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Vertical oscillations of magnetized particles in complex plasmas

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Up to now, oscillatory modes of magnetized particles in a gas discharge plasma were considered only in homogeneous magnetic fields [1], which implies that a magnetic force levitation was not discussed as well as the role of inhomogeneous magnetic field on the particle dynamics. In this paper, we will focus on the grain levitation in an external magnetic field and vertical vibrations of the paramagnetic particles around their equilibrium. As an application we consider the conditions used in the recent magnetic experiment on the magnetic levitation [2].

Vertical oscillations of a single dust particle. The equation describing the vertical motion of a particle in gas discharge plasmas can be written in a classical form

$$\ddot{z} + 2\gamma \dot{z} = \frac{F}{M} - g , \tag{1}$$

where F is the electromagnetic force acting on the particle of mass M, g denotes the gravitational acceleration, and γ the damping rate due to neutral gas friction. In the experiments, which do not invoke a magnetic field, the particles are trapped in the vertical direction z, by the balance of gravitational and electrostatic forces $F_E = Q_0 E_0$, where Q_0 is the particle charge, and E_0 is the electric field at the equilibrium. When the particle is perturbed from its equilibrium position, the characteristic frequency of vertical vibrations is given by [3,4]

$$\Omega_E^2 = -\frac{(QE)'_0}{M}. \tag{2}$$

Consider now the case, when the grain levitation occurs mainly due to the magnetic force. A spherical particle of radius a and magnetic permeability μ immersed in an external magnetic field B parallel to the vertical $z-$axis gets a magnetic moment $m = \alpha B$, where $\alpha = a^3(\mu - 1)/(\mu + 2)$. A magnetic force, associated with the vertical magnetic field gradient is

$$F_m = -\frac{\partial(mB)}{\partial z} = -2\alpha B \frac{\partial B}{\partial z}. \tag{3}$$

Expanding $F_m \cong F_0 + F_0' z$ around equilibrium yields

$$\ddot{z} + 2\gamma \dot{z} + \frac{2\alpha}{M}\left(B_0'^2 + B_0 B_0''\right)z = 0 \, .$$

So that a new frequency of vertical oscillations is determined as

$$\Omega_m^2 = \frac{2\alpha}{M}\left(B_0'^2 + B_0 B_0''\right) = \frac{6(\mu - 1)}{4\pi\rho \;\mu + 2}\left(B_0'^2 + B_0 B_0''\right) \, , \tag{5}$$

(ρ is the material density). It follows immediately that all magnetized grains have the same frequency of vertical vibrations independent of size (5), but specified by the vertical profile of the external magnetic field and the magnetic properties of the grain material. As a result, the value Ω_m^2 can be positive or negative depending on B_0, B_0' and B_0'' at the levitation height. Since these quantities are easily controlled in experiments, Ω_m^2 can be always made positive to stabilize vertical particle vibrations.

To obtain a numerical estimate of the frequency Ω_m, we use typical conditions for the magnetic experiments [2]. Particle levitation was observed for average magnetic fields $B_0 \sim 0.1 = 0.15T$ and field gradients of $|B_0'| \sim 0.75 = 1.15 T/m$. Taking $B_0'' \sim 5 - 7.5 T/m^2$, this yields $\Omega_m \sim 50 - 70 \; s^{-1}$. Such values are similar to the frequencies of vertical vibrations in a sheath electric field Ω_E [3,4] and can be easily measured.

Finally, when both the magnetic and electric forces contribute to balancing gravitation in Eq. (1), the frequency of vertical oscillations can be written as combinations of (2) and (5)

$$\Omega_v^2 = \Omega_m^2 + \Omega_E^2 = \frac{6(\mu - 1)(B'B)'_0}{4\pi\rho \;\mu + 2} - \frac{(QE)'_0}{M} \, . \tag{6}$$

While the electrostatic term $\Omega_E^2 \sim -(QE)'_0$ is either positive or negative within the sheath region [4], Ω_m^2 can be always made positive and even exceeding $|\Omega_E^2|$, stabilized the vibrations, which would be unstable in the absence of the magnetic field.

The question now is what complex plasma parameters can be recovered from measurements of the characteristic frequencies (5) or (6). If the frequencies of the vertical vibrations have been accurately measured and since values of B_0, B_0' and B_0'' are determined numerically, expression (5) immediately gives the value of the magnetic permeability μ and thus the magnetic moment. If the levitation is due to the combination of the electrostatic and magnetic forces, one can determine the $(QE)'_0$ profile at given μ, B_0, B_0' and B_0''. Such measurements can be of importance to understand how the particle charge or the sheath structure is affected by an external magnetic field.

