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On the theory of dust in a plasma using matched asymptotic expansions

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1. Introduction

The well-known ABR theory [1], originally developed for Langmuir probes, involves cold ions being accelerated radially towards a dust grain by a spherically symmetric potential. Using matched asymptotic expansions we exploit the smallness of ϵ_p (the size of the dust particle compared to λ_D) to find the electrostatic potential as a matched asymptotic expansion in ϵ_p. This method gives new insights into the structure of the solution.

We consider a single dust grain suspended in an electropositive, collisionless plasma, with Maxwellian electrons. It is bombarded by ions and electrons from the plasma, and in equilibrium attains a negative charge.

2. The ABR Theory

Under an electrostatic potential $V(\rho)$, cold ions move radially towards the dust grain with velocity v_i and density n_i. The governing equations are:

Ion continuity	$\nabla \cdot \left(N_i \underline{v_i}\right) = \frac{1}{\rho^2}\frac{\partial(\rho^2 N_i v_{ir})}{\partial \rho} = 0$
Ion energy	$0 = \frac{1}{2}Mv_i^2 + eV$
Poisson	$\frac{d^2V}{d\rho^2} + \frac{2}{\rho}\frac{dV}{d\rho} = -\frac{e}{\epsilon_0}\left[N_i - N_0 \exp\left(\frac{eV}{kT_e}\right)\right]$

We use the normalisations:

$$\phi = \frac{eV}{kT_e};\ \underline{q} = \frac{\underline{v_i}}{v_B};\ n = \frac{N_i}{N_0};\ r = \frac{\rho}{\lambda_D}; \tag{1}$$

where $v_B = \sqrt{\frac{kT_e}{M}}$. The above equations then reduce to:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = e^{\phi} - \frac{J}{r^2\sqrt{-\phi}} \tag{2}$$

where J is a constant, denoting the normalised ion current, given by:

$$J = \frac{I_i}{n_0 e\sqrt{2kT_e/M}4\pi\lambda_D^2}$$

The boundary conditions are:

$$\phi, \phi' \to 0 \text{ as } r \to \infty \tag{3}$$

and on the dust surface, $r = \epsilon_p$:

$$\phi = \phi_p; \; J = \alpha \epsilon_p^2 e^{\phi_p} \tag{4}$$

where $\alpha = \sqrt{\frac{M}{4\pi m_e}}$. (4) represents the floating condition, i.e. balance of the ion and electron currents to the dust surface.

Points to note:

- We are interested in small dust particles, and the floating condition suggests this corresponds to small J. Therefore we write $J = \epsilon^2$ for some $\epsilon \ll 1$.
- We will show *a posteriori* that in fact $\epsilon \sim \epsilon_p \sqrt{\alpha}$. This is consistent with values typically found in experiment.

The following theorem is fundamental:

Uniqueness theorem [3] Given the equation (2) and the boundary conditions (3), then for every $J > 0$ there exists a *unique* solution $\phi(r)$ for $r \in (0, \infty]$.

Note that the floating boundary condition (4) does *not* feature in this theorem. Instead, its role is to relate ϵ_p to J.

3. Results

The following structure emerges:

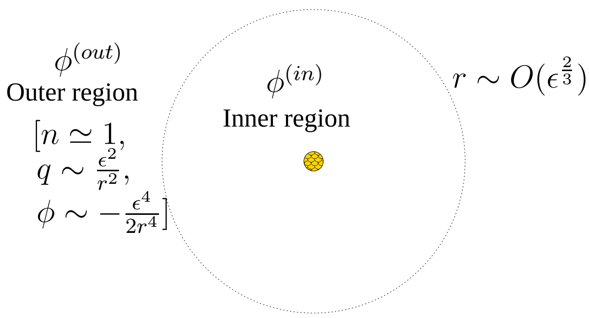


FIGURE 1: Spatial structure of the asymptotic ABR solution for small $\epsilon \sim \epsilon_p \sqrt{\alpha}$.

where:

$$\phi^{(out)} \sim -\frac{\epsilon^4}{2r^4} + \frac{\epsilon^8}{r^8}\left(-2 + \frac{24}{r^2}\right) + O(\epsilon^{12}) \tag{5}$$

