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Absorption of EM wave by the Mei-Debye scattering of a dust plasma

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Abstract: Absorption of electromagnetic waves by the dust particles in a plasma was studied based on a Mie-Debye scattering mode. The longitudinal field of the Debye scattering has been derived and the wave energy loss from it has been calculated. It is shown that the lower the temperature of the plasma is and the higher the density of the plasma is, the larger the absorption cross section due to the longitudinal scattering will be. For the low frequency waves scattered in a dusty plasma are mainly in the form of Debye scattering. In this case the energy loss due to the longitudinal scattering will affect the wave propagation seriously.

1. Longitudinal scattered field of Debye scattering

Let the Debye scattered field $\vec{E}^D = \vec{E}_R^D + \vec{E}_L^D$, where the superscript D denotes the Debye scattering and the subscripts R and L denote the transversal part and longitudinal part respectively. From the Maxwell's equations we have

$$\nabla \times \nabla \times \vec{E}_R^D - k^2 \vec{E}_R^D = i\mu_0 \omega \vec{J}_R \tag{1a}$$

$$\vec{E}_L = -\frac{i}{\varepsilon(\omega)\varepsilon_0 \omega} \vec{J}_L \tag{1b}$$

where $\vec{J} = \vec{J}_L + \vec{J}_R$ is the nonlinear current in the Debye sphere. Under the condition of cold electrons the current can be

$$\vec{J}(\vec{r}) = J_0 \frac{\exp(-r/\lambda_D)}{r} U(r-a)\vec{E}_0 \tag{2}$$

where $J_0 = i(ze^2 / 4\pi\omega m \lambda_D^2)\exp(a/\lambda_D)$ and $U(r-a)$ is the step function. An eigenfunction expansion of the current of equation（2）using the spherical vector wave functions $\vec{M}, \vec{N}, \vec{L}$:

$$\vec{J}_L(\vec{r}) = \int_0^\infty dh \sum_n c_n(h) \vec{L}_{1,n}(h,\vec{r}) \tag{3}$$

After some manipulations the final expression of the longitudinal part of the Debye scattered field is (the incident wave is assumed to be of unit amplitude)

$$\vec{E}_L^D(r) = r_0 \frac{ze^{a/\lambda_D}}{k^2 \lambda_{De}^2} \sum_n i^{n+1} \left\{ \vec{B}_n(\theta,\phi)\Delta - (n+1)\vec{P}_n(\theta,\phi)\Theta \right\} \tag{4}$$

where $$\Delta = \int_a^r dxe^{\;x/\lambda_D} j_{n\ 1}(kx)\frac{x^n}{r^{n+2}} - \int_r^{\infty} dxe^{\;x/\lambda_D} j_{n+1}(kx)\frac{r^{n-1}}{x^{n+1}},$$

$$\Theta = \int_a^r dxe^{\;x/\lambda_D} j_{n\ 1}(kx)\frac{x^n}{r^{n+2}} \quad n\int_r^{\infty} dxe^{\;x/\ \ _D} j_{n+1}(kx)\frac{r^{n-1}}{x^{n+1}} - \frac{2n+1}{kr^2} j_n(kr)e^{\;r/\ \ _D},$$

$$\vec{B}_n(\theta,\phi) = \hat{\theta}\tau_n\cos\phi - \hat{\phi}\pi_n\sin\phi,$$

$$\vec{P}_n(\theta,\ \ ) = \hat{r}P_n^1(\cos\ \ )\cos\ \ ,$$

$$\pi_n = \frac{P_n^1(\cos\theta)}{\sin\theta},$$

$$\tau_n = \frac{dP_n^1(\cos\theta)}{d\theta},$$

and $r_0 = e^2/4\pi\varepsilon_0 mc^2$ is an electronic classical radius, and c is the speed of light.

An asymptotic expression of the field For $r \to \infty$ from Eq. (4) is:

$$\vec{E}_L^D(\vec{r}) \approx \frac{1}{1+\delta}\frac{zr_0}{(kr)^3\lambda_D}\frac{2}{\sqrt{1+(k\lambda_D)^2}}\sin(ka - \tan^{-1}(k\lambda_D))(\vec{P}_1(\ ,\ ) - \frac{\vec{B}_1(\theta,\phi)}{2}) \qquad (5)$$

It shows that the longitudinal field $E_L^D \propto r^{-3}$ is an evanescent field which in agreement with the result of Guerra and Medonca's paper. But In the region nearby the central grain, the longitudinal field $\vec{E}_L$ occupies the major part of energy.

In the case of Raleigh limit $k\lambda_d << 1$

$$\vec{E}_L^D(\vec{r}) \approx \frac{1}{1+\delta}\frac{zr_0}{(kr)^3\lambda_{De}^2}\Big\{ [\vec{B}_1(\theta,\phi) - 2\vec{P}_1(\ ,\phi)][(r+\lambda_D)e^{-(r-a)/\lambda_D} - (a+\lambda_D)]$$

$$+r\vec{P}_1(\ ,\ )e^{-(r-a)/\lambda_D} - \frac{8}{5}[\vec{B}_1(\theta,\phi) + \vec{P}_1(\ ,\phi)]\,[(r+\lambda_D)e^{-(r-a)/\lambda_D} - (a+\lambda_D)]\Big\} \qquad (6)$$

The transversal part of the Debye scattering is：

$$\vec{E}_R^D(\vec{r}) = i\frac{e^{ikr}}{kr}\sum_n \frac{2n+1}{n(n+1)}[a_n^D\vec{B}_n(\theta,\phi) + b_n^D\vec{C}_n(\theta,\phi)] \qquad (7)$$

where $a_n^D = -ir_0k\frac{ze^{a/\lambda_D}}{\lambda_{De}^2}I_{2,n}(k)$, $b_n^D = -ir_0k\frac{ze^{a/\lambda_D}}{\lambda_{De}^2}I_{1,n}(k)$ and $\vec{C}_n(\theta,\phi) = \hat{\theta}\pi_n\cos\phi - \hat{\phi}\tau_n\sin\phi$.