Vibrational modes of a horizontal string of paramagnetic grains. In order to describe the collective vertical modes in complex plasmas with magnetized grains, we consider simplified model of a one-dimensional string of particles, immersed in the vertical magnetic field. The particles have charge Q, mass M, and magnetic moment m. The energy of electrostatic and magnetic interactions between the n-th and m-th grains of the string is

$$U_{n,m} = \frac{Q_n Q_m}{|\mathrm{r}_{n,m}|} \exp\left(-\frac{|\mathrm{r}_{n,m}|}{\lambda_D} \right) - \frac{\mu_0}{4\pi}\left[\frac{\mathrm{m}_n \mathrm{m}_m}{|\mathrm{r}_{n,m}|^3} - 3\frac{(\mathrm{m}_n \cdot \mathrm{r}_{n,m})(\mathrm{m}_m \cdot \mathrm{r}_{n,m})}{|\mathrm{r}_{n,m}|^5} \right], \tag{7}$$

where the first term in the right hand side of Eq. (7) describes the interactions between two charges, while the second one accounts for interactions of two magnetic dipoles [5], and $\mathrm{r}_{n,m}$ is the distance between the grains. The equation of motion for the n-th particle in the nearest neighbor approximations can be written as

$$\ddot{\mathrm{r}}_n + 2\gamma \dot{\mathrm{r}}_n = M^{-1}\left(\mathrm{F}_{n,n+1} + \mathrm{F}_{n,n-1} + \mathrm{F}_{n,ext} \right) \tag{8}$$

Here $\mathrm{F}_{n,n\pm1} = \mathrm{F}(\mathrm{r}_{n,n\pm1} - \mathrm{r}_n)$, $\mathrm{F}_{n,m} = -\partial U_{n,m} / \partial \mathrm{r}_n$, and $\mathrm{F}_{n,ext} = -\partial U_{ext} / \partial \mathrm{r}_n$ are the forces acting on the n-th grain in the external magnetic and electric fields. The external potential $U_{ext}(\mathrm{r})$ is approximated by a parabolic potential $U_{ext} = \frac{1}{2} M (\Omega_m^2 + \Omega_E^2) z^2$. Using (7)-(8) in the case of small vertical oscillations $z_n \sim \exp(-i\omega t + ikn\Delta)$, gives a dispersion relation describing transverse dust lattice waves in the external electric and magnetic fields

$$\ddot{z}_n + 2\gamma \dot{z}_n = -(\Omega_m^2 + \Omega_E^2) z_n + \left(\Omega_\perp^2 + 9\frac{m_0^2}{\Delta^5} \right)(2z_n - z_{n+1} - z_{n-1}) = 0\,. \tag{9}$$

Here z_n is the vertical displacement of the n-th particle, the quantity $\Omega_\perp$ denotes the frequency of the transverse dust-lattice waves [4] and m_0 stands for the equilibrium magnetic moment of the grains. The relation (9) is similar to the dispersion dependence for the vertical vibrational mode found in the sheath region [3]: the wave is characterized by the optical dispersion in the long-wavelength range. The main difference is in the increase of the total characteristic frequency of vertical vibrations $\Omega_v^2 = \Omega_m^2 + \Omega_E^2$ for positive Ω_m^2 or its decrease if $\Omega_m^2 < 0$. It is even possible for Ω_v^2 to become negative ($\Omega_E^2 < |\Omega_m^2|$) and then the vertical mode reveals instability due to magnetic interactions. An opposite situation is also possible, when the magnetic term Ω_m^2 exceeds the absolute value of negative Ω_E^2 and thus plays the stabilizing role. When particle levitation occurs mainly due to the magnetic force (3) outside the sheath region, $\Omega_E^2 \to 0$, and then only the term proportional to $\Omega_\perp^2$ remains from the electrostatic interactions of dust charges in Eq. (9) describing a vertical dust-lattice mode. Finally note, that just as in the case of vertical vibrations of a single particle, the measurements of the wave dispersion $\omega(k)$ corresponding to Eq. (9) give an opportunity to estimate a few important complex plasma parameters: an equilibrium charge in the pre-sheath or bulk plasmas (via determining $\Omega_\perp$), magnetic moment of the grain (magnetic permeability), or even the vertical profile of $(QE)'$ (by means of estimations of Ω_E^2 at different values of the magnetic field).

Conclusions

We have demonstrated the appearance of a novel type of vertical vibrations of a single magnetized particle in discharge plasmas when an inhomogeneous external magnetic field is applied. Such vibrations can be stable or unstable depending on the distribution of the magnetic field inside the discharge chamber. Moreover it is shown that the vertical oscillations of a one-dimensional string of grains supported by the magnetic force give rise to a new low-frequency mode, which is characterized by the optic-like dispersion when the wavelength far exceeds the intergrain distance. The identifications of a vertical mode can provide a tool for determining the electric field profile, particle charge or magnetic moment of the grains.

References

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