31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, O-4.19 (2004)

Quasi-linear model of non-local magnetic field generation in laser-plasmas

R.J. Kingham[1], A.R. Bell[1]

1. Plasma Physics Group, Imperial College London, London SW7 2AZ, UK

A new magnetic field generation mechanism in laser-plasmas was recently reported [1]. This source, which was first seen in Vlasov-Fokker-Planck simulations using the simulation code IMPACT, works in a semi-collisional regime in contrast to conventional mechanisms such as the $\nabla n_e \times \nabla T_e$ source (collisional) or ponderomotive mechanisms (collisionless). In essence it is a non-local analogue of the $\nabla n_e \times \nabla T_e$ mechanism but can operate without the need for a density gradient. Analytical models are being developed to further our understanding of how this B-field source works and also to identify the key parameters and their scalings. One model predicts the early-time growth (i.e. over the first few collision times) of B-field from arbitrary temperature, density and Z (degree of ionization) profiles. The newest model employs a quasi-linear like approach and is valid for arbitrarily long times. It applies to pairs of temperature perturbations (with n_e and Z uniform) and shows that non-parallel perturbations with *different* wavelengths drive B-field generation. Although this quasi-linear model doesn't include heating – it is developed in terms of decaying temperature perturbations – we find that it does a very good job at describing B-field growth seen in simulations where the plasma is heated.

The $\nabla n_e \times \nabla T_e$ or thermoelectric mechanism is a well known source of B-fields in laser-plasmas under collisional conditions [2]. Strong thermoelectric magnetic fields are believed to be responsible for the hot spots measured in the (ionized) gas fill of hohlraums [3] since they suppress thermal transport. The thermoelectric B-field mechanism originates from the curl of the ambipolar electric field that maintains quasineutrality, i.e., $\underline{E} = -\nabla P_e/(en_e)$, where $P_e = n_e T_e$ is the electron pressure. The pressure gradient is one term appearing in Ohm's law, which describes momentum balance of the electron fluid. The local approximation (restricting f_0, the isotropic part of the electron distribution function, to a Maxwellian; $f_0 = f_M$) is conventionally made when obtaining the versions of Ohm's law (e.g. Braginskii s version) from the Vlasov-Fokker-Planck (VFP) equation which are then used to derive the $\dot{\underline{B}} = -(k_b/en_e)\nabla n_e \times \nabla T_e$ source. By solving the Vlasov-Fokker-Planck equation directly and thereby dispensing with the local approximation we have found new B-field source terms which are non-local analogues of the $\nabla n_e \times \nabla T_e$ mechanism. It is well known that non-local effects play a crucial role in determining heat flow down steep temperature gradients; they explain the severe reduction of heat flow compared to the Spitzer-Härm value (which uses the local approximation) when the temperature scale length approaches λ_c the collisional mean-free-path. We find that non-local effects are also relevant to magnetic field generation and in this context they also become important when the characteristic temperature, density and/or Z scale-lengths approach λ_c. Such conditions are found, e.g., in the gas fill of hohlraums where the high temperature ~ 5keV plus low density $\sim 10^{21}\text{cm}^{-3}$ and Z result in a mean-free-path (for thermal electrons) that is a substantial fraction of the dimensions of the hohlraum container.

Kinetic simulations of a uniform density plasma undergoing strong heating, carried out using the VFP code IMPACT [4], have shown that magnetic fields strong enough to seriously affect heat flow out of the heating region can spontaneously grow via the non-local mechanism. In [1] heating with a rippled profile (described by $1 - \tanh[(x - \{x_o + \Delta\cos(ky)\})/L])$ was applied at the left edge of a computational domain of size $x_{len} = 12\lambda_c$ by $y_{len} = 2\lambda_c$. In [5] the degree of ionization was modulated with the same rippled profile and the heating was planar instead (with $x_{len} = 60\lambda_c$ by $y_{len} = 10\lambda_c$). In both cases the plasma became magnetized with $\omega_{ge}\tau_c$ reaching values of 10 in places. The absence of anisotropic pressure (IMPACT uses the diffusion approximation), rules out mechanisms like Weibel as the cause of the B-fields observed.
The general basis for this non-local B-field source can be understood by obtaining a more general version of Ohm's law that does away with the local approximation. Under the assumptions/simplifications used in the simulations, Ohm's law becomes $e\underline{E}/m_e = -\nabla(n_e\langle v^5\rangle)/(6n_e\langle v^3\rangle)$ when the current and magnetic field are negligible (as occurs initially). This is valid for arbitrary f_0 and the moments are defined as $n_e\langle v^m\rangle = 4\pi\int_0^\infty f_0 v^{2+m}dv$. The curl of the electric field therefore involves the cross product $\nabla(n_e\langle v^3\rangle) \times \nabla(n_e\langle v^5\rangle)$. When $f_0 \to f_M$ the moments become $\langle v^m\rangle \propto T^{m/2}$ and the familiar result, $\nabla n_e \times \nabla T_e$, is recovered. But if f_0 is not Maxwellian a B-field can be generated even when $\nabla n_e = 0$ if the gradients in the $\langle v^3\rangle$ and $\langle v^5\rangle$ moments are not parallel. Non-local transport which leads to non-Maxwellian f_0 can achieve this.

