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Regular and stochastic motion of ions in the presence of Alfvén waves

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Introduction. The interaction of Alfvén waves and the ions plays an important role in both space and laboratory plasmas. In particular, the acceleration of the ions by Alfvén waves is a possible source of the energetic particles in solar-terrestrial environment, various mechanisms of such an acceleration being proposed, see, e.g., Ref. [1–3]. In this work, we consider the particle motion influenced by Alfvén waves of finite amplitude through non-linear resonances on sub-harmonics of the ion gyrofrequency. Note that these resonances are known as long as for more than 45 years [4], but only recently it was recognized that they can lead to the stochastic motion [5]. In contrast to Ref. [5], we study the particle motion in the presence of the elliptically polarized Alfvén waves, $E_y/E_x = -i(\omega/\omega_B)(k_\parallel^2/k_\perp^2)$, where ω is the wave frequency, ω_B is the particle gyrofrequency, $k_\parallel$ and $k_\perp$ are the longitudinal and transverse wave numbers, respectively, $\mathbf{E}$ is an electric field. The finite ratio of ω/ω_B (we are interested in $\omega/\omega_B \approx 1/4 - 1/2$) makes the E_y component non-negligible and affects the features of the particle motion.

Basic equations. We begin with a derivation of an equation of the particle motion in the electromagnetic field. With this purpose, we use the Lagrangé equation, with the Langrangean

$$L = \frac{m\dot{r}_j^2}{2} + \frac{e}{c}\dot{r}_j A_j - e\Phi(\mathbf{r}), \tag{1}$$

where $\mathbf{r} = (x, y, z)$, $\mathbf{A}$ is the vector potential of the electromagnetic field, Φ is the field scalar potential, two repeating subscripts (j) in a product mean the summation, $\dot{r}_j^2 = \dot{\mathbf{r}}\cdot\dot{\mathbf{r}}$, and $\dot{r}_j = dr_j/dt$. The Lagrangé equation with L given by Eq. (1) can be written as

$$m\ddot{r}_i + \frac{e}{c}\dot{A}_i = \frac{e}{c}\dot{r}_j\frac{\partial A_j}{\partial r_i} - e\frac{\partial \Phi}{\partial r_i}. \tag{2}$$

We assume that the electromagnetic field consists of the equilibrium homogeneous straight magnetic field, $\mathbf{B}_0 = (0, 0, B_0)$ and the waves propagating in the (x, z) plane. In this case $\mathbf{A}_0 = (-B_0 y, 0, 0)$ and $\Phi_0 = 0$, where the subscript "0" denotes the equilibrium (unperturbed) quantities and the waves are described by the perturbed vector potential $\widetilde{\mathbf{A}}$ and scalar potential $\widetilde{\Phi}$. We take a perturbed quantity, $\widetilde{X}$, in the form

$\widetilde{X} = \sum_k X_k = \sum_k \hat{X}_k exp(i\Psi_k)$ with $\Psi_k = k_x x + k_z z - \omega_k t$, where $\mathbf{k}$ is the wave vector and ω_k is the corresponding wave frequency. Then Eq. (2) yields:

$$m\ddot{x} - \frac{eB_0}{c}\dot{y} + \frac{e}{c}\dot{\widetilde{A}}_x = \sum_{\mathbf{k}} \frac{e}{c} ik_x(\dot{x}A_{kx} + \dot{y}A_{ky} + \dot{z}A_{kz}) - \sum_{\mathbf{k}} eik_x\Phi_k, \tag{3}$$

$$m\ddot{y} + \frac{e}{c}\dot{\widetilde{A}}_y = \frac{e}{c}\dot{x}(-B_0), \tag{4}$$

$$m\ddot{z} + \frac{e}{c}\dot{\widetilde{A}}_z = \sum_{\mathbf{k}} \frac{e}{c} ik_z(\dot{x}A_{kx} + \dot{y}A_{ky} + \dot{z}A_{kz}) - \sum_{\mathbf{k}} eik_z\Phi_k. \tag{5}$$

If ω were negligible compared to ω_B, the ideal MHD approximation could be used. Then Alfvén waves would be characterized by vanishing longitudinal magnetic field, $\widetilde{B}_\parallel = 0$, and they would be linearly polarized, $[\widetilde{\mathbf{E}} = (\widetilde{E}_x, 0, 0)]$. Due to the condition $\widetilde{B}_\parallel = 0$ in the ideal MHD one can take $\widetilde{\mathbf{A}_\perp} = 0$, in which case Eqs. (3)-(5) lead to Eq. (12) of Ref. [5]. However, we are interesting in waves with finite ω/ω_B. For these waves, $\widetilde{B}_\parallel \neq 0$. Therefore, only one component of the perturbed vector potential (or $\widetilde{\Phi}$) is arbitrary. We use the gauge $\widetilde{A}_x = 0$ in order to describe elliptically polarized Alfvén waves, $E_{ky}/E_{kx} = \alpha$, with $\alpha = -i\omega k_\parallel^2/i\omega_B k_\perp^2$. The longitudinal component of the electric field of these waves is small, $E_{kz} \sim \beta\omega^2/\omega_B^2$, ($\beta$ is the ratio of the plasma pressure to the magnetic field pressure) and will be neglected in our analysis.

Let us express all the perturbed quantities through the y component of the wave magnetic field. This can be done by using the following equations: $A_{ky} = -\widetilde{\omega}k_z B_{ky}/k_x^2$, $A_{kz} = -B_{ky}/ik_x$, $\Phi_k = i\omega B_{ky}/ck_z k_x$. Then integrating (4) one time and taking into account that $\dot{A}_{kz} = i\dot{\psi}_k A_{kz} = i(k_x\dot{x} + k_z\dot{z} - \omega)A_{kz}$ we obtain:

$$\ddot{x} + x - x_0 - \dot{y}_0 = \sum_k \left[\widetilde{V}_A\left(1 + \frac{k_\parallel^2}{k_\perp^2}\right) - \dot{z}\right]\hat{b}_k \sin\Psi_k + \{x - x_0 - \dot{y}_0$$
$$-\left[\widetilde{V}_A \sum_k \frac{k_\parallel^2}{k_\perp^2}\hat{b}_k(\sin\Psi_k - \sin\Psi_{k0})\right]\Bigg\}(\widetilde{V}_A \sum_k k_\parallel \rho_* \frac{k_\parallel}{k_\perp}\hat{b}_k\cos\Psi_k) - \widetilde{V}_A\sum_k \frac{k_\parallel^2}{k_\perp^2}\hat{b}_k \sin\Psi_{k0}, \tag{6}$$

$$\dot{y} = x_0 - x + \widetilde{V}_A \sum_k \frac{k_\parallel^2}{k_\perp^2}\hat{b}_k(\sin\Psi_k - \sin\Psi_{k0}) + \dot{y}_0, \tag{7}$$

$$\ddot{z} = \dot{x}\sum_k \hat{b}_k \sin\Psi_k - \dot{y}\widetilde{V}_A \sum_k k_\parallel \rho_* \frac{k_\parallel^2}{k_\perp^2}\hat{b}_k\cos\Psi_k, \tag{8}$$

where the normalized quantities are introduced: $\hat{b}_k = \hat{B}_{ky}/B_0$, $\tau = \omega_B t$, space coordinates are normalized to ρ_*, $\rho_* = V_*/\omega_B$, $\widetilde{V}_A = V_A/V_*$, $\Psi_k = k_x x\rho_* + k_z z\rho_* - \widetilde{\omega}_k\tau$, ρ_* and V_* are some characteristic Larmor radius and particle velocity, respectively.

These equations differ from the corresponding equations of Ref. [5] not only by the presence of additional terms proportional to the wave amplitude but also by the presence of a term proportional to the square of the wave amplitude. The latter term affects the wave-particle interaction. To see it we linearize Eqs. (6)-(8) by following the procedure of Ref. [5]. As a result, we obtain an equation of the following type:

$$\ddot{s} + as = \epsilon \exp(2i\tau)s + p\epsilon \exp(2i\tau) + q\epsilon^2 \exp(4i\tau), \tag{9}$$

where $s = k_x(x - x_0) \ll 1$, $\ddot{s} = d^2s/dT^2$, $2T = k_x x_0 + k_z z_0 + (k_z v_z - \omega)t$, $\epsilon \propto \hat{b}_k$, a, p, q are constant coefficients. $v_z = const$. We solve Eq. (9) perturbatively by using a small parameter ϵ. Writing $s = \sum_i s_i$, with $s_i \propto \epsilon^i$ and $i = 0, 1, 2...N..$, we obtain the resonances $a_N^{(1)} = N^2$, which corresponds to a Mathieu equation [5] and, in addition, the resonances $a_N^{(2)} = 2N^2$. Taking into account that $a = 4\omega_B^2/\omega^2$ we have the following resonance frequencies: $\omega^{(1)}/\omega_B = 2/N$ (Mathieu resonances) and $\omega^{(2)}/\omega_B = 1/N$ (new resonances). We conclude that the presence of the quadratic term provides additional resonances (for given N).

Analysis of the particle motion. We choose $v_* = v_{Ti}$, where v_{Ti} is the ion thermal velocity, $\widetilde{V}_A = 10$ (which corresponds to $\beta_i = 1\%$). In order to choose the wavenumbers of interest we take into account that when $\omega_k \approx k_\parallel v_A$, $k_\parallel \rho_* \approx \widetilde{\omega}_k/\widetilde{v}_A$, where we take $\widetilde{\omega}_k$ satisfying a resonance condition. We made calculations for two cases: first, when the wave is monochromatic, and second, when there are several waves with the same amplitude. The results are presented in Figs. 1-4. We observe that the motion is regular in the case of a monochromatic wave with $\hat{b} = 0.1$ (Fig. 1) and in the presence of several waves with $\hat{b} = 0.001$ (Fig. 2b). However, it becomes chaotic when there are several waves with the same amplitude as in Fig. 1, see Fig. 2a.

Summary and conclusions. We have derived the equations of the particle motion in the presence of elliptically polarized Alfvén waves. It is shown that these equations contain more resonances at the given approximation order than the Mathieu type equation considered in Ref. [5]. A numerical code solving the derived equations is developed. Using this code it was found that the motion in the presence of a monochromatic wave with $\hat{b} = 0.1$ is regular whereas it is chaotic in the presence of several waves of the same amplitude. The developed numerical tool will be used to analyze the motion of the thermal ions during Alfvén instability caused by the energetic ions with an anisotropic velocity distribution [6].

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