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Discrete breather transverse modes in complex plasma crystals in the presence of an electric and/or magnetic field

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Abstract

Highly localized structures (Discrete Breathers) involving vertical (transverse, off-plane) oscillations of charged dust grains are shown to exist in a dust lattice, accounting for the lattice discreteness and the sheath electric field nonlinearity. These structures correspond to either extremely localized pulses or dark excitations composed of dips/voids. Explicit criteria for different modes are presented.

1. Introduction. Discrete Breathers (DBs) (or Intrinsic Localized Modes, ILMs) are highly localized oscillatory modes known to occur in Wigner crystals (e.g. atomic or molecular chains) characterized by nonlinear inter–site coupling and/or substrate potentials and a highly discrete structure [1]. The properties of DB modes have recently gathered an increasing interest in modern nonlinear science. An exciting paradigm of such a nonlinear chain is provided by dust lattices, i.e. chains of mesoscopic size heavily charged, massive dust particulates, gathered in a strongly coupled arrangements spontaneously formed during plasma discharge experiments. In earth laboratory experiments, these crystals are subject to an (intrinsically nonlinear [2]) external electric and/or magnetic field(s), which balance(s) gravity at the levitated equilibrium position, and are held together by electrostatic (Debye type) interaction forces. Although the linear properties of these crystals now seem quite well understood, the elucidation of the *nonlinear* mechanisms governing their dynamics is still in a preliminary stage. This study is devoted to an investigation, from first principles, of the existence of DB excitations, associated with transverse (off-plane, optical mode) dust grain oscillations in a dust mono-layer.

2. The model. The vertical (off-plane) grain displacement in a dust crystal (assumed quasi-one-dimensional, composed from identical grains of charge q and mass M, located at $x_n = n\,r_0,\ n \in \mathcal{N}$) obeys an equation of the form

$$\frac{d^2\delta z_n}{dt^2} + \omega_0^2\,(\,\delta z_{n+1} + \,\delta z_{n-1} - 2\,\delta z_n) + \omega_g^2\,\delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3 = 0\,, \qquad (1)$$

where $\delta z_n = z_n - z_0$ denotes the small displacement of the $n-$th grain around the (levitated) equilibrium position z_0, in the transverse ($z-$) direction. The characteristic frequency $\omega_0 = [-q\Phi'(r_0)/(Mr_0)]^{1/2}$ results from the dust grain (electrostatic) interaction potential $\Phi(r)$, e.g. for a Debye-Hückel potential: $\Phi_D(r) = (q/r)\,e^{-r/\lambda_D}$, one has $\omega_{0,D}^2 = q^2/(Mr_0^3)\,(1 + r_0/\lambda_D)\,\exp(-r_0/\lambda_D)$ (where λ_D denotes the effective DP Debye length). The gap frequency ω_g and the nonlinearity coefficients α, β are defined via the overall vertical force $F(z) = F_{e/m} - Mg \approx -M[\omega_g^2\delta z_n + \alpha\,(\delta z_n)^2 + \beta\,(\delta z_n)^3] + \mathcal{O}[(\delta z_n)^4]$, which has been expanded around z_0 by formally taking into account the (anharmonicity of the) local form of the sheath electric and/or magnetic field(s), as well as, possibly, grain charge variation (refer to the definitions in [2, 3], not reproduced here). The electric/magnetic levitating force(s) $F_{e/m}$ balance(s) gravity at z_0. Notice the difference in structure from the usual nonlinear Klein-Gordon equation used to describe 1d oscillator chains: 'phonons' in this chain are stable *only* thanks to the field force $F_{e/m}$ (via ω_g). Collisions with neutrals and coupling anharmonicity are omitted, at a first step, in this simplified model.

Eq. (1) leads to the discrete dispersion relation $\omega^2 = \omega_g^2 - 4\omega_0^2 \sin^2(kr_0/2)$. The wave frequency ω decreases with increasing wavenumber $k = 2\pi/\lambda$ (or decreasing wavelength λ), implying that transverse vibrations propagate as a *backward wave* (see that $v_g = \omega'(k) < 0$), in agreement with recent experiments [4]. We notice the *gap frequency* ω_g, as well as the *cutoff frequency* $\omega_{min} = (\omega_g^2 - 4\omega_0^2)^{1/2}$ (obtained at the end of the first Brillouin zone $k = \pi/r_0$), which is *absent in the continuum limit.*