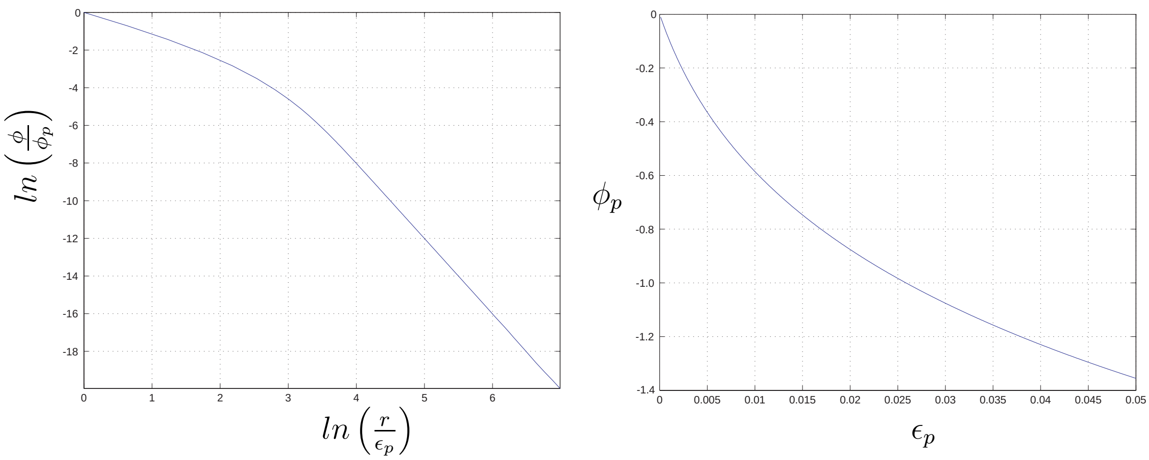
$$\phi^{(in)} \sim \epsilon^{\frac{4}{3}}\left(\frac{c_0}{r_1} + c_1 + c_2 r_1^{\frac{1}{2}}\right) + \cdots \text{ as } r_1 \to 0, \tag{6}$$

with $r = \epsilon^{\frac{2}{3}} r_1$, and:

$$c_0 \simeq -1.3844;\ c_1 \simeq 2.1699;\ c_2 = \frac{4}{3}(-c_0)^{-\frac{1}{2}}$$

Using the floating condition (4), it is seen that the dust surface lies in the $r_1 \to 0$ limit of the inner region. This confirms that indeed $\epsilon \sim \epsilon_p \sqrt{\alpha}$, allowing us to transfer the parameterisation of the solution from ϵ to ϵ_p.

For the typical case of Argon ($\alpha \simeq 76.5$), fig.2a shows the potential profile thus obtained for $\epsilon_p = 0.01$ whilst fig.2b shows the variation of the floating potential with grain size:



(a) (b)

FIGURE 2: Plots of (a) potential profile and (b) dust floating potential vs.dust size for Argon.

In physical terms, if r_p is the dust radius in unnormalised units we have a region of order $(r_p^{\frac{2}{3}} \lambda_D^{\frac{1}{3}} \alpha^{\frac{1}{3}})$ around the dust particle. This represents the shielding distance around the dust grain.

- Outside this region, as expected we have plasma-like behaviour, where $n_i, n_e \gg \nabla^2 \phi$, i.e. quasineutrality.
- In the inner region, however, the space-charge becomes more important. The dust surface lies in the *inner* inner region, where ϕ satisfies Laplace's equation to lowest order.

4. Work in progress

We are now in a position to investigate the problem of a dust particle immersed in a flowing plasma. Let v_0 be the ion flow speed normalised by the Bohm velocity.

- For $v_0 \ll 1$, we can treat the solution as a perturbation of the solution found here. In fact, for $v_0 < \epsilon^{\frac{2}{3}}$, we find that the inner region remains unchanged to lowest order in ϵ; only the outer solution loses spherical symmetry.

- For $v_0 \sim \epsilon^{\frac{2}{3}}$ the problem decomposes into the same regions as before but the spherical symmetry of the inner region is broken by the stronger ion flow.

- For $1 > v_0 \gg \epsilon^{\frac{2}{3}}$ we expect a very different structure altogether.

This method will also be applied to the separate problem of a dust particle in the sheath, a typical scenario in terrestrial experiments. Here, the ion streaming velocity is *at least* v_B and the boundary conditions are changed.

5. References

[1] J.E.Allen, R.L.F.Boyd & P.Reynolds, 1957 The collection of positive ions by a probe immersed in a plasma. *Proc. Phys. Soc* **B70**, 297-304

[2] M.Van Dyke (1964), *Perturbation methods in Fluid Mechanics* Academic Press

[3] J.B.McCleod, [Private communication]