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# Charge properties of fine dispersed dust grains in space plasma

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**Introduction.** Traditional approaches to the calculation of the equilibrium charge on a single dust grain use the condition that electron and ion currents toward the grain equal to each other [1,2]. To describe charge fluctuations on the grains we apply the discrete population balance (DPB) method used earlier for Fokker-Plank description of particle charging in ionized gases [3]. The proposed method takes adequately into account the discrete charges of plasma particles (ions and electrons) and enables us to obtain a probability of finding some definite charge on a dust grain. We are interested in finding *exact solutions* for the stationary charge distribution on a grain and the asymptotes of the distributions in the limiting cases of the parameter $\eta = e^2/aT_e$ ($e < 0$ is the elementary electron charge, $a$ is the radius of the dust grain, and $T_e$ is the temperature of electron plasma component in energy units).

**Basic equations.** The main idea of the DPB method is as follows. Suppose that $P_n$ is the probability of finding a charge $ne$ (state $n$) on the dust grain. Then the rate of the departure from a state $n$ is proportional to the product of $P_n$ and the total flux $\psi_e(n) + \psi_i(n)$ entering the grain. On the other hand, the rate of the income to the state $n$ is proportional to the sum of $P_{n-1}\psi_e(n-1)$ (the income from the state $n-1$) and $P_{n+1}\psi_i(n+1)$ (the income from the state $n+1$). It follows that the infinite system of the balance equations can be written in the following form

$$\frac{d}{dt}P_n = -P_n[\psi_e(n) + \psi_i(n)] + P_{n-1}\psi_e(n-1) + P_{n+1}\psi_i(n+1), \quad -\infty < n < \infty, \quad (1)$$

providing the normality condition $\sum_{-\infty}^{\infty} P_n = 1$ for any time $t$. The expressions for the fluxes of electrons and ions entering the dust grain can be taken from the well-known orbit-limited motion (OLM) approach [1-3], namely

$$\psi_e(n) = n_0 a^2 \left(\frac{8\pi T_e}{m_e}\right)^{1/2} \begin{cases} \exp(-\eta n), & n \geqslant 0 \\ 1 - \eta n, & n < 0 \end{cases}, \quad (2)$$

$$\psi_i(n) = n_0 a^2 \left(\frac{8\pi T_i}{m_i}\right)^{1/2} \begin{cases} 1 + \eta n/\tau, & n \geqslant 0 \\ \exp(\eta n/\tau), & n < 0 \end{cases}, \quad (3)$$

where $\tau = T_i/T_e$.

The solutions of the time-dependent system (1) complemented by a normalized initial distribution give us the time evolution of the charge to the steady-state distribution which is a solution of time independent balance equations

$$P_n[\psi_e(n) + \psi_i(n)] = P_{n-1}\psi_e(n-1) + P_{n+1}\psi_i(n+1), \quad -\infty < n < \infty. \quad (4)$$

Similar equations have been used in [3] for obtaining the Fokker-Plank equation at $\eta \ll 1$.

**Solutions.** Note that the solutions of Eq.(4) are the solutions of more simple equation

$$P_n = P_{n-1}\frac{\psi_e(n-1)}{\psi_i(n)}, \quad -\infty < n < \infty. \tag{5}$$

If we use Eq.(5) as a recursion, the probability $P_n$ can be expressed through the probability $P_0$ of finding the dust grain uncharged. With help of (2) and (3) we can get

$$\frac{P_n}{P_{n-1}} = \frac{\sqrt{\tau\mu}\, e^{-\eta(n-1)}}{\tau+\eta n}, \quad (n>1), \qquad \frac{P_{n-1}}{P_n} = \frac{\sqrt{\tau/\mu}\, e^{\eta n/\tau}}{1-\eta(n-1)}, \quad (n<-1). \tag{6}$$

Since the parameter $\mu = m_i/m_e$ (the ratio between the mass of ion and electron) is very high, it is evident from (6) that the probabilities $P_{n<0}$ of finding a positive charge $ne$ on the grain is negligible small with respect to $P_{n>0}$ of finding a negatively charged grain. Thus, the stationary charge distribution can be finally written in the *simple analytical form*

$$P_n = P_0(\tau\mu)^{n/2}\exp\left[-\eta n(n-1)/2\right]\left[\prod_{k=1}^{n}(\tau+\eta n)\right]^{-1}, \tag{7}$$

where the probability $P_0$ is determined by the normality condition $\sum_0^\infty P_n = 1$.

