

Finite amplitude electrostatic disturbance in a inhomogeneous plasma

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Abstract

The propagation of a spatially localized time-dependent charge density perturbation in a inhomogeneous unmagnetized collisionless plasma is investigated by means of a Vlasov-Ampere numerical code with open boundary conditions. The aim of the work is to study the kinetic plasma response to the electrostatic field far from the source when the typical spatial scale-lengths of the density gradients are comparable with the excited wavelengths, for arbitrary field amplitude.

Introduction

Self-consistent electrostatic (ES) fields in spatially non-uniform plasmas represent one of the fundamental aspects of plasma physics with several implications in both microwave and laser based experiments. In the '70s extensive theoretical [1-4] and experimental [5-7] researches have been devoted to investigate ponderomotive effects, electron acceleration, ion acceleration, wavebreaking, resonant absorption in plasmas with density gradients. The problem concerns magnetized plasmas, as well, when Bernstein waves are excited and propagate in the vicinity of hybrid plasma resonances. Recently, a renewed interest for electron Bernstein wave physics has appeared due to the possibility of implementing an attractive emission diagnostic in fusion plasmas [8-10]. Then, it is interesting to investigate the kinetic aspects of the propagation of ES fields in a non uniform plasma with arbitrary density scale lengths and electric field amplitudes. To this aim, we have used a Vlasov-Maxwell numerical code with open boundary conditions, which are particularly suitable to treat inhomogeneous problems. This paper deals with the kinetic simulations of ES waves in a unmagnetized plasma, in 1-dim geometry. The source of the ES perturbation is an oscillating electron charge density, localized at one edge of the spatial interval of integration. This work is intended as preparatory to the investigation of the case of a inhomogeneous magnetized plasma.

The physical model

Let us consider an unmagnetized plasma localized in the region $0 < x < L$, homogeneous in the plane (y,z) , with unperturbed density $n_{a0}(x)$, where $a = e, i$. The relevant dimensionless Vlasov equations take the form

$$\frac{\partial f_a}{\partial t} + v \frac{\partial f_a}{\partial x} + (\delta_a E + F_a) \frac{\partial f_a}{\partial v} = 0, \quad (1)$$

where $t \rightarrow t\omega_{pe}$, $x \rightarrow x/\lambda_{de}$, $v \rightarrow v_x/v_{te}$, $E \rightarrow eE\lambda_{de}/T_e$, $v_{te} = \lambda_{de}\omega_{pe} = (T_e/m_e)^{1/2}$. A non uniform equilibrium, in the absence of a magnetic field, can be realized by introducing ‘‘fictitious forces’’ F_a acting on both plasma species, which in dimensionless units write $F_e \rightarrow F_e\lambda_{de}/T_e = d(\ln n_{e0})/dx$ and $F_i \rightarrow F_i\varepsilon\lambda_{de}/T_e = (\varepsilon/r_T)d(\ln n_{i0})/dx$. Finally, $\delta_e = -1$, $\delta_i = Z\varepsilon$, $\varepsilon = m_e/m_i$, $r_T = T_e/T_i$, Z is the ion charge, m_a and $n_{a0}(x)$ are the mass and the equilibrium density profiles of the a -species, respectively. Eqs.(1,2) are solved with initial conditions

$$E(x, t=0) = 0, \quad f_{e0}(x, v) = n_{e0}(x)/\sqrt{2\pi} \exp(-v^2/2),$$

$f_{i0}(x, v) = (r_T/\varepsilon)^{1/2} n_{i0}(x)/\sqrt{2\pi} \exp[-(r_T/\varepsilon)v^2/2]$. As a boundary condition, we impose a harmonic oscillation on the electron distribution function (EDF), at $x=0$, in the form $f_e(x=0, v, t) = f_{e0}(x, v)[1 + a \cdot \sin(\omega_0 t)]$, where a is a constant amplitude and ω_0 the perturbation frequency. The electric charge perturbation generates an ES field, according to the Ampere equation for zero magnetic field

$$\frac{\partial E}{\partial t} = - \int_{-\infty}^{+\infty} dv v f_i + \int_{-\infty}^{+\infty} dv v f_e, \quad (2)$$

