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On the role of the electromagnetic counter-streaming instability in GRBs

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Abstract. The dynamics of two counter-streaming electron-positron-ion unmagnetized plasma shells with zero net charge is analyzed in the context of magnetic field generation in GRBs due to the Weibel instability. Hot plasma, arbitrary mixture of plasma components and space charge effects are taken into account. We show that, although thermal effects slow down the instability, baryon loading leads to a non-negligible growth rate even for large temperatures and very different shell velocities, thus guaranteeing the robustness and the occurrence of the Weibel instability for a wide range of scenarios.

1. INTRODUCTION

One of the unsolved problems in the astrophysics of extreme events is magnetic field generation in Gamma-Ray Bursts (GRBs). It is almost universally accepted the hypothesis that the internal and the external shocks in relativistic ejecta represent the most likely mechanism for ≈-ray production, with synchrotron emission as the main radiation process in GRBs [1], thus indicating the presence of strong particle acceleration and near-equipartition magnetic fields. Although the latter has been confirmed by synchrotron radiation measurements, the origin of the required magnetic fields is still under strong debate [2]. The Weibel instability [3] seems to be a good candidate to explain magnetic fields in GRBs [4].

In this paper, we examine the dynamics of two counter-streaming electron-positron-ion unmagnetized plasma shells with zero net charge using relativistic kinetic theory. We focus our discussion on the growth rate (≈) of the electromagnetic beam-plasma instability or Weibel instability, since the saturation level b_{sat} of the magnetic field is related to ≈ according to $b_{sat}\ \Gamma\ \Gamma^2$ [5]. In previous analytical works on the Weibel instability in GRBs, the relativistic plasma is assumed cold and only purely electromagnetic transverse modes are excited; moreover, no arbitrary mixture of plasma components has been considered. In this work, we employ relativistic kinetic theory to include thermal effects, with arbitrary temperatures, space charge effects and an arbitrary mixture of leptons and baryons. In fact, it is well known that thermal effects can shutdown and strongly suppress the Weibel instability [6]. Furthermore, a non-relativistic analysis [7] has showed that the component of the electric field $\boldsymbol{E}$ along the wave vector $\boldsymbol{k}$ can also be important for two inter-penetrating current beams having different temperatures. We show that, although thermal effects slow down the instability, the thermal spread does not shutdown the Weibel instability: baryon loading leads to a non-negligible growth rate even for colliding shells with large temperatures, thus guaranteeing the robustness and the occurrence of the instability.

2. THEORETICAL MODEL

In order to obtain the dispersion relation for waves propagating in the *z* direction with wave vector $\boldsymbol{k}=k\boldsymbol{e}_z$, we use the relativistic Vlasov's and Maxwell's equations, following the technique outlined in [6] yielding:

$$\left(\omega^2 - k^2c^2 + C_{xxz}\right)\left[\left(\omega^2 - k^2c^2 + C_{yyz}\right)\left(\omega^2 + D_z\right) - D_y C_{zyz}\right] -$$
$$C_{xyz}\left[C_{yxz}\left(\omega^2 + D_z\right) - D_y C_{zxz}\right] + D_x\left[C_{yxz}C_{zyz} - \left(\omega^2 - k^2c^2 + C_{yyz}\right)C_{zxz}\right] = 0 \quad (1)$$

with

$$C_{lmn} = \sum_j \omega_{pj0}^2 m_j \int d\boldsymbol{p}\,\frac{\omega v_l}{\omega - k v_z}\left\{\left(1 - \frac{k v_z}{\omega}\right)\partial_{p_m} + \frac{k v_m}{\omega}\partial_{p_n}\right\}F_{j0},\; D_l = \sum_j \omega_{pj0}^2 m_j \int d\boldsymbol{p}\left\{\frac{\omega v_l}{\omega - k v_z}\partial_{p_z}\right\}F_{j0} \quad (2)$$

where the *j* component of the plasma is described by the corresponding unperturbed normalized distribution function F_{j0}, the unperturbed density n_{j0}, the mass m_j and the plasma frequency $\approx_{pj0}$, We choose a waterbag distribution function $F_{j0} = \left[1/\left(2p_{zj0}\right)\right]\delta\left(p_x - p_{xj0}\right)\delta\left(p_y\right)\left\{\Theta\left(p_z + p_{zj0}\right) - \Theta\left(p_z - p_{zj0}\right)\right\}$, where p_{xj0} is the momentum in the *x* direction and p_{zj0} is the momentum thermal spread in the *z* direction of the *j* plasma component, while $\approx(x)$ is the Heaviside step function. Thus, we consider a cold plasma shell along the *y* direction, propagating along the *x* direction, with *z* defining the perpendicular direction. To assure charge neutrality in each shell, we assume an electron density n_{e^-0}, while the positron density n_{e^+0} varies with $- = n_{e^+0}/n_{e^-0}$, and the ion density is $n_{io} = (1 - -)n_{e^-0}$. Our calculations are performed in the center of mass reference frame. Using the subscripts "1" and "2" for the shell moving in the $+\boldsymbol{e}_x$-direction and $-\boldsymbol{e}_x$-direction, momentum conservation along the *x* direction yields $\sum|v_1|\sum\sum_{j=1}^{3} m_{j0}n_{j0}\sum = \sum|v_2|\sum\sum_{k=1}^{3} m_{k0}n_{k0}\sum$,

