*31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-1.036 (2004)*

The Plasma Boundary in Electronegative Gases

J.E.Allen

University College, Oxford, OX1 4BH, UK.

Abstract. It is well known that the Bohm criterion for electronegative gases yields a multi-valued solution for certain plasma parameters. The prescription for the determination of the correct solution, in the original paper by Braithwaite and Allen [1] was to choose the smaller value of the two possible potential drops in the quasi-neutral plasma. A number of authors have recently questioned this procedure. The present poster paper affirms the correctness of the original method. In some cases an outer plasma may surround an inner plasma, a space charge region separating the two. In this case the Braithwaite-Allen prescription refers to the edge of the inner plasma; further calculations are required to obtain the flux of positive ions leaving the entire system.

In the derivation of the Bohm criterion for sheath formation in electronegative plasmas a multi-valued function was found by Braithwaite and Allen [1]. Figure 1 illustrates the normalised sheath edge potential, as a function of the ratio of the negative ion density to electron density, for various values of γ, the ratio of the electron temperature to negative ion temperature. The multi-valued solution was found when $\gamma > 5+\sqrt{24}$, (~9.9). Braithwaite and Allen proposed that the lower value should be chosen when the function becomes multi-valued, as illustrated in Figure 1. This procedure has been questioned by Sheridan, Chabert and Boswell [2]and more recently by Fernández Palop et al [3], who have made alternative proposals based on the positive ion flux.

To fix our ideas let us first consider the latter paper. The authors have written down Poisson's equation for the sheath region and have integrated it once analytically. It is of interest to note that the equation so obtained has a clear physical meaning; it is a *stress-balance* equation, as discussed by Allen [4] for the electropositive case. It could have been written down directly. The square of the electric field, given there in dimensionless units, is the only term of the Maxwell stress tensor that enters into the present problem. Fernández Palop et al. find that when the lower "Bohm value" is employed, as proposed by Braithwaite and Allen, the square of the electric field term becomes negative at some point in the sheath. Fernández Palop et al. took this to indicate that they had arrived at an unacceptable solution. This is not the case, the Bohm criterion refers only to a particular position in space, the plasma-sheath boundary, and not the entire sheath region. At this point I cannot do better than to quote from a review lecture by Riemann [5]: " It was derived from a local expansion at the sheath edge and allows no statements on the global sheath structure. In particular we emphasize that the Bohm criterion is not a sufficient condition for a stable sheath and not a sufficient condition for a monotonic sheath". The interpretation of the Bohm criterion as postulated by Braithwaite and Allen corresponds to the breakdown of quasi-neutrality and the initial stages of sheath formation. In some situations space charge layers will form surrounded by another plasma. In these cases the said Bohm result refers to only one part of the system; further calculations may be required, for example to calculate the ion flux to the wall or a probe, but that does not mean that the Bohm criterion has been incorrectly interpreted.

An early relevant paper is by Schott [6] who considered two groups of electrons, rather than electrons and negative ions. He identified the lower value of the multi-valued function as the location of the sheath edge, a fact noted by Braithwaite and Allen. Schott went on to calculate the subsequent behaviour in the sheath and found a series of spatial oscillations of potential. In this work a so-called "fluid model" was employed, of a kind that has been widely employed by other workers. In this model the ions are assumed to be monoenergetic, i.e. the ion stress tensor has simply been discarded. The mathematical formulation describes a situation in which the positive ions, on travelling through an element of space, "share out" their momentum with the newly produced ions in that space; in this way ionization enters into the momentum equation.

A more realistic model has been studied by Sato and Miyawaki [7] who carried out a free-fall analysis, essentially extending the classical work of Tonks and Langmuir to include two sets of electrons, with different temperatures. The plane case was considered. Sato and Miyawaki found a multi-valued function, as did Braithwaite and Allen. Their kinetic model for a collisionless plasma gave a critical value of $\gamma = 10.8$, not very different from $\gamma = 5+\sqrt{24}$ (~9.9). They too chose the smaller root as an indication of where the sheath begins. Sato and Miyawaki described the formation of a double layer, followed by a second plasma region and then a further sheath. These authors noted a close similarity to the case of the laser-plasma corona, where the existence of two electron temperatures led to the critical value of $\gamma = 5+\sqrt{24}$ [8]. The paper of Sato and Miyawaki undoubtedly represents an important step in the development of the subject, although a simple double layer may represent an oversimplification [9]. Sato and Miyawaki present a diagram showing two distinct groups of ions, one group is produced by ionization in the inner plasma and then accelerated across the double layer, the other group is produced by ionization in the outer plasma. The model has been further explored by Sheridan, Braithwaite, and Boswell [10], who have calculated the ion flux leaving the second (outer) plasma, for small, but finite, values of λ_D.

