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Scaling and a Fokker-Planck model of the fast and slow solar wind as seen by WIND spacecraft.

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Introduction

Statistical properties of velocity field fluctuations recorded in fluid experiments and in solar wind observations exhibit a range of similarities [2, 16]. Common features found in these fluctuations include fractal or multi-fractal scaling[1] and the clear departure of their Probability Density Function (PDF) from the Normal distribution when small spatial scales are considered[5]. The approach is then to treat the solar wind fluctuations as arising in a manner similar to that of hydrodynamic turbulence [4, 15, 6].
The observed statistical features of turbulent systems can not be derived from the original Kolmogorov theory of turbulence[10] with self-similar energy cascade. Indeed, the multi-fractal statistics can only be obtained when self-similarity of the cascade is broken by the introduction of intermittency[11]. A number of models has been proposed to incorporate the intermittency in velocity differences[3, 13]. These predict a functional form of the fluctuation PDFs based on a number of assumptions about the energy transfer rate that can not be easily verified experimentally. Complimentary way of modeling turbulence is to look for the set of variables that are self-similar and as such can offer a simplified description of the plasma flows. In this context it is important to identify these quantities and verify that their behavior is distinct from that of multi-fractal parameters. Recently systems has been identified where statistical intermittency coincides with statistical self-similarity in fluctuations of certain quantities which also exhibit leptokurtic PDFs[12, 7].
Here, we will apply two model-independent and generic techniques to extract scaling properties of fluctuations in the kinetic energy density, ρv^2 directly from the data. The aim is to determine a set of plasma parameters that exhibit statistical self-similarity and to verify the nature of the PDF for their fluctuations. We find that the PDFs of fluctuations in ρv^2 exhibit self-similar scaling, when conditioned to 10 standard deviations, in the slow wind while fast wind fluctuations do not exhibit scaling. The self-similarity of slow wind fluctuations leads immediately to a Fokker-Planck approach.

The Dataset and Methods

The time series investigated here was collected at the L1 point using the SWE instruments of the WIND spacecraft. We will examine fluctuations in the kinetic energy density defined as $\delta[\rho v^2(t,\tau)] = \rho v^2(t+\tau) - \rho v^2(t)$, where τ is a temporal scale. The slow solar wind is defined by condition $v_{sw} \leq 500$ km/s and all data with $v_{sw} > 500$km/s is taken as a fast wind. For the slow (fast) solar wind stream we require both $x(t+\tau)$ and $x(t)$ to have speeds below (above) the threshold.

Generalized structure functions (GSF) S_m can be defined for fluctuations $\delta(\rho v^2)$ as $S_m(\tau) \equiv \langle|\delta(\rho v^2)|^m\rangle$. When these moments exhibit scaling with respect to τ we have $S_m \propto \tau^{\zeta(m)}$. If $\zeta(m) = \alpha m$ (α constant) then the time series is self-similar with single scaling exponent α. Recently, conditioned GSFs, S_m^c, have been used to quantify the impact of intermittency on fluctuations of different sizes [16]. Practically, the method relies on eliminating the largest fluctuations, where the statistical errors are the greatest. In our case, this threshold will be based on the standard deviation of the fluctuation time series for a given τ, $A(\tau) = 10\sigma(\tau)$.

Generalized structure functions can be related to the PDF rescaling technique [12, 7]. If the process under investigation exhibits statistical self-similarity on some temporal scales than PDFs $P(\delta x, \tau)$ can be collapsed onto a single curve P_s using the formula $P(\delta x, \tau) = \tau^{-\alpha} P_s(\delta x \tau^{-\alpha})$. It can be shown that, for self-similar fluctuations, scaling exponent obtained from structure functions and that required to perform successful PDF rescaling are identical[8]. The fluctuation PDFs are computed with non-overlapping intervals of while structure functions use overlapping intervals for better convergence of higher moments.

Results and discussion

We start the discussion of our results with comparison of the second order conditioned structure functions S_2^c of the kinetic energy density fluctuations in slow and fast solar wind. Figure 1 shows a log-log plot of standard deviation $\sigma(\tau) = [S_2^c(\tau)]^{1/2}$, with τ. The slope of fitted line provides an estimate of the Hurst exponent $H_\sigma = \zeta(2)/2$. The slow solar wind fluctuations in ρv^2 exhibit scaling up to ~ 20 hours with $H_\sigma = 0.37 \pm 0.02$ while the fast stream data do not show a clear region of scaling. The main panel of figure 2 shows scaling exponents $\zeta(m)$ of structure functions with order $0 \leq m \leq 6$. The inset shows slopes of fitted lines for S_1^c to S_6^c where we recover a family of straight up to order $m = 6$. The good quality of linear fits to $S_m^c(\tau)$ is the main indication of successfully recovered scaling after the conditioning process. When conditioned to $10\sigma(\tau)$, exponents $\zeta(m)$ can be approximated by the linear function $\zeta(m) = m\alpha$ with $\alpha = 0.32 \pm 0.03$. Although one could, in principle, fit a very weak multi-fractal model to those points, the straight line fit is within the errors and thus captures the essential behavior of the data. The mono-scaling of the slow solar wind fluctuation in ρv^2 can then be verified by applying the PDF rescaling, with the rescaling exponent obtained

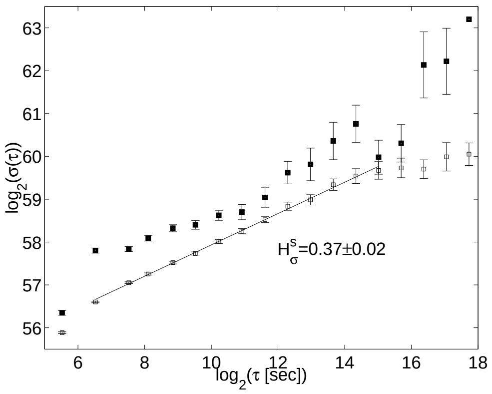

Figure 1: Standard deviation scaling of the ρv^2 for slow (□) and fast (■) solar wind.