Simple estimations show that for the near-Earth space plasma ($T_e = T_i \approx 200°$K) the parameter $\eta = e^2/aT_e$ may vary from 0.001 up to 10 in the range of $a$ =[10$\mu$m,10nm]. For this reason it would be interesting to find the asymptotes of the charge distribution in two cases $\eta \ll 1$ and $\eta \gg 1$.

**Case** $\eta \ll 1$. In this case it is convenient to start with well-known transcendent equation

$$\sqrt{\tau\mu}\exp(-\eta N_0) = \tau + \eta N_0, \tag{8}$$

determined the particle charge $Q = N_0 e$ from the condition $\psi_e(n) = \psi_i(n)$, see (2) and (3). Since the solution of Eq.(8) with respect to $N_0$, generally, is not an integer, we put $N_0 = N + \delta$, where $N$ is the integer part of $N_0$. Using (2), (3) and (5) we can express the probabilities $P_{N\pm k}$ ($k \geqslant 1$) in terms of $P_N$

$$P_{N+k} = P_{N+k-1}\frac{e^{-\eta(k-\delta-1)}}{1+\eta(k-\delta)/(\tau+z)}, \quad P_{N-s} = P_{N-s+1}\left[1-\frac{\eta(s+\delta-1)}{\tau+z}\right]e^{-\eta(s+\delta)}, \tag{9}$$

for $k \geqslant 1$ and $1 \leqslant s \leqslant N$, respectively. Assuming $\eta \ll 1$ and using (9) as a recursion we can finally get the distribution

$$P_n = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(n-N_0-\Delta)^2}{2\sigma^2}\right], \quad 0 \leqslant n < \infty, \tag{10}$$

which *does not contain parameter* $\delta$. Here we used the following notations

$$\sigma^2 = \eta^{-1}\frac{\tau+\eta N_0}{\tau+\eta N_0+1}, \qquad \Delta = \frac{\tau+\eta N_0-1}{2\left(\tau+\eta N_0+1\right)}. \tag{11}$$

Thus, in the region $\eta \ll 1$ the steady-state charge distribution is Gaussian one, but the peak position of the distribution is slightly deviated from early known distribution [3]

$$P_n = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(n-N_0)^2}{2\sigma^2}\right], \quad 0 \leqslant n < \infty. \tag{12}$$

With help of (10) we can find the following expression for important dusty plasma parameter

$$Z \equiv e\varphi_d T_e^{-1} = \eta\overline{N} = \eta\sum\nolimits_0^\infty nP_n = \eta(N_0 + \Delta) = \mathrm{const} + \eta\Delta, \tag{13}$$

where $\varphi_d = e\overline{N}/a$ is the surface potential of the grain. Thus, $Z$ is a linear function of $\eta$ while the traditional approach predicts in this case the independence $Z$ of $\eta$.

**Case** $\eta \gg 1$. In this case we can leave only the dominant probabilities

$$P_0 = (\tau+\eta)\left(\tau+\eta+\sqrt{\tau\mu}\right)^{-1}, \qquad P_1 = \sqrt{\tau\mu}\left(\tau+\eta+\sqrt{\tau\mu}\right)^{-1}, \tag{14}$$

in (7), setting $P_{n>1}$ equal to zero. As a result, we obtain the following equality

$$Z \equiv e\varphi_d T_e^{-1} = \eta\overline{N} = \eta\sum\nolimits_0^\infty nP_n = \eta P_1 = \eta\sqrt{\tau\mu}\left(\tau+\eta+\sqrt{\tau\mu}\right)^{-1}, \tag{15}$$

revealing the *non-linear dependence* between $Z$ and $\eta$.

**Numerical calculations.** The results of calculations carried out at arbitrary $\eta$ show that solutions (7) are almost indistinguishable from the normalized solutions obtained directly from Eqs. (2)-(4). The figures below demonstrate good correspondents of approximated curves to the exact solutions obtained numerically from (7).