The 'early time model' [5] describes this distortion of f_0 due to non-local transport of electrons and the subsequent spontaneous generation of B-field. We have developed an analytical expression for the magnetic field that is generated at early times by prescribed, initial, electron temperature, density and Z profiles. When the density and temperature are uniform

$$\frac{e\ddot{\underline{B}}}{m_e} = -\left(\frac{\lambda_c^4}{\tau_c^3}\right)\frac{\nabla T}{T} \times \left[154\,\frac{\nabla(\nabla^2 T)}{T} + 620\,\frac{\nabla(\,|\nabla T|^2\,)}{T^2}\right] \tag{1}$$

where $\lambda_c \propto T_e^2/Zn_e$ and $\lambda_c = v_t\tau_c$ as usual and the numerical coefficients are given to 3s.f. The model assumes that f_0 starts out as Maxwellian (with the relevant local density and temperature) at $t = 0$. Other assumptions are no time dependent ionization, no ion motion, no electron inertia and only e–i collisions. It is valid in the limit when B and j are negligibly small. Its range of validity extends over the first few collision times only. Next, adding in spatially varying Z introduces 4 extra terms involving factors like $(\nabla Z/Z) \times (\nabla T/T)$ (see [5]). Finally introducing density gradients reveals another 14 terms with factors like, e.g., $(\nabla n/n) \times (\nabla T/T)$ and $(\nabla n/n) \times (\nabla Z/Z)$. Note that a similar expression can be obtained in the local limit but has 9 terms, none of which vary like $(\nabla T/T)\times$ or $(\nabla Z/Z)\times$, and the numerical coefficients are 4–15 times smaller.

The quasi-linear model is based on linearization of the f_0 and f_1 VFP equations for harmonic, decaying temperature-perturbations (but uniform density and Z) with $\delta T_j \propto \cos(\underline{k}_j \cdot \underline{x}) \exp(-\gamma_j t)$. Electron-electron collisions are included in the evolution equation for f_0 but we use a Krook collision operator, $(\partial_t f_0)_c = -\tilde{\nu}_{ee}/v^p(f_0 - f_M)$ (where p can be changed to specify how collision rates scale with velocity) rather than the full integro-differential operator (involving Rosenbluth coefficients etc.) used in IMPACT. The first order f_0 and f_1 equations are combined and using two appropriate constraints (usually $j = 0$ and $n = 0$ to first order in δT_j), we are able to obtain a dispersion relation which relates the decay rate γ_j to that perturbation wavenumber k_j. This shows that $k\lambda_{ed}$ is the key parameter where $\lambda_{ed} = \sqrt{\lambda_{ee}\lambda_{ei}}$ is the electron delocalization length [6]. Proceeding onto the second order f_1 equation it is possible to derive the following expression for non-local B-field growth

$$\underline{\dot{\omega}} = \tau_c^{-2} \psi_{NL} \; (\underline{k}_1 \lambda_c \times \underline{k}_2 \lambda_c) \left(\frac{\delta T_1}{T}\right) \left(\frac{\delta T_2}{T}\right) \sin(\underline{k}_1 \cdot x) \sin(\underline{k}_2 \cdot x) \tag{2}$$

where ψ_{NL} is a function that essentially encapsulates the degree of non-locality and is a (complicated) function of $k_1\lambda_{ed}$ *and* $k_2\lambda_{ed}$. It turns out that $\psi_{NL} \to 0$ when $k_2\lambda_{ed} = k_1\lambda_{ed}$. Additionally, it is immediately apparent from equation (2) that the temperature perturbations must not be parallel either. Figure 1 shows how ψ_{NL} varies with $k_1\lambda_{ed}$ when $k_2\lambda_{ed} = 2k_1\lambda_{ed}$. It is apparent that degree of non-locality is within a factor of ~ 3 of the optimum over a wide range, $0.01 < k_1\lambda_{ed} < 1$ (i.e. $6 < \lambda_1/\lambda_{ed} < 600$). Figure 1 (left) shows that the quasi-linear model (with its simplified treatment of e–e collisions) recovers the essence of the non-local source. Furthermore if *heating* is applied with a dual $1 + \cos(\underline{k} \cdot x)$ spatial dependence (right figure) instead of starting with temperature perturbations and letting them decay, the ψ_{NL} parameter measured in VFP simulations is still in good agreement with the quasi-linear model. Therefore the mechanism is robust. The blue curve on the right figure corresponds to $\lambda_1 = 380\lambda_{ed}$, $\lambda_2 = \lambda_1/2$, $Z = 10$, and heating at a (peak) rate (per mode) that raises T_e by the initial temperature T_o in 10^4 e–i collision times (τ_{ei}). Time progresses along the curve starting at the lower left. The non-locality rapidly develops (the 'stem' of the curve) and then over time the system gradually heats