where $-_n = \left(1 - -^2_{x,n}\right)^{-1/2}$ is the shell-*n*'s Lorentz factor and *j*,*k*=1,2,3 is for electrons, positrons and ions. We can define $\approx_n$ as

$$\delta_n = \frac{\left(\sum_{j=1}^3 m_{j0}n_{j0}\right)_n}{\left(\sum_{k=1}^3 m_{k0}n_{k0}\right)_2} = \left(\frac{\gamma_n|v_n|}{\gamma_2|v_2|}\right)^{-1} = \left(\frac{\gamma_n|\beta_n|}{\gamma_2|\beta_2|}\right)^{-1}. \quad (3)$$

Assuming without loss of generality, that the shell-1 is faster than the shell-2, the value of $\approx_n$ can vary within the range $0 \leq \leq_n \leq 1$ ($\approx_2$=1). After normalizing time to $1/\omega_{pe0}$ and space to the collisionless skin depth c/ω_{pe0}, the dispersion relation (1) can be written:

$$\left(\omega^2 - k^2 + [C_{xxz}]\right)\left(1 + [D_z]\,\frac{1}{\omega^2}\right) - [D_x]^2\,\frac{1}{\omega^2} = 0 \tag{4}$$

where the terms in square brackets correspond to the normalization of the coefficients from equation (2), such that

$$[C_{xxz}] = -\left\{\sum_{n=1}^{2}\delta_n\left\{\sum_{j=1}^{3}\omega_{pj0}^2\left[\left(\frac{\bar{1}}{\gamma_{j,n}}\right) - \frac{u_{xj0,n}^2}{1+u_{xj0,n}^2} + \frac{1}{\gamma_{j0,n}}\frac{k^2\beta_{xj0,n}^2}{\omega^2 - k^2\beta_{thj0,n}^2}\right]\right\}\right\},$$

$$[D_z] = -\left\{\sum_{n=1}^{2}\delta_n\left[\sum_{j=1}^{3}\omega_{pj0}^2\left(\frac{1}{\gamma_{j0,n}}\frac{1}{\omega^2 - k^2\beta_{thj0,n}^2}\right)\right]\right\}\omega^2,\ [D_x] = -\left\{\sum_{n=1}^{2}\delta_n\left[\sum_{j=1}^{3}\omega_{pj0}^2\left(\frac{1}{\gamma_{j0,n}}\frac{k\beta_{xj0,n}}{\omega^2 - k^2\beta_{thj0,n}^2}\right)\right]\right\}\omega, \tag{5}$$

where $\approx_{pj0}$ is the normalized plasma frequency of the j plasma component, $\approx_{xj0}$ is the velocity along the x direction and $\approx_{thj0}$ is the perpendicular thermal velocity of the j plasma component, with the standard definitions $-_{j0} = \left(1 - -^2_{xj0} - -^2_{thj0}\right)^{-1/2}$, $u_{xj0} = \beta_{j0}\beta_{xj0}$, complemented by the definition $\left(\bar{1}/\int_j\right) = \int d\boldsymbol{p}\left(F_{j0}/\int\right)$.

3. DISCUSSION

When the colliding shells have the same velocity and temperature, the dependence of the maximum growth rate of the Weibel instability with the electron Lorentz thermal factor is shown in Fig. 1a; the largest value of $\approx_{\mathrm{the0}}$ corresponds to the value of the motion Lorentz factor $\approx_{int}$=4 [4]. For both shells, three different plasma component mixtures are examined ($\approx$=1, 0.5, 0). As expected, the maximum growth rate decreases as temperature increases, but for all plasma component mixtures the Weibel instability does not shutdown. The ion presence slightly reduces $\approx_{\mathrm{max}}$, but does not strongly affect the occurrence of the instability. We observe a similar behavior of the maximum growth rate in the case with shells having same velocity but different temperature (Fig. 1b); like in the previous case, the