At each time integration step, it is verified that the Poisson equation is satisfied by the current values of $f_a(x, v, t)$ and $E(x, t)$. We have chosen the equilibrium density distributions in the form $n_{e0}(x) = Zn_{i0}(x) = n_{e00} + \Delta n \cdot \{1 + \tanh[\kappa(x - x_m)]\}$, where n_{e00} is the electron density at the boundary $x = 0$, $|2\Delta n|$ is the amplitude of the density variation, $\kappa = 1/L_n$ is the inverse of the density scale length, and x_m is the position where $n_{e0}(x_m) = n_{e00} + \Delta n$. Moreover, $n_c \equiv n_{e0}(x_c)$, that is at $x = x_c$ the electron density equals the critical density at the frequency ω_0 , $n_c = m_e\omega_0^2/(4\pi e^2)$. From now on the densities will be normalized to n_{e00} . Then, the dimensionless forms of the equilibrium forces acting on the electrons and ions are

$$F_i(x) = \frac{\varepsilon}{r_T} F_e(x) = \frac{\varepsilon}{r_T} \frac{\Delta n \kappa \operatorname{sech}^2[\kappa(x - x_m)]}{1 + \Delta n \{1 + \tanh[\kappa(x - x_m)]\}}. \quad (3)$$

Eqs.(1-3), with the relevant boundary conditions, allows us to study the plasma evolution in a spatial region far from the source of an ES disturbance, when the typical spatial scale-length of the density gradient, L_n , is comparable with or even smaller than the consistently excited wavelengths, for arbitrary field amplitude.

Discussion of the results

An extensive check of the linear theory of ES waves in a slightly non uniform plasma (with $\lambda_0 \ll L_n$, being $10 < \lambda_0 < 70$ and $L_n = 250$) has been carried out, recovering the properties of the small amplitude ($a \leq 0.1$ corresponding to $E_{\max} \leq 0.02$) Langmuir wave propagation, and the electron trapping in phase space at the phase velocity $v_\phi = \omega_0/k_0$. Then, large amplitude charge perturbations ($a=1$, corresponding to $E_{\max} \cong 0.4$) with steep density gradients ($L_n = 50$), for two values of the pump frequency $\omega_0 = 1.1$ and $\omega_0 = 1.01$ (with $\lambda_0 \cong 24$, $v_\phi \cong 4.2$, and $\lambda_0 \cong 77$, $v_\phi \cong 12.3$, respectively), have been investigated. All the simulations have been performed for $r_T = Z = 1$, and integrating in the intervals $0 < x < 1000$, and $0 < t < 1000$. In this paper, we consider the case $\omega_0 = 1.1$, only, with $\Delta n = 0.25$, being $x_c \cong 465$.