Turning to the paper by Sheridan, Chabert and Boswell [2], these authors employed the so-called "fluid theory", as described above, and found spatial oscillations in potential similar to those found by Schott. They then went on to propose a different choice of Bohm velocity when dealing with the multi-valued function, i.e. one defined in terms of positive ion flux. Their plots of potential, however, clearly show that a sheath begins to form, i.e. a space charge develops, when the *lower* Bohm condition is satisfied. We have seen that two plasmas separated by a space charge region are sometimes to be found. In such a case the Bohm criterion for the boundary of the first (inner) plasma does not lead to a correct expression for the ion flux leaving the entire system. Further calculations are required to determine the overall structure of the discharge and the resulting fluxes of ions produced in both inner and outer plasmas. In the opinion of the writer the much-used "cold fluid model", which is always a crude approximation, is hardly the right one to employ in this instance.

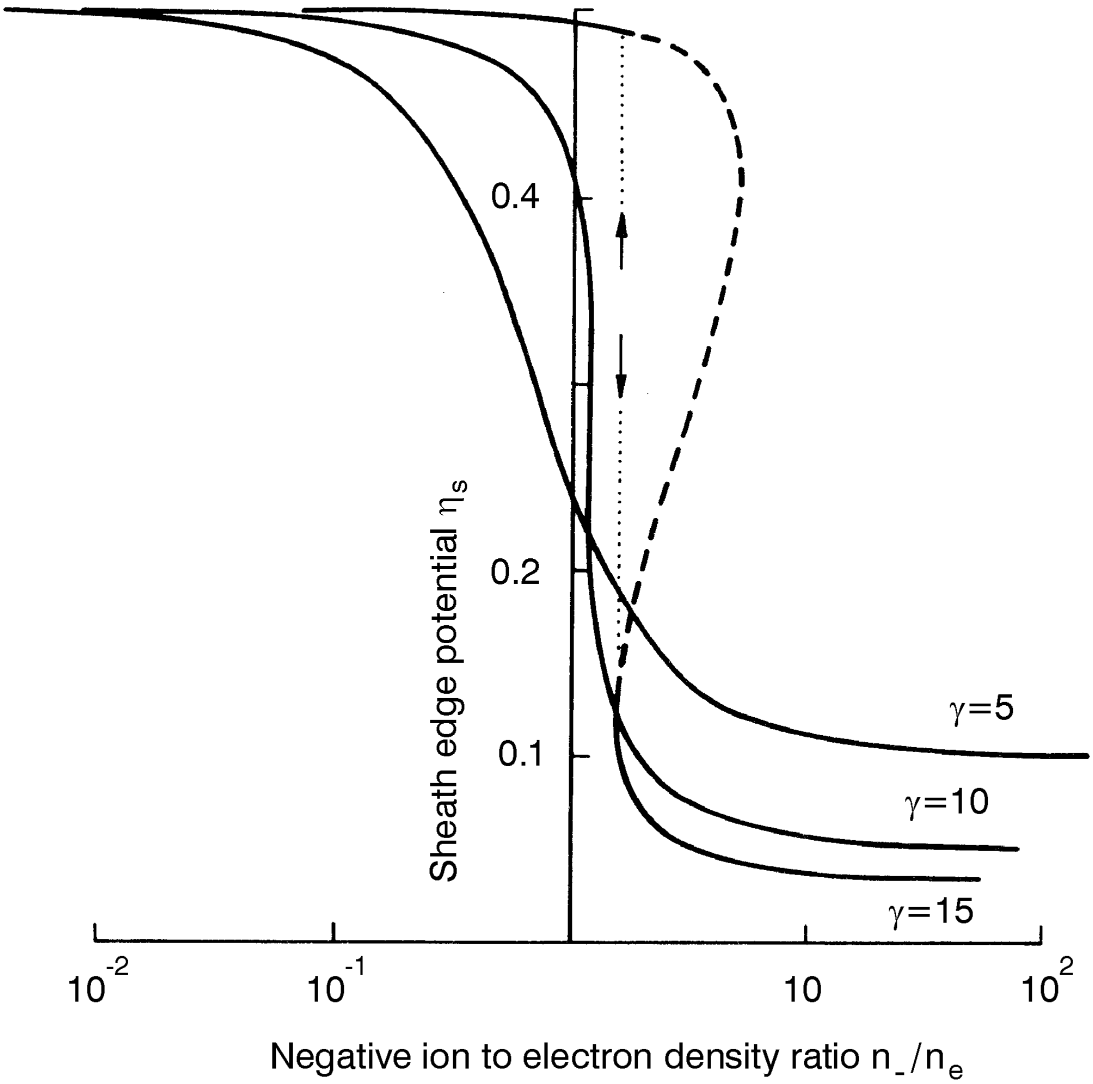
The simpler case of the sheath structure around a spherical probe has been studied by Kono [11]. The simplification arises because a plasma solution can be found without including ionization; the plane case requires ionization to exist throughout the plasma. Spatial oscillations of potential were found once more in this case; the "fluid model" employed, however, is again one in which the positive ions all have the same energy at any point in space. The oscillations that were found are almost certainly due to the limitations of the model; the *diffusion* of positive ions is not allowed to take place, a fact discussed by Franklin and Snell [12]. More relevant to the purpose of this note, however, is that the beginning of the space charge region occurs at, or near, the lower of the two values of the potential drop in the plasma given by the Bohm criterion for electronegative gases. This is seen to be the case when the Debye distance is small compared with the probe radius. The Bohm criterion is valid, or more precisely it is an excellent approximation, only for thin sheaths, where the plasma boundary is well-defined. An interesting exception is the special case of high voltage sheaths, referred to in a different context by Braithwaite and Allen [13].