3. A discrete envelope evolution equation. Following Ref. [5] (and drawing inspiration from the quasi-continuum limit [2, 3]), one may adopt the ansatz

$$\delta z_n \approx \epsilon\,(u_{1,n}^{(1)}\, e^{-i\omega_g t} + \text{c.c.}) + \epsilon^2\,[u_{2,n}^{(0)} + (u_{2,n}^{(2)}\, e^{-2i\omega_g t} + \text{c.c.})] + \dots, \tag{2}$$

where we assume: ω_g^2, α, $\beta \sim 1$ and $d/dt, \omega_0^2 \sim \epsilon^2$, implying a high ω_g/ω_0 ratio (this condition is clearly satisfied in recent experiments; cf. [4]). Inserting into Eq. (1), one obtains the discrete nonlinear Schrödinger equation (DNLSE)

$$i\,\frac{du_n}{dt} + P\,(u_{n+1} + u_{n-1} - 2u_n) + Q\,|u_n|^2\,u_n = 0\,, \tag{3}$$

where $u_n = u_{1,n}^{(1)}(t)$, along with the relations $u_{2,n}^{(2)} = [\alpha/(3\omega_g^2)]\,u^2$ and $u_{2,n}^{(0)} = -(2\alpha/\omega_g^2)\,|u|^2$ and the definitions $P = -\omega_0^2/(2\omega_g)$ and $Q = (10\alpha^2/3\omega_g^2 - 3\beta)/2\omega_g$ for the discreteness and nonlinearity coefficients P and Q, respectively. Note that $P < 0$; the sign of Q, on the other hand, depends on the sheath characteristics and cannot be prescribed.

Eq. (3) yields a plane wave solution $u_n = u_0 \exp(i\theta_n) \equiv u_0 \exp[-i(n\tilde{k}r_0 - \tilde{\omega}t)]+$ c.c., where the envelope frequency $\tilde{\omega}$ obeys the dispersion relation

$$\tilde{\omega}(\tilde{k}) = -4P\sin^2(\tilde{k}r_0/2) + Q|u_0|^2\,. \tag{4}$$

The (envelope) frequency band lies between $\tilde{\omega}_{min} = |Q||u_0|^2$ and $\tilde{\omega}_{max} = |Q|u_0|^2 - 4P|$.

The envelope stability may be studied by setting $u_0 \to u_0 + \xi\hat{u}_0 \exp[-i(n\hat{k}r_0 - \hat{\omega}t)]$ and $\theta_n \to \theta_n + \xi\hat{\theta}_0 \exp[-i(n\hat{k}r_0 - \hat{\omega}t)]$, where $\xi \ll 1$, and then linearizing in ξ; one thus obtains the *perturbation* dispersion relation

$$[\hat{\omega} - 2P\sin\hat{k}r_0 \sin\tilde{k}r_0]^2 = 4\,P\,\sin^2(\hat{k}r_0/2)\,\cos\tilde{k}r_0\,[4P\sin^2(\hat{k}r_0/2)\,\cos\tilde{k}r_0 - 2Q|u_0|^2]\,.$$

The wave envelope will be *unstable* to external perturbations (viz. $\text{Im}\hat{\omega} \neq 0$) if $PQ\cos\tilde{k}r_0 > 0$ *and* (in the same time) the amplitude u_0 exceeds some critical value $u_{cr}(\tilde{k})$. Otherwise, the wave envelope will be *stable*.

4. Bright-type localized gap modes. It is known that modulational instability, here possible e.g. for $Q < 0$ *and* $0 < \tilde{k} < \pi/2r_0$, may result in the formation of localized modes in the linear frequency gap region. Following previous works (see Refs. in [5]), single-mode (fixed ω) *odd-parity* localized periodic solutions may be sought in the form

$u_n(t) = f_n\, u_0 \cos \Omega t$, assuming $f_0 = 1$, $f_{-n} = f_n$ and $|f_n| \ll f_1$ for $|n| > 1$. One thus obtains a localized lattice pattern of the form:

$$u_n(t) = u_0 \, \cos \Omega t \, \{..., 0, 0, \eta, 1, \eta, 0, 0, ...\} \,, \tag{5}$$

(see Fig. 1a) where $\Omega \approx |Q| u_0^2/4$ and $\eta = 4P/(Q u_0^2) \ll 1$ $\;(\eta > 0)$. We see that Ω lies outside the (amplitude) frequency band prescribed by Eq. (4).

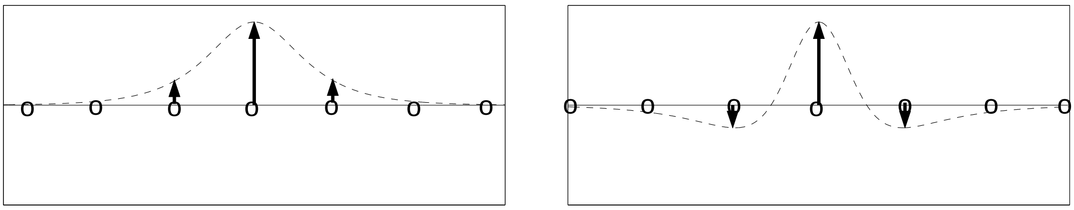

Figure 1: Localized discrete breather dust lattice excitations of the *bright* type, obtained for $Q < 0$; the successive lattice site displacements are depicted at maximum amplitude: (a) odd-parity solution, as given by Eq. (5); (b) even-parity solution, given by Eq. (6).