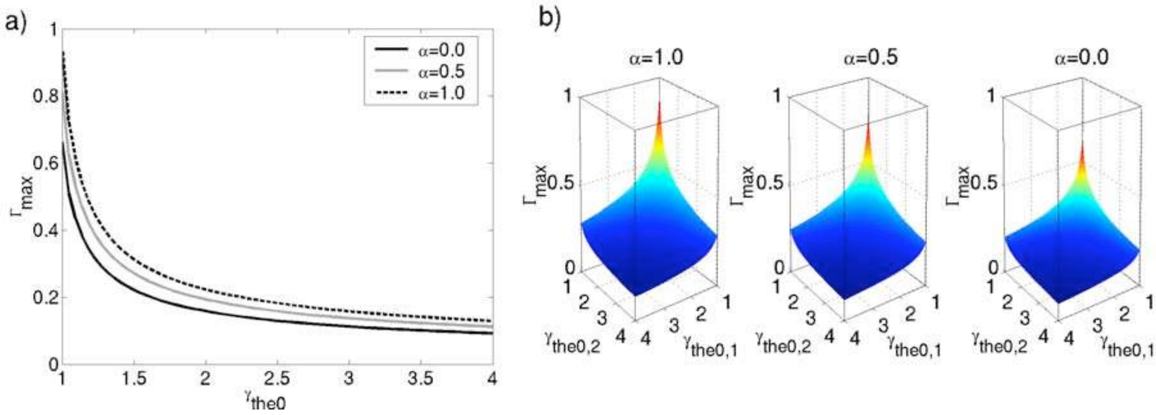

FIG. 1: Evolution of the max growth rate of the Weibel instability: a) shells with same temperature and velocity; b) shells with different temperature and same velocity.

ion presence leads to a very small reduction of $\approx_{max}$. As for the analysis with shells having same temperature but different velocity, we consider the shell-1, moving with $\approx_{int}$, to be faster than the shell-2. By varying the parameter $\approx_{1}$ defined through equation (3), it is straightforward to calculate the corresponding shell-2's velocity. We have found that even for fairly different velocities, the Weibel instability occurs for every plasma component mixtures with the same features of the previous scenarios. Still, when we keep on decreasing $\approx_{1}$, we find a velocity limit below which the baryon loading becomes fundamental to guarantee the occurrence of the instability. For such a velocity limit, a temperature exists above which the Weibel instability in the electron-positron plasma cannot develop anymore, whereas when ions are included, they lead to a small but non-negligible growth rate. This is shown in Fig. 2a, where the growth rate for all wave numbers is plotted as function of the electron thermal Lorentz factor, for $\approx_{1}$=0.17. The role played by baryons is illustrated more clearly in Fig. 2b, using again $\approx_{1}$=0.17. Here, the maximum growth rate for the electron-positron plasma goes to zero for a $\approx_{the0}\approx 1.4$, while the Weibel instability does not shutdown for the other plasma component mixtures due to the ion presence.

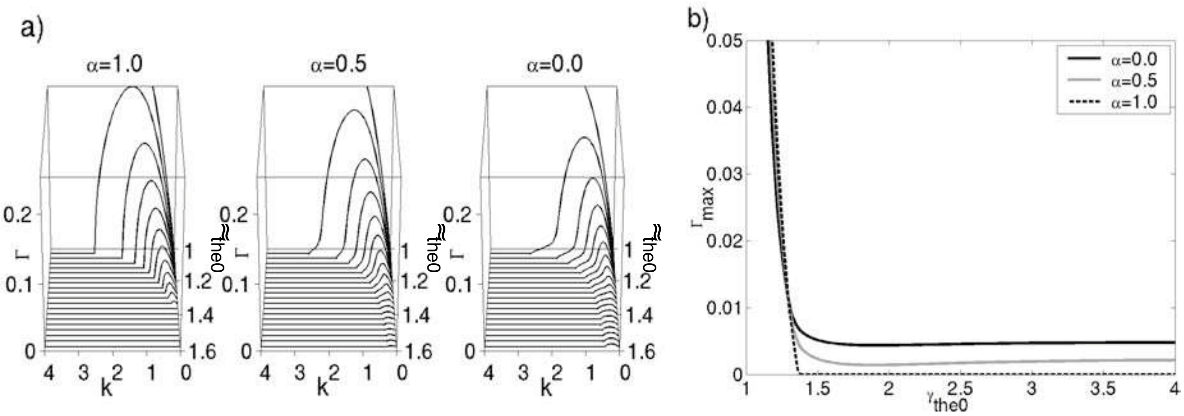

FIG. 2: Evolution of the growth rate of the Weibel instability for shells with same temperature and different velocity ($\approx_{1}$=0.17): a) $\approx$ for all wave numbers; b) $\approx_{max}$ as function of the electron Lorentz thermal factor.

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