In conclusion we can state that the interpretation of the Bohm criterion for electronegative gases, given by Braithwaite and Allen, is indeed the correct one. It signifies the beginning of the space charge formation at the boundary of the plasma. It does not, by itself, enable us to calculate the flux of ions leaving the entire system, as in many other cases. Further analysis is required to achieve that objective. We can note that Braithwaite and Allen deduced the criterion for an electronegative plasma in several different ways, not only as the singular limit of a monotonic "plasma solution". The results are compatible with the fact that both the lower and the higher Bohm values might be attained in a discharge under some circumstances, the first being associated with the inner plasma and the second with the outer plasma. In other words the Bohm criterion may be applicable at more than one position in a discharge [14]. The present discussion of the Bohm criterion is also relevant to the case of plasmas that contain two groups of electrons with different temperatures.

This Conference contribution is a corrected version of a recently published note [15]. In that note it was erroneously stated that Sato and Miyawaki considered negative ions, rather than two different sets of electrons.

References

[1] Braithwaite, N.St.J. and Allen, J.E., J.Phys.D:Appl. Phys., 21, 1733, (1988).
[2] Sheridan,T.E., Chabert, P and Boswell.R.W., Plasma Sources Sci.Technol., 8, 457, (1999).
[3] Fernández Palop, J.I., Ballesteros, J., Hernández, M.A. and Morales Crespo, R., J.Appl.Phys., 91, 2587, (2002).
[4] Allen, J.E., Plasma Physics, Ch. 5, (ed. Keen, B.E., London and Bristol: Institute of Physics, 1974).
[5] Riemann, K.-U., J.Tech.Phys., 41, 89, (2000).
[6] Schott,L., Phys.Fluids, 30, 1795, (1987).
[7] Sato,K. and Miyawaki, F., Phys.Fluids B, 4, 1247, (1992).
[8] Wickens, L.M., Allen, J.E. and Rumsby, P.T., Phys. Rev. Letters, 41, 243, (1978). [9] Sheridan,T.E., Braithwaite, N.St.J. and Boswell, R.W., Phys.Plasmas, 6, 4375, (1999).
[11] Kono, A., J.Phys.D:Appl.Phys., 32, 1357, (1999).
[12] Franklin,R.N. and Snell, J., J.Phys.D: Appl.Phys., 33, 1990, (2000).
[13] Braithwaite, N.St.J. and Allen, J.E., Phys.Fluids, 27, 203, (1984).
[14] Franklin,R.N.,J.Phys.D: Appl.Phys., 36, R309, (2003).
[15] Allen, J.E., Plasma Sources Sci.Technol.,13, 48, (2004).



Caption for the diagram

FIG. 1. Plot of the normalised sheath edge potential against the ratio of the negative ion density to electron density in the bulk plasma for three values of γ, the ratio of the electron temperature to negative ion temperature. The arrowed dotted line indicates the jump in the solution proposed by Braithwaite and Allen [1].