31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-1.097 (2004)

Vlasov description of space charge inversion close to the wall in collisionless plasma sheath at grazing incidence of the magnetic field

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We present in this work a study of the problem of the formation of a collisionless plasma sheath at a plasma-wall transition with grazing incidence of the magnetic field. The simulation is done for deuterium. We show that below a critical angle $\alpha_c = \sqrt{T_i m_e / (T_e m_i)}$ (with T_i/T_e=2 and a ratio of ion gyroradius over Debye length equal to 20 in the present simulation), the electrons moving parallel to B can no longer follow the ions gyrating perpendicular to B. The relevant physics in this case will be studied. For angles larger than the critical angle we recover the standard sheath which is determined by the Bohm condition.

Kinetic model for the sheath

We consider a one-dimensional slab geometry in which the inhomogeneous direction is in the direction x normal to the wall. The y and z directions are assumed homogeneous. The constant magnetic field is located in the (x,y)-plane, and makes an angle α with the y axis. The magnetized electrons are restricted to move with a velocity $\upsilon_\parallel$ along the magnetic field and are described using a kinetic equation in the direction along the magnetic field, with a distribution function $f_e(x,\upsilon_\parallel)$ obeying the Vlasov equation:

$$\frac{\partial f_e}{\partial t} + \upsilon_\parallel \sin\alpha \frac{\partial f_e}{\partial x} - \frac{m_i}{m_e} E_x \sin\alpha \frac{\partial f_e}{\partial \upsilon_\parallel} = 0 \qquad (1)$$

Time is normalized to the ion plasma frequency ω_{pi}^{-1},velocity is normalized to the acoustic velocity $c_s = \sqrt{T_e / m_i}$, and length is normalized to Debye length $\lambda_D = c_s / \omega_{pi}$. The potential is normalized to T_e / e, and the density to the peak initial central density n_o. The ions are treated with a fully kinetic Vlasov equation in 1D, which is written for the distribution function $f_i(x,\upsilon_x,\upsilon_y,\upsilon_z)$ as:

$$\frac{\partial f_i}{\partial t} + \upsilon_x \frac{\partial f_i}{\partial x} + (E_x - \upsilon_z \omega_{ci} \cos\alpha)\frac{\partial f_i}{\partial \upsilon_x} + \upsilon_z \omega_{ci} \sin\alpha \frac{\partial f_i}{\partial \upsilon_y} + \omega_{ci}(\upsilon_x \cos\alpha - \upsilon_y \sin\alpha)\frac{\partial f_i}{\partial \upsilon_z} = 0 \qquad (2)$$

The electric field is calculated from the Poisson equation: $\frac{\partial^2 \phi}{\partial x^2} = -(n_i - n_e)\,;\, E_x = -\frac{\partial \phi}{\partial x}$ (3)

where $n_{i,e} = \int f_{i,e} d\vec{\upsilon}$. We assume the initial distributions to be Maxwellian with: $n_{i,e} = n_o$.

$$f_e(x,\upsilon_{\|}) = n_o \sqrt{\frac{\frac{m_e}{m_i}}{2\pi \frac{T_e}{T_i}}} e^{-(m_e/m_i)\upsilon_{\|}^2/2(T_e/T_i)}; \qquad f_i(x,\vec{\upsilon}) = \frac{n_o}{(2\pi)^{3/2}} e^{-\upsilon^2/2} \qquad (4)$$

with $T_i / T_e = 2$ in the present calculations and $m_e / m_i = 0.5/1836$ for deuterium, and with $\omega_{ci} / \omega_{pi} = 0.1$. The wall is located at $x = 0$, and L is the length of the system. For the boundary conditions of the distribution functions it is assumed that at the sheath entrance at the right boundary at $x = L$ the plasma extends to an identical plasma, so that the input particle flux with negative velocities is entering from a similar plasma (essentially a Maxwellian for a sufficiently long system), and the return particle outflow is allowed to evolve freely, leaving the boundary, at $x = L$ with positive velocities also to an identical plasma. At the left boundary, particles hitting the plate are lost from the system and collected through the current delivered at the plate:

$$\frac{\partial E_x}{\partial t}\Big|_{x=0} = -J_x\Big|_{x=0} = -(J_{xi} - J_{xe})\Big|_{x=0}; \text{ from which } E_x\Big|_{x=0} = -\int_0^t J_x\Big|_{x=0} dt \equiv -\frac{\partial \phi}{\partial x}\Big|_{x=0} \qquad (5)$$

Equations (1)-(3) are solved using a method of fractional steps [1]. We run the code and let the initially neutral plasma evolve to a steady state. The motion of the ions and the motion of the electrons are advanced with a time step $\Delta t = 0.02$. The total length L of the system is taken to be 200 Debye lengths. We use 300 grid points in space and 60 points in each velocity space direction, with velocity maxima for the ions equal to ± 5 ion thermal velocities ($\pm 5\sqrt{2}$ acoustic velocities c_s) and also for the electrons the velocity maxima are ± 5 electron thermal velocities. See ref. [2] for more details.

