}$. As a result we have

$$\nabla_\perp \cdot \left((n/M\Omega_i^2) d_t \nabla_\perp (e\varphi)\right) = (2n/L_{con})\sqrt{T_e/M}\{1 - \sqrt{M/2\pi m}\exp(-e\varphi/T_e)\}, \quad (11)$$

where L_{con} is the connection length. In addition to Eq. (12) we need to use the continuity equation and the similar equation for the advection of electron temperature, $d_t T_e = 0$, which describes the energy balance. Notice that although the plasma contact with end plate is crucially important for the vorticity equation (11), this contact can be neglected from the view point of transport of the plasma density and electron temperature.

To simplify the problem even more we will assume constant plasma density and relatively small variation of plasma temperature so that $T_e = T_0 + \delta T_e$ and $\varphi = \varphi_0 + \delta\varphi$, where $T_0 \equiv \mathrm{const.}$, $\varphi_0 \equiv \mathrm{const.}$, and $|\delta T_e| < T_0$ and $|\delta\varphi| < \varphi_0$. Then from Eq. (11) and advection of electron temperature we find

$$(\rho_s^2 L_{con} / 2C_s) d_t \nabla_\perp^2 \phi = \phi - \vartheta, \qquad d_t \vartheta = 0, \quad (12)$$

where $\phi = e\delta\varphi / T_0$ and $\vartheta = 0.5\ln(M/2\pi m)\delta T_e$. From (12) in a long wavelength approximation, assuming that $\partial_t(...) << \mathbf{V}_E \cdot \nabla(...)$, we find

$$(2C_s/\rho_s^2 L_{con})\partial_t\phi = \mathbf{V}_E \cdot \nabla\{\mathbf{V}_E \cdot \nabla(\nabla_\perp^2\phi)\}. \tag{13}$$

From Eq. (13) we recover corresponding linear growth rate, $\gamma = (\rho_s^2 L_{con}/2C_s)(\mathbf{V}_E \cdot \mathbf{k})^2 k^2$, as in Ref. 4. We will search for a nonlinear solution of Eq. (13) assuming $\partial_y(...) << \partial_x(...)$ and using the ansatz (9). Substituting (9) into (13) we find $\beta = 1$, $\alpha = 5/2$, and

$$w\left(2.5F - \eta d_\eta F\right) = d_\eta^2 F\left(d_\eta F\right)^2 - 2.5F\left(d_\eta^2 F\right)^2 + 5F d_\eta F d_\eta^3 F + 12.5F^2 d_\eta^4 F, \tag{14}$$

where $W = 2U/(\rho_s^4 L_{con} C_s)$. The analysis of Eq. (14) shows that F approaches zero at some point $\eta = \eta_0$ as $F(\eta) \propto (\eta_0 - \eta)^{3/2}$, which can be matched with unperturbed solution $\delta\varphi = 0$ outside the wedge. In order to find such solution we solve Eq. (14) numerically assuming at $\eta = 0$ that $dF(\eta)/d\eta = d^3F(\eta)/d\eta^3 = 0$ and varying $d^2F(\eta)/d\eta^2$. In Fig. 2 we present both $F(\eta)$ and $dF(\eta)/d\eta$ found numerically from (14). As one can see from Fig. 2, in accordance with expected asymptotic we recover $-dF(\eta)/d\eta|_{\eta\to\eta_0} \propto (\eta_0 - \eta)^{1/2}$. As a result we get the following expression for convective velocity

$$U \approx U_{\nabla T_e} \equiv 45 C_s(\phi_0(x))^2 \eta_0 \rho_s^4 L_{con}(y_0(x))^{-5}, \tag{15}$$

where $\phi_0(x)$ and $y_0(x)$ are the normalized potential $\phi = e\delta\varphi/T_0$ at y=0 and the width of a wedge as a function of x. Notice, that for our solution $\phi_0(x) \propto (y_0(x))^{5/2}$ and we made an approximation $\partial_y(...) << \partial_x(...)$, which results in the inequality $\eta_0 < 1$.