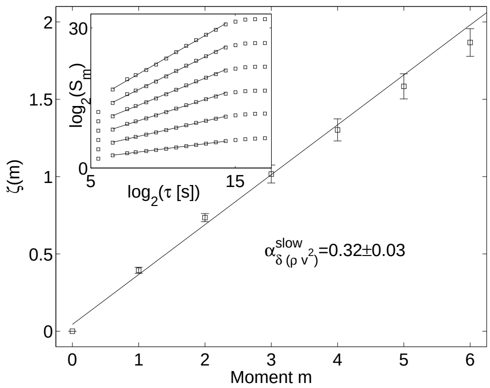

Figure 2: Conditioned structure functions of fluctuations in ρv^2 for the slow solar wind.

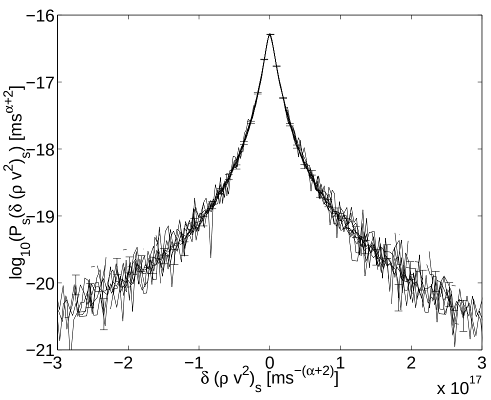

Figure 3: One parameter rescaling of the fluctuation PDFs for the slow solar wind ρv^2 with $\alpha = H = 0.37$. Curves represent temporal scales between ~ 2 minutes and ~ 20 hours.

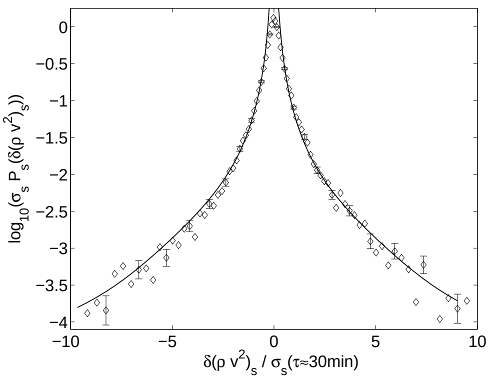

Figure 4: Rescaled normalized PDF for $\delta(\rho v^2)$ on temporal scale $\tau \approx 30$ minutes in slow solar wind. The solid line shows solution (1) with parameters given in the text.

from the GSF analysis. In practice, we found that the Hurst exponent $H_\sigma = 0.37$ gives the best collapse of the PDFs. Figure 3 show these collapsed curves for temporal scales from ~ 2 minutes to ~ 20 hours. The χ^2 test was applied to these collapsed PDFs by comparing the PDF for $\tau = 2$ minutes with all other curves. All collapsed curves lie within $3-5\%$ error band for the slow solar wind ρv_s^2 fluctuations. The self-similar scaling of the fluctuation PDF allows the development of a Fokker-Planck (F-P) model for the PDF dynamics[7]. The model predicts the following functional form of the PDF[7]:

$$P_s(\delta x_s) = \frac{a_0}{b_0}\frac{C\exp\left(-\frac{\alpha^2}{b_0}(\delta x_s)^{1/\alpha}\right)}{|\delta x_s|^{a_0/b_0}} \times \int_0^{\delta x_s} \frac{\exp\left(\frac{\alpha^2}{b_0}(\delta x_s')^{1/\alpha}\right)}{(\delta x_s')^{1-a_0/b_0}} d(\delta x_s') + k_0 H(\delta x_s), \quad (1)$$

where k_0 is a constant and $H(\delta x_s) = [1/(|\delta x_s|)^{a_0/b_0}] \times \exp\left(-\alpha^2(|\delta x_s|)^{1/\alpha}/b_0\right)$. This solution captures both the self-similar character of fluctuations and statistical intermittency revealed through the "heavy tails" of the PDF. Figure 4 shows PDF for slow wind fluctuations in ρv^2 on temporal scale of ~ 30 minutes. The solid line represents solution (1) obtained with parameters $a_0/b_0 = 2.0$, $b_0 = 10$, $C = 0.00152$, $k_0 = 0.0625$ and $\alpha = 0.37$. In the model we assumed that scaling $\tau^{-\alpha}$ extends to the smallest fluctuations and this gives a singularity in the solution P_s as $\delta x_s \rightarrow 0$. This singularity, however, is integrable so that $\int_{-\infty}^{\infty} P_s d(\delta x_s)$ is finite. We see that the functional form (1) fits slow wind fluctuations quite well.

Summary

We have examined differences in scaling properties of the fluctuations in kinetic energy density ρv^2 for slow and fast solar wind. We found scaling in structure functions of the slow solar wind that extended to about $10 - 20$ hours. Fast stream fluctuations do not exhibit such scaling. From these structure functions we have obtained a scaling exponent α and we used it to perