Numerical researches of formation of jet stream of plasma in large-scale geophysical experiment


1 Moscow Institute of Physics and Technology, Russia
2 Central Institute of Physics and Technology, Sergiev Posad, Russia

In a large-scale geophysical experiment "Starfish" (weight of plasma $10^6$ [g], energy $6 \times 10^{22}$ [erg]) the formation of an ascending plasma jet was observed where the most of the mass was distributed in the plane of a magnetic meridian.

This kind of stream can be formed as the result of flutter mode growth on the front of the plasma stream. We created a physical model, which shown that this growth is determined by dissipation processes, and the main of them is viscosity, it’s affect is significant after the time $\tau_\eta \sim 1/k^2$, where $k = \frac{2\pi R}{\lambda}$ is a wave number. Viscosity destroys disturbances of high frequency, which grow (in ideal plasma) for the time $\tau \sim 1/\sqrt{k}$. As the result, increment of perturbation growth is about $\gamma \sim \sqrt{k}$ for low $k$ and about $\gamma = \frac{\tau_\eta}{\tau} \sim 1/k$ for high $k$. Maximum of this function is at $k_m = 0.5 \left( \frac{\pi R \sqrt{p \rho}}{\sqrt{2} \eta} \right)^{2/3}$

where: $R, p, \rho, \eta$ - radius, pressure, density and viscosity of plasma. Our estimations shown, that in the moment of plasma braking, $k_m = 5\pm6$. Numerical modeling of such a large-scale three-dimensional stream of plasma with sated physical contents is an extremely difficult problem. In the same time, due to extremely low pressure of surrounding ionized atmosphere, which exists on the heights over 400 km (we have no reason to look at jet stream formation on the lower heights), speed of Alfven waves is about $600\pm1000$ [km/s] which exceeds initial plasma speed $\sim 500$ [km/s]. MHD-perturbation reaches the boarder of the area very fast, so we must choose in this moment – either we need to follow MHD-wave and increase the computational area very fast, or we want to work with main plasma streams, and grow the area bit slower. In this work we chose second approach, which led us to quite a small mistakes taking place due to absence of information about stream structure behind the front of MHD-perturbation, which has already run away from the area.
Let’s describe the system of equations used in the code. We used conservative form of equations, taking into account tensor of viscous strains ($\sigma_{ik}$), because kinetic and internal energy on this phase of motion are comparable.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}\left( \rho v_k \right) = 0.
\]

\[
\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_k}\left[ \rho v_i v_k + \left( p + \frac{B_i^2}{8\pi} \right) \delta_{ik} - \frac{B_i B_k}{4\pi} - \sigma_{ik} v_i \right] = g_i \rho.
\]

\[
\frac{\partial e}{\partial t} + \frac{\partial}{\partial x_i}\left[ \left( e + p + \frac{B_i^2}{8\pi} \right) v_k - \frac{B_i (\vec{v} \cdot \vec{B})}{4\pi} - \sigma_{ik} v_i \right] = 0.
\]

Where:

\[
e = \rho \left( \frac{\varepsilon + \frac{\vec{V}^2}{2}}{2} \right) + \frac{\vec{B}^2}{8\pi}, \quad p = \frac{\rho kT}{m} (1 + \alpha)
\]

\[
\varepsilon = \frac{3}{2} \frac{kT}{m} (1 + \alpha) + \frac{\alpha}{m}, \quad \vec{V}^2 = u^2 + v^2 + w^2, \quad \vec{B}^2 = B_x^2 + B_y^2 + B_z^2
\]

We assumed that gas is not relaxing, so:

\[
\sigma_{ik} = \eta \left( \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_i}{\partial x_i} \right)
\]

The field is described by equation:

\[
\frac{\partial \vec{B}}{\partial t} = D \Delta \vec{B} + \text{rot}[\vec{V} \times \vec{B}]
\]

Viscous coefficients $\eta, D$ take the following forms:

\[
\eta = 1.4 \cdot 10^{-7} \frac{T^{5/2} (eV) \sqrt{\mu}}{z^4 \ln \Lambda}, \quad D = 0.776 \cdot 10^6 \frac{\ln \Lambda \alpha + 0.186 \cdot T^{2} (eV) (1 - \alpha)}{\alpha T^{3/2} (eV)} = 1.2 \cdot 10^7 / T^{3/2} (eV), \quad \text{sm}^2 / \text{s} = D_0 / T^{3/2}
\]

Due to existence of strong discontinuities, we had to create a new method of entropy correction of high-accurate monotonous difference schemes for multidimensional MHD-equations, which allowed us to perform calculations of strong discontinuities up to hundreds of seconds, while the strong plasma stream can be observed.

We performed calculations using the conditions on the heights about 600÷700 [km]. In the initial state plasma was assumed to form a growing spherical area with disturbances of the surface along the lines of external geomagnetic field. We varied the
initial speed of plasma growth, initial radius of the sphere and difference between internal and external pressure.

By now, only preliminary calculations are performed. These calculations allowed us to debug the program and find main features of plasma stream.

If the initial speeds are low ($V=1 \text{ [km/s]}$) and radius of the sphere $R=12 \text{ [km]}$, external magnetic pressure slightly overcomes internal, so we can observe slow compression of plasma with strong growth of plasma density in the center of the area. But initial perturbations of the surface don’t decrease – they grow (see Fig.1).

If initial speed is taken from the range $V = 10 \div 100$, plasma keeps growth until breaking, approximately saving the shape of the perturbations. Than begins the compression and appearance of the jets phase. In the moment of breaking plasma is extremely sensitive to external affects and small internal perturbations. If external affects are too strong (this take place in situations, when we have problems with board conditions), we can even see the results where the number of jets differs from number of initial perturbations. When jets are formed, big part of plasma mass is moved into jets.

If non-uniform distribution of the density of external field is taken into account, we can observe decrease on jet intensity in the direction of positive gradients of $\rho$ and $B$. It means that jets directed down and horizontally, if real distribution of $\rho$ and $B$ is taken into account, should decline, and their mass should move into single up-directed jet. Though we were dealing with big heights, where Alfven wave speed in the external plasma is high, and energy of each jet is much lower than energy of all plasma, we had to perform a special analysis of impact of the computational net and board conditions on the solution’s behavior. On the Fig.2 you can see an example of jet stream with $k=5$, where perturbations are situated at random angle to axes of computational network.

In addition, if the initial distribution is strongly deviated from balance, when internal pressure $p > \frac{B^2}{8\pi}$ and initial speed of plasma front $V < V_A$, and in the same time, initial perturbations of plasma surface are quite big ($\sim 0.1R$), each jut of the plasma emit a wave into surrounding atmosphere. This wave, interfering with analogous wave from another jut, forms a very thin area between the juts, where the speed of plasma is much bigger, than in surrounding areas. Developing further, this area shoots up from the surrounding plasma in the moment of equilibrium and forms a jet.

In the same time, the calculations we performed shown that the algorithm we created has a very big range of abilities.
Fig. 1 – Distribution of density in the area of calculations in different moments ($t = 0.3; 3.9; 5.7$ s.)

Fig. 2 – Distribution of density and internal energy in another calculation when $t = 3.4$ s.  
Gradient of external density is horizontal