Discussion. We have shown that in a long wavelength approximation the drives, associated with both parallel shear and $\nabla_\perp T_e$, can cause the cross-field advection of nonlinear structures. However, these large scale structures can be unstable against relatively short wavelength fingering instabilities caused, correspondingly, by parallel shear and $\nabla_\perp T_e$, which growth rates are proportional, respectively, to k^2 and k^4 for $k > k_*$, where k_* is the wave number where long wave length approximation, $\mathbf{V}_E \cdot \mathbf{k} > \gamma$, is marginal. Such short wavelength instability was found [3] for large-scale blobs. But simultaneously it was also found that blobs with spatial scale $\sim k_*^{-1}$ is very stable and for a tokamak conditions can travel as a coherent structure at distances ~10 cm and more [3]. The reason for "super-stability" can be explained as follows. While the large $(> k_*^{-1})$ scale structures are subjected to fingering instability, the small ones, for which inertia term dominates, experience the development of mushroom-like shapes similar to the shapes that usually develop on nonlinear stage of the Rayleigh-Taylor instability of stratified fluid. For the scales $\sim k_*^{-1}$ these two effects, fingering and mushrooming, compensate each other. Since in the limit of large k the main terms for both parallel shear and $\nabla_\perp T_e$ instabilities are the same inertial terms, we can expect similar stabilizing effects for nonlinear structures considered with the scale length $\sim k_*^{-1}$. Then estimating $\mathbf{V}_E \cdot \nabla(...) \sim V_E/y_0(x) \sim C_s\phi_0(x)\rho_s/y_0^2(x)$ and $\partial_t(...) \sim u/y_0(x)$ we find both $\delta_{(...)} \sim k_*^{-1}$ and the expressions for the velocities $U_{||}(\delta_{||})$ and $U_{\nabla T_e}(\delta_{\nabla T_e})$ which correspond to these scales. After some algebra, from (10) and (15) we find

$$\delta_{||}/\rho_s \approx \left(\phi_0(L_b/\rho_s)\beta_p^{1/2}\right)^{1/2}, \qquad U_{||}(\delta_{||})/C_s \approx \left(\phi_0\beta_p^{-1/2}(\rho_s/L_b)\right)^{1/2}, \tag{16}$$

$$\delta_{\nabla T_e}/\rho_s \approx (\phi_0\eta_0(L_{con}/\rho_s))^{1/4}, \qquad U_{\nabla T_e}(\delta_{\nabla T_e}) \approx \left((\phi_0^3/\eta_0)(\rho_s/L_{con})\right)^{1/4}, \tag{17}$$

where β_p is the plasma beta and ϕ_0 is the characteristic perturbation of normalized electrostatic potential. For $\phi_0 \sim 1$, $\eta_0 \sim 1$, $\beta_p \sim 10^{-3}$, and $L_b/\rho_s \sim L_{con}/\rho_s \sim 3\times10^4$ from (16), (17) we find $\rho_s/\delta_{||} \sim U_{||}(\delta_{||})/C_s \sim 0.03$ and $\rho_s/\delta_{\nabla T_e} \sim U_{\nabla T_e}(\delta_{\nabla T_e})/C_s \sim 0.1$, which is consistent with experimental observations.

We also notice that in the SOL region, the inhomogeneity of plasma parameters along the magnetic field lines and, correspondingly, the drive associated with parallel shear of $\mathbf{E}\times\mathbf{B}$ drift velocity, naturally occurs due to plasma recycling at the end plates providing that coulomb mean free path, $\lambda_c \propto T^2/n$, is smaller than connection length $L_{con} \propto qR$, where R is the tokamak major radius and q is the safety factor. In order to avoid the effects of this instability we need to have $\lambda_c > L_{con}$. But, requirement $\lambda_c > L_{con}$ results in inequality $n < CT^2/qR$ (where C is the constant), which is somewhat similar to the density limit (see for example Ref. 7 and the references therein). Moreover, since temperature drops quickly while we move from the LCFS into the SOL, λ_c reaches L_{con} first in the far SOL. Then, with increasing plasma density both the region $\lambda_c \sim L_{con}$ and appearance of nonlinear convective structures, associated with parallel shear of $\mathbf{E}\times\mathbf{B}$ drift velocity, propagate toward the LCFS. This qualitative picture fits very well experimental observations [7] of spreading of radial convective transport from far SOL to the core while plasma density increases. Thus, it is plausible that the instability related to parallel shear of $\mathbf{E}\times\mathbf{B}$ drift velocity is responsible for the onset of density limit and related processes like MARFE and divertor detachment.

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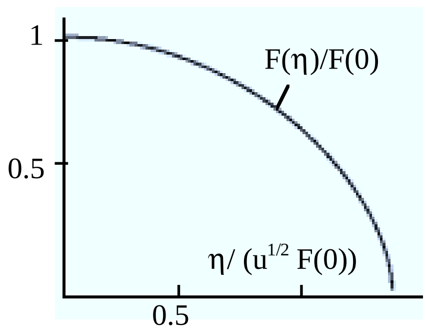

Fig. 1. Numerical solution of Eq. (9) for $\alpha = 3/2$.

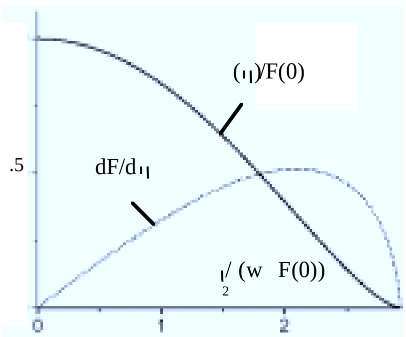

Fig. 2. Numerical solution of Eq. (14).