For $Q > 0$, the modulational stability profile is reversed (instability occurs for $\tilde{k} > \pi/2r_0$). The same procedure then leads to an *even parity* solution of the form

$$u_n(t) = u_0 \, \cos \Omega t \, \{..., 0, 0, -\eta, 1, -\eta, 0, 0, ...\} \,, \tag{6}$$

where $\eta = -P/(Q u_0^2)$ satisfies $0 < \eta \ll 1$ and $\Omega \approx -4P + Q u_0^2$ (see Fig. 1b).

5. Dark/grey-type localized gap modes. Dark-type solutions (voids) may also be sought. For $Q < 0$, for instance, one should look into the region $\tilde{k} > \pi/(2r_0)$, e.g. near the cutoff frequency $\tilde{\omega}_{cr,2}$. One thus finds [6] the discrete pattern

$$u_n(t) = 2A \, \cos \Omega t \, \{..., 1, -1, 1, -(1-\eta), \, 0, \, 1-\eta, -1, 1, -1, ...\} \,, \tag{7}$$

where $\Omega = 4P - Q|A|^2$ and $\eta = P/(QA^2)$ satisfies $0 < \eta \ll 1$; see Fig. 2a.

Focusing in the middle of the Brillouin zone, i.e. at $k = \pi/2r_0$ (where *odd* sites remain at rest, while *even* ones oscillate out of phase; cf. the analysis in [5, 7]), one obtains [5, 6] the pattern

$$u_n(t) = 2A \, \cos \Omega t \, \{..., \, 1, \, 0, \, -1, \, 0, \, 1-\eta, \, 1-\eta, \, 0, \, -1, \, 0, \, 1, ...\} \,, \tag{8}$$

where $\Omega = 2P - Q|A|^2$ and $\eta = P/2QA^2$ satisfies $0 < \eta \ll 1$. This *grey-type* discrete lattice excitation (known in solid lattices [5]), is depicted in Fig. 2b.

6. Breather control. The stability of a breather excitation may be *controlled* via external feedback, as known from one-dimensional discrete solid chains [8]. The method consists in using the knowledge of a reference state (unstable breather), say $\delta z_n^{(0)} = \hat{z}_n(t)$, e.g. obtained via an investigation of the homoclinic orbits of the 2d map obeyed by the main Fourier component [9], and then perturbing the evolution equation (1) by adding a term $+K[\hat{z}_n(t) - \delta z_n]$ in the right-hand side (*rhs*), in order to stabilize breathers via tuning of the control parameter K. This method relies on the application of the

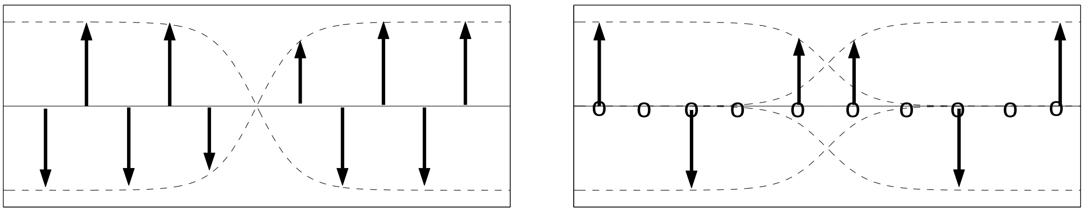

Figure 2: Localized discrete breather dust lattice excitations of the (a) *dark* type; (b) *grey*-type (obtained for $Q > 0$): successive dust grain displacements.

continuous feedback control (*cfc*) formalism (see the Refs. in [9]). Alternatively, as argued in [9], a more efficient scheme should instead involve a term $+Ld[\hat{z}_n(t) - \delta z_n]/dt$ in the *rhs* of Eq. (1) (*dissipative cfc*), whence the damping imposed results in a higher convergence to the desired solution $\hat{z}_n(t)$. Preliminary work in this direction is being carried out and progress will be reported later.

The highly localized excitations discussed here correspond to a localization of the energy stored in the lattice. The perspective is thus open of the generation and subsequent manipulation of localized modes in a DP crystal, in view of applications.

7. Conclusions. We have examined, from first principles, the conditions for the occurrence of intrinsic localized modes (discrete breather excitations) in a dust mono-layer. This preliminary study needs to be confirmed by a rigorous investigation of the stability and dynamical properties of these excitations. An extended study, complementing these first results [6], is under way and will be reported soon. These theoretical predictions may hopefully motivate appropriately designed experimental studies of pulse localization phenomena in DP crystals.

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