Results

Figure 1 shows the potential (full curves) and the electric field (broken curves) for the case $\alpha = 5°$.For the present set of parameters $\alpha_c = (T_i m_e / T_e m_i)^{1/2} = 1/42.85$ or equivalently 1.34°. In this case $\alpha > \alpha_c$ negative potential and negative electric field are monotonously increasing (in absolute value) toward the wall, and so does the positive charge density when approaching the wall. A constant electric field profile is appearing in front of the plate, with a small oscillation away from the plate of period around 60 (the gyroperiod). Coming from the right boundary, ions are essentially maxwellian, then accelerate in front of the wall to the acoustic speed (velocities have to be divided by $(T_e + T_i)^{1/2} / T_e^{1/2}$, if we want to normalize them to the acoustic speed $(T_e + T_i)^{1/2} / m_i^{1/2}$), to form what is essentially a classical sheath in front of the wall [3]. Figure 3 shows a similar result for the case when $\alpha = 1.3°$ (around the critical angle), where we notice a more important oscillation of the potential, still the gyroperiod. We also observe that the electric field is already much flatter close to the wall, which is reflected in a broader space charge density distribution. Figures 3 and 4 obtained with $\alpha = 0.75°$ and $\alpha = 0.5°$ (with $\alpha < \alpha_c$) show during about half a period the large amplitude oscillations for the potential. The period is about 100 for $\alpha = 0.75°$ and 150 for $\alpha = 0.5°$. We show in fig. 5 the charge density $(n_i - n_e)$ for the angles $\alpha = 0.5°$, (dash-dotted curve at $t = 640$), $\alpha = 0.75°$ (broken curve at $t = 360$), $\alpha = 1.3°$ (dotted curve at $t = 240$), and $\alpha = 5°$ (full curve at $t = 205$). For small angles, the electrons moving parallel to the magnetic field B can no longer follow the ions gyrating perpendicular to B. Thus electrons are accumulating in front of the plate, whereas ions with large gyroradius are scraped off by the wall. This creates a negative space charge close to the wall. With our collisionless model, undamped oscillations with periods much larger than the gyroperiod are observed for the small angles. The electrons take over and determine the characteristic times for information propagation. In front of the plate, we have a sheath inversion with a negative space charge density (see fig. 5 for $\alpha = 0.5°$), and an electric field becoming less negative at the plate (instead of decreasing monotonously towards the plate, as in a classical sheath). Figure 6 shows the electric field (full curve) for $\alpha = 0.5°$ and at $t = 690$, together with the pressure force $\nabla P_i / n_i$, $P_i = 0.5\, n_i\, (T_{ix} + T_{iz})$ (dotted curve), and the Lorentz force term $0.1\, j_x / (n_i \cos\alpha)$ (dash-dot curve). The combined force term $\nabla P_i / n_i + 0.1\, j_x / (n_i \cos\alpha)$ is given by the broken curve, which follows nicely the electric field term, except in front of the plate where n_i tends to zero and the curve shows a divergence. The ion density $n_i / 4$ is given by the dash – 3dots curve. According to our results, it seems that a minimum of the particle flux and the heat transmission factor occurs for $\alpha \approx \alpha_c / 2 = 0.67°$, which is close to the case $\alpha = 0.75°$. For smaller α the sheath transmission becomes again greater. Similar effects have been observed experimentally in [4].