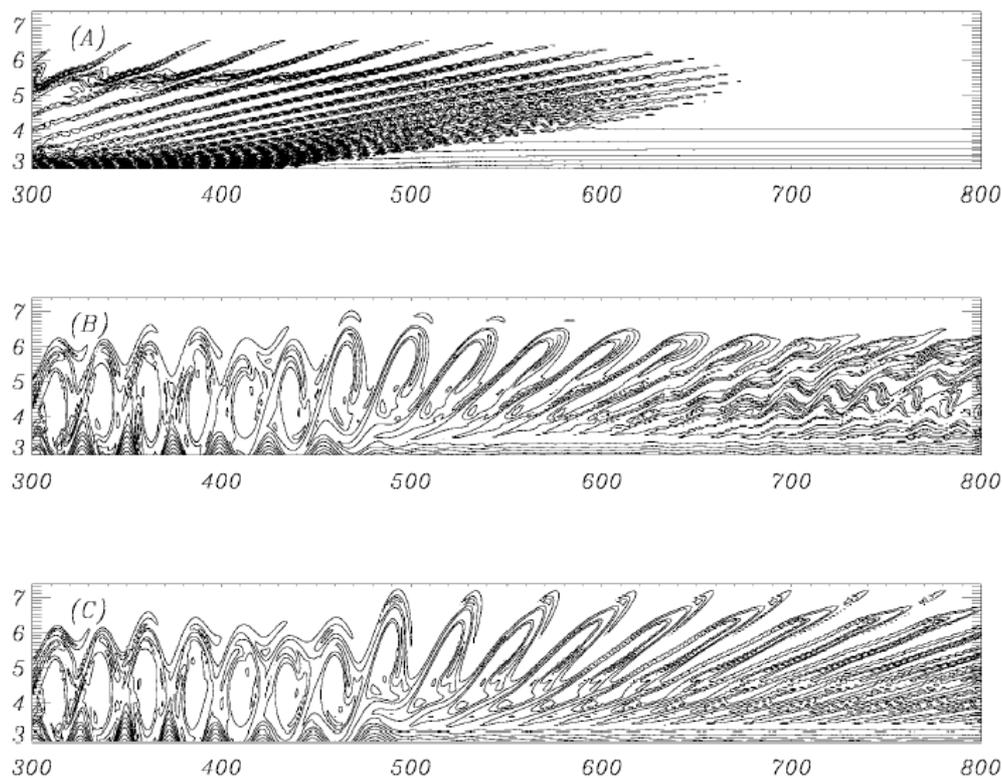


Fig.1

In Fig.1 the contour lines of the EDF are plotted in the phase space (x, v) , at $t = 150$ (A), 600 (B), 1000 (C). At early times (A), ballistic streams of electrons are produced, which propagate forward in x . After several bounce times (B), electron trapping takes place for $x < x_c$, where the large amplitude field exists (see Fig.2A).

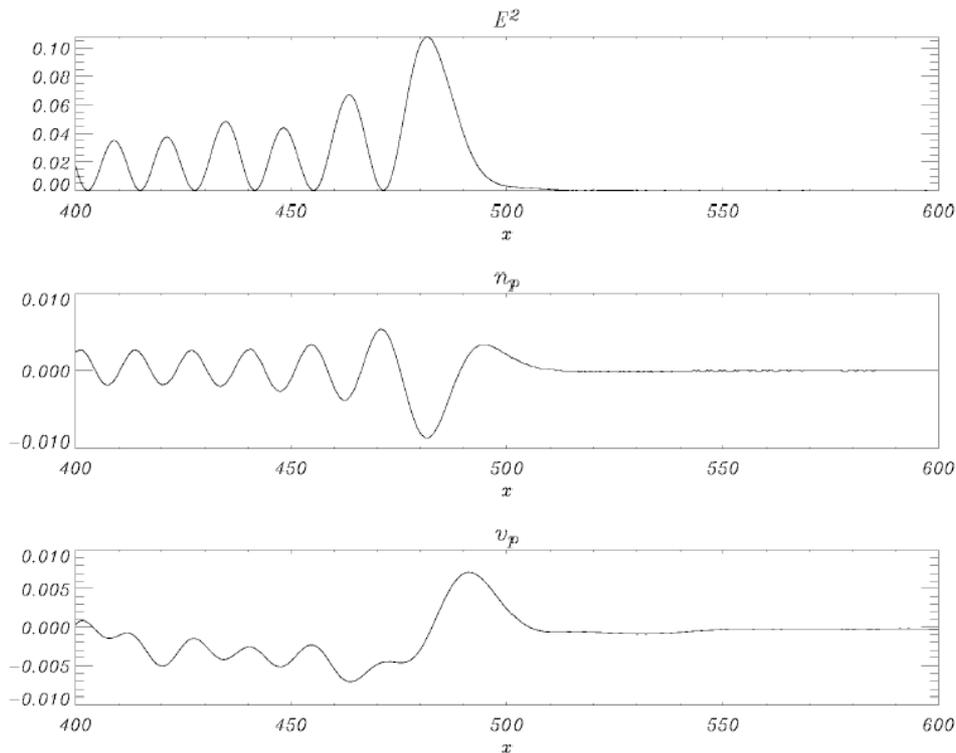


Fig.2

At $x > x_c$, the ES field cannot propagate and the electron vortices become more and more stretched due to the ballistic motion of trapped electrons (B,C). In Fig.2 the electric field (A), the ion density perturbation $\delta n_i(x,t) = n_i(x,t) - n_{i0}(x)$ (B), the ion fluid velocity $U_i(x,t)$ (in units of v_{ii}) (C), as functions of x , at $t=1000$, are plotted. When the ES disturbance has reached the region of the density gradient, part of the wave is reflected, and a stationary ES field pattern is produced. Then, the ion density distribution is affected, in particular around $x = x_c$, where a quasistationary troughs start to be produced due to the expulsion of ions. At the two edges of the main density cavity (at $x \cong 477$), the outflow of ion density is clearly observed (C). The process of cavitation ($\delta n_i \cong -0.01$) has not saturated when the run is stopped ($t=1000$).

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