2. Analysis of the cross section

The total cross s is $\sigma_t = \ _{at} + \ _{st}$.

The total absorbed section is $\sigma_{at} = \sigma_{am} + \sigma_{ad}$. The absorbed section produced by the longitudinal part of the Debye scattering is

$$\sigma_{ad} = \int_V k \, |\vec{E}_L^D|^2 \, dV \ ,$$

and the absorbed section given by the imaginary part of the grain's dielectric constant ε'' is

$$\sigma_{am} = k\varepsilon_r'' \, \left|\frac{3}{m^2+2}\right|^2 V$$ (V is the volume of the grain , and $m^2 = \varepsilon_r$)

The total scattering section is $\sigma_{st} = \sigma_{sm} + \sigma_{sd} + \sigma_{smd}$, where the Mie scattering section is given by

$$\sigma_{sm} = \frac{8\pi(ka)^4 a^2}{3} \left|\frac{m^2-1}{m^2+2}\right|^2,$$

the Debye scattering section is given by

$$\sigma_{sd} = \frac{2\pi}{k^2} \sum_n (2n+1)[|a_n^D|^2 + |b_n^D|^2] \ ,$$

and their interference scattering section in the case of Raleigh limit is given by

$$\sigma_{smd} = -8\pi z k^4 a^3 \frac{r_0 e^{a/\lambda_D}}{(k\lambda_D)^2} I_{2,1}(k) \, \mathrm{Re}[\frac{m^2-1}{m^2+2}] .$$

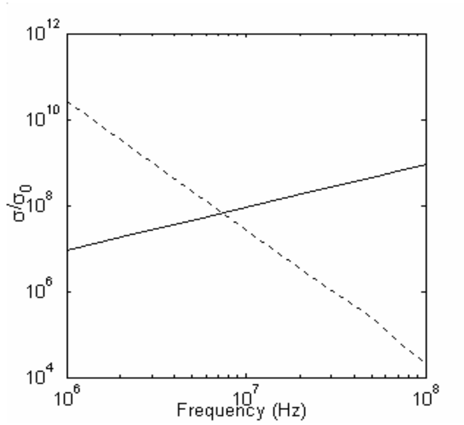

FIG. 1 Absorbed section (normalized to the grain's surface $\sigma_0 = 4\pi a^2$) as a function of frequency of the incident wave for $a = 1\mu m$, λ_D =1m , Z=1000 and m=2+i. The solid line for σ_{am}, dotted line for σ_{ad}

Figure 1 shows that (1) in the region of $k\lambda_D << 1$, $\sigma_{ad} \propto f^{-3}$ while $\sigma_{am} \propto f$; (2) in lower frequency region σ_{ad} becomes dominant while in the higher frequency region σ_{am} becomes dominant. It can be concluded from figure 2 that for lower temperature and higher electronic density, the dust grain is of bigger abortion section σ_{ad} for the waves.

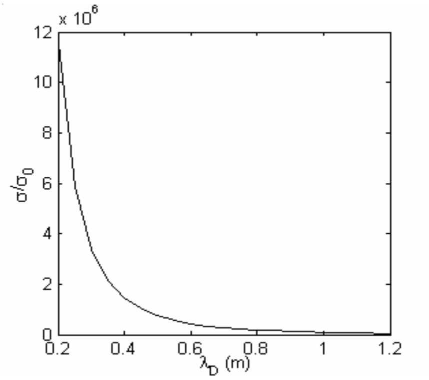

FIG. 2 Absorbed section from the longitudinal scattering (normalized to the grain's surface $\sigma_0 = 4\pi a^2$) for $a = 1\mu m$, Z=1000, f=10^8Hz as a function of the radius of the Debye sphere.

Figure 3shows that when the frequency rises higher absorbing section, the scattering section and the total section decrease. However for $f < 10^8\,Hz$ the absorbing section becomes dominant and is a major factor for the wave propagation.

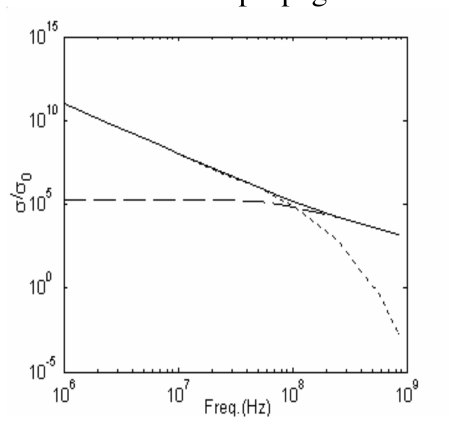

FIG. 3 The cross section (normalized to the grain's surface $\sigma_0 = 4\pi a^2$) of a dust grain in a plasma as a function of frequency of the incident wave for case $a = 1\mu m$, λ_D =1m , Z=1000, m=2. The dotted line is for σ_{at}, the dashed line for σ_{st} and the solid line for σ_t.

3. Conclusion

It was pointed out by Guerra and Medonca that the Mie scattering dominates for the waves of high frequency while the Debye scattering dominates for the waves of low frequency. The results of this study show that when the Debye scattering dominates, the effect of absorption from the longitudinal scattering cannot be ignored.

References

1. Guerra R, Medonca J T. Mie and Debye scattering in dusty plasmas, Phys. Rev. E, 62, pp1190—1201 (2000)