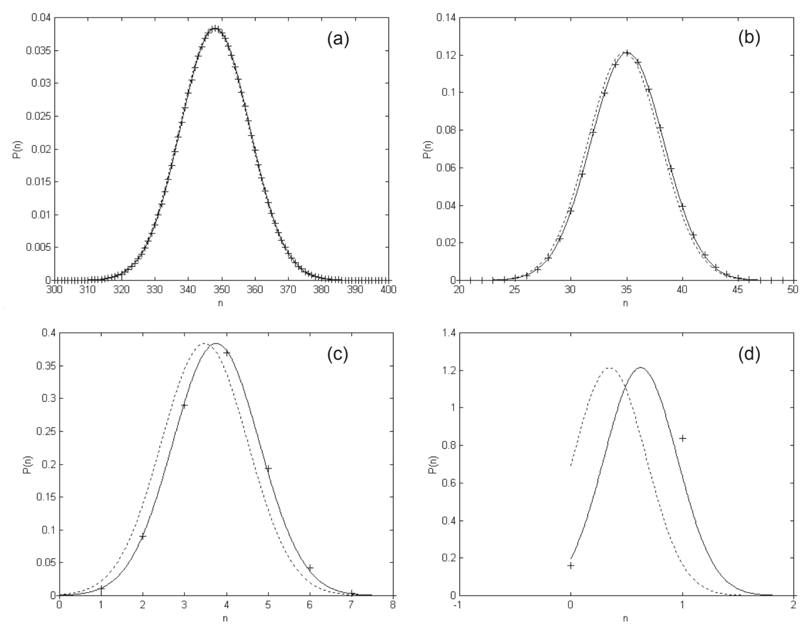


Figure 1. Stationary charge distribution $P_n$ at $T_e = T_i = 200°\mathrm{K}$ for a hydrogen plasma with different radius $a$ of the grain: $10\mu$m (a), $1\mu$m (b), 100nm (c), and 10nm (d). The marks "+" correspond to exact solution (7), solid line — predicted Gaussian (10), dashed line — early obtained Gaussian (12).

**Conclusions.** We have found that for the region $\eta \ll 1$ the steady-state charge distribution is Gaussian one, but the peak position is a little deviated from the mean particle charge obtained from the OLM theory on the basis of equilibrium charge alone. The floating potential of the grain in this region is linear function of inverse grain radius while the OLM theory predicts the independence of this potential from the grain radius.

In the case $\eta \gg 1$ the grain charge distribution couldn't be written in Gaussian-like form and the floating potential of the grain is non-linear function of the grain radius and other plasma parameters.

The results of the theory are in good agreement with numerical calculations.

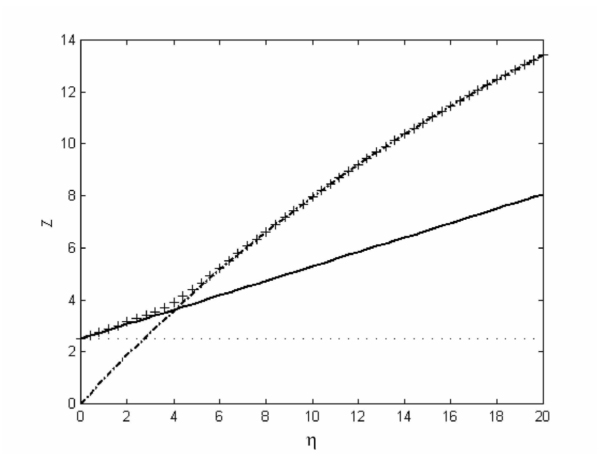


Figure 2. The variation of $Z$ with $\eta$. The marks "+" correspond to exact solution, solid line — predicted linear curve (13), dashed line — predicted curve (15), dotted line — early obtained solution [1-3].

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**References.**

1. P.Shukla, A.Mamun, Introduction to dusty plasma physics, IoP Publishing Ltd, 265p. (2002).

2. V.Tsytovich, Physics-Uspekhi, **40**, 53 (1997).

3. T.Matsoukas, M.Russel, *Phys. Rev. E*, **55**, 991 (1997).