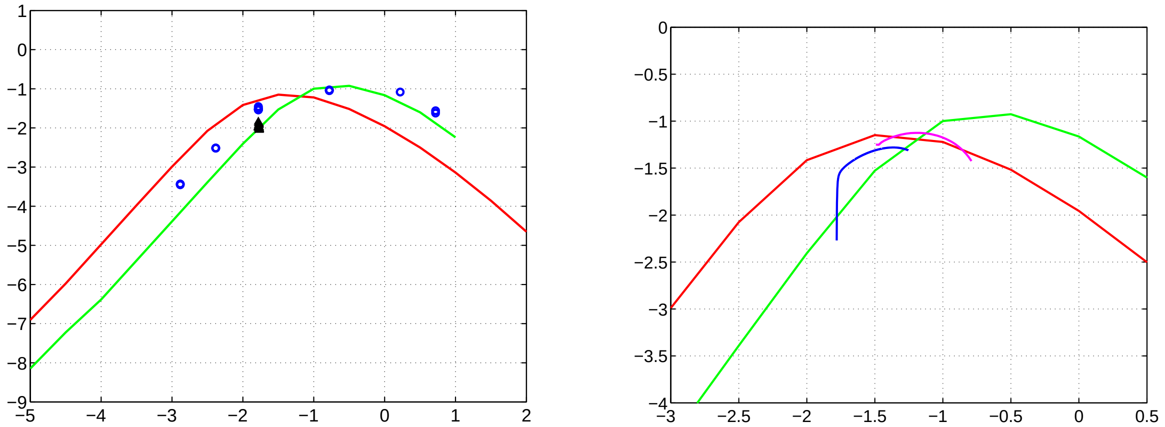

Figure 1: Scaling of the non-locality parameter for B-field generation, $\psi_{NL}(k_1\lambda_{ed}, k_2\lambda_{ed})$, with temperature perturbation wavenumber $k_1\lambda_{ed}$ (when $k_2\lambda_{ed} = 2k_1\lambda_{ed}$). Left: quasi-linear model with $\nu_{ee} \propto v^{-3}$ (red curve) and $\nu_{ee} \propto v^0$ (green), VFP simulations with full e–e collision operator (blue points) and the simplified Krook collision operator with $\nu_{ee} \propto v^0$ (black points). Right: shows ψ_{NL} measured from two VFP simulations of *heated* plasma (blue and magenta curves) next to the same quasi-linear results. The x and y scales are logarithmic (base 10).

up and $k\lambda_{ed}$ increases (the 'cap' of the curve) since $\lambda_{ed} \propto T^2$. The simulation ends after $8500\tau_{ei}$. The magenta curve corresponds to $\lambda_1 = 200\lambda_{ed}$, $\lambda_2 = \lambda_1/2$, $Z = 1$, $t_{max} = 450\tau_{ei}$ and heating at T_o per $180\tau_{ei}$. For very strong heating rates approaching T_o/τ_{ei} we expect significant departures from the quasi-linear model. Finally, it should be said that simple scaling arguments suggest that non-local B-field generation by 2-δT_e will exceed the (local version of the) $\nabla n_e \times \nabla T_e$ source when $L_T < L_n \psi_N$ where $L_T = |\nabla T/T|$. Thus for sufficiently compact hot spots where the main Fourier components fall within $0.01 < k_1\lambda_{ed} < 1$, non-local B-field generation is expected to exceed the conventional $\nabla n_e \times \nabla T_e$ source.

This work is supported by the U.K. EPSRC under Grants No. GR/M38988 and GR/R71979.

[1] R. J. Kingham and A. R. Bell, Phys. Rev. Lett. **88**. 045004 (2002)

[2] J. A. Stamper *et al.* Phys. Rev. Lett. **26**, 1012 (1971).

[3] S. H. Glenzer *et al.* Phys. Plasmas **6**, 2117 (1999).

[4] R. J. Kingham and A. R. Bell, J. Comput. Phys. **194**, 1 (2004)

[5] R. J. Kingham *et al.* CLF Annual Report 2002/2003, pg. 86 (www.clf.rl.ac.uk/Reports/2002-2003/contents.htm)

[6] E. M. Epperlein Phy. Rev. Lett. **65**, 2154 (1990).