References

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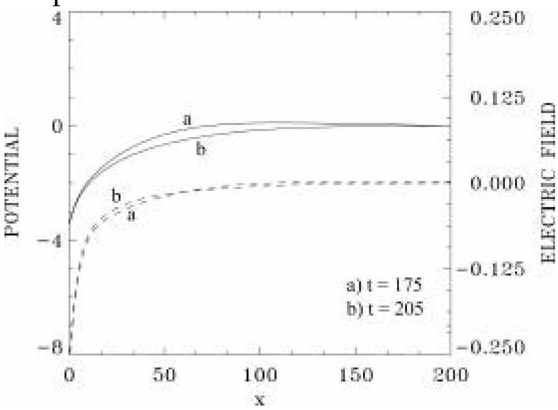

Figure 1 : $\alpha = 5°$

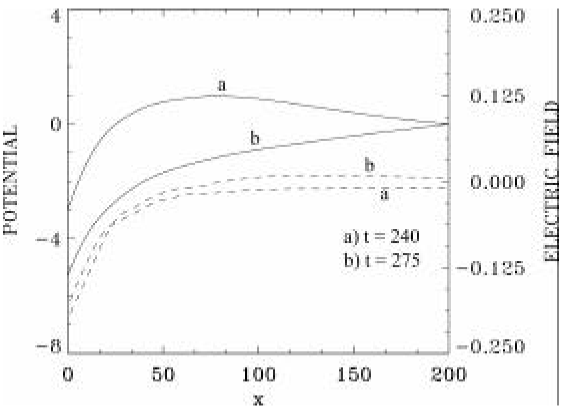

Figure 2 : $\alpha = 1.3°$

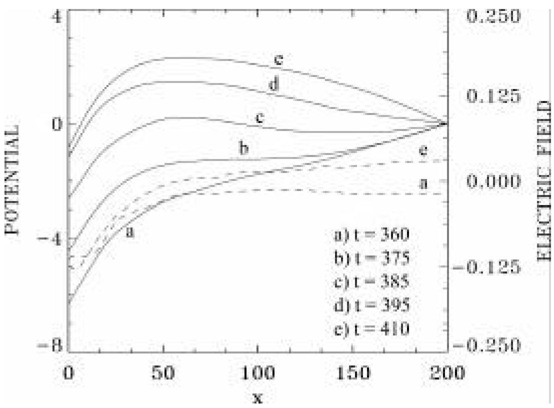

Figure 3 : $\alpha = 0.75°$

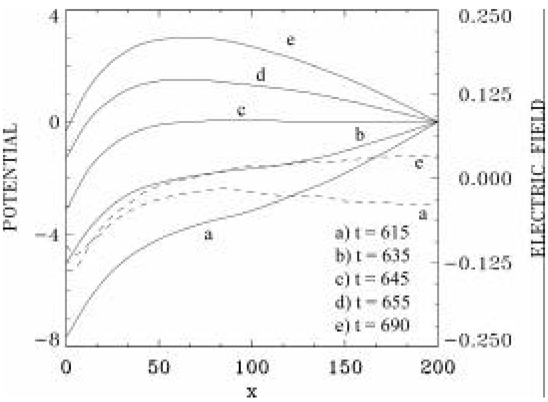

Figure 4 : $\alpha = 0.5°$

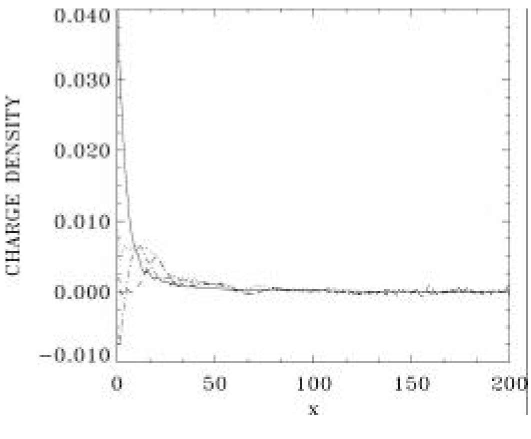

Figure 5: Space charge density

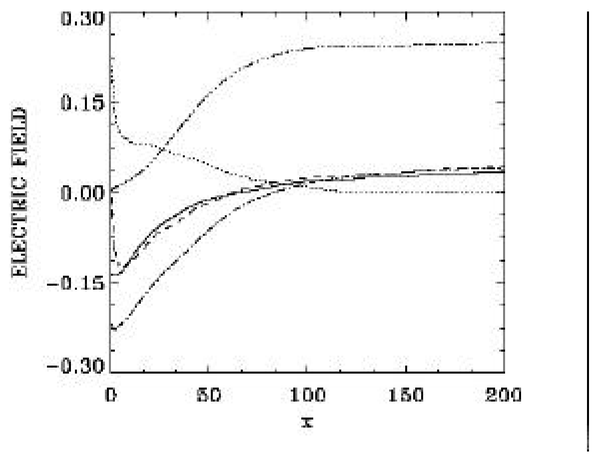

Figure 6: Forces for $\alpha = 0.5°$