Connecting shock velocities to electron injection mechanisms

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1. Introduction

The interaction between electrostatic (ES) waves and electrons is the likely electron injection mechanism at supernova remnant (SNR) shocks [1]. The waves are driven by the shock-reflected ion beam. This injection must produce electrons that are fast enough to cross the shock to then undergo Fermi acceleration which is believed to produce the observed electron synchrotron radiation [2]. Here we model the injection by particle-in-cell (PIC) and by Vlasov simulations. We find that, for ion beam speeds representative for SNR shocks, the life-time of the saturated ES wave, a BGK mode [3], is considerably longer in the Vlasov simulation than in the PIC simulation for identical plasma parameters [4]. We relate this observation to differences in how both simulation methods approximate the plasma. As a consequence electron injection by surfing acceleration that requires a stable BGK mode may be more important than previously thought [5]. By increasing the beam speeds above a critical speed which we estimate here, both simulation codes exclude surfing acceleration due to a rapid BGK mode collapse.

2. Equations and numerical solution

A warm collision-less plasma consisting of multiple species with index $j$ can be approximated by the phase space density (PSD) $f_j(x, v, t)$. The electron charge to mass ratio is $e/m_e$ and $q_j/m_j$ is that for the species $j$. We consider the unit-less Vlasov equation Eq. (1) in an unmagnetized plasma with only ES fields.
\[
\frac{\partial f_i}{\partial t} + v \frac{\partial f_i}{\partial x} + \left( q_j m_e / e m_j \right) E \frac{\partial f_i}{\partial v} = 0
\] (1)

The corresponding ES fields evolve according to Eq. (2).

\[
\frac{\partial E}{\partial x} = \sum_i \int f_i(x, (v - \delta_i), t) dv - \int f_e(x, v, t) dv
\] (2)

The PSD \( f_j(x, v, t) \) is approximated by PIC simulations as in Fig. 1 while Vlasov simulations approximate it as in Fig. 2.

![Fig. 1: The PIC code PSD function.](image1)

![Fig. 2: The Vlasov code PSD function.](image2)

We perform numerical simulations in which we model two counter-propagating proton beams with an equal density moving through a plasma. The beams have a speed \( \delta_{b1} = v_b \) and \( \delta_{b2} = -v_b \). We use 800 grid cells for the simulation box to resolve a length of \( 4\lambda_u \) where \( \lambda_u = 2\pi v_b / \omega_p \) is the wave length of the most unstable wave and where \( \omega_p \) is the plasma frequency. The electron thermal speed is \( v_{th} = (k_B T_e / m_e)^{0.5} \), where \( k_B \), \( T_e \) and \( m_e \) are the Boltzmann constant, the electron temperature and the electron mass. The temperature of the background proton species and that of beam 1 is \( 10 \times T_e \) while that of beam 2 is \( 100 \times T_e \). The initial conditions are discussed in more detail in [4].

We perform a Vlasov simulation (Case 1) with \( v_b = 21 \times v_{th} \), a Vlasov simulation (Case 2) with \( v_b = 15v_{th} \) and a PIC simulation (Case 3) with \( v_b = 15v_{th} \). Fig. 3 shows the amplitude of the \( \lambda_u \)-wave in the Vlasov simulation with \( v_b = 21v_{th} \) (1) while (2) and (3) show the amplitudes for the Vlasov and the PIC simulation for \( v_b = 15v_{th} \) respectively.
The phase speed of the ESW is \( v_{ph} \approx \omega_p \lambda_u / 2\pi \) and \( E_c = m_e \omega_p^2 \lambda_u / 2\pi e \) is the field at which the wave traps electrons that move at the speed \( v_{ph} \) relative to it.

All curves show an exponential growth and a non-linear saturation. The saturated curve 1 is stable for the time \( T_1 \approx 200/\omega_p \), curve 2 is stable for \( T_2 \approx 500/\omega_p \) while curve 3 collapses instantaneously. Eventually all waves collapse due to the sideband instability [6] driven by the beam of trapped electrons.

Fig. 3: The amplitudes of the waves with \( \lambda_u \) as a function of \( t\omega_p \).

Fig. 4 shows the PSD corresponding to the time \( t_0 \) at which the electric field of the curves (2) and (3) in Fig. 3 first crosses \( E/E_c = 0.46 \). The speed unit is \( v_{th} \).

After that the wave in the Vlasov simulation begins to saturate while it continues to grow in the PIC simulation. We see from Fig. 4 that the earlier wave saturation in the Vlasov simulation is due to the much better developed island of trapped electrons. In the PIC simulation this island is developed at a later stage by which the wave is already much stronger, pushing the plasma into a more non-linear regime. In Fig. 5 we show the PSDs integrated over \( x \) at the end of the Vlasov simulation (2) and the PIC simulation (3). After the proton-beam driven wave has collapsed, the electrons are distributed across the same velocity interval in the PIC and in the Vlasov simulation despite of the substantial
difference in the time evolution of the waves. The top speed the electrons reach is 3-4 $v_b$.

3. Conclusions

PIC simulations and Vlasov simulations show the same initial exponential growth rates for the proton-beam driven ES waves. The waves in the Vlasov simulations saturate at lower amplitudes than in the PIC simulations. The larger dynamical range for the PSD in the Vlasov simulation is the reason since, for an initially Maxwellian electron distribution, the trapped electron island can form earlier. The ES wave collapses instantaneously in the PIC simulation. In the Vlasov simulations the waves have a long life-time if $v_b < 20v_{th}$.

If the ES wave propagates orthogonally across a magnetic field, the trapped electrons are accelerated by their cross-field transport (electron surfing acceleration) [5]. For $v_b = 15v_{th}$ the Vlasov simulation indicates that the life-time of the ES wave is sufficient to accelerate electrons to the injection threshold of $10^5$ eV if the plasma were magnetized as that in [5].

As an example, take a supernova remnant shock that moves at $v_s = 1.8 \times 10^7$ m/s. Specularly reflected protons will move at a speed $v_b = 2v_s$ in the upstream frame of reference. To get a relation $v_b = 15v_{th}$, the upstream electrons must have the thermal speed $v_{th} \approx 2.4 \times 10^6$ m/s (30 eV) or above. The life-time of the ES wave would then be sufficient to accelerate the electrons beyond the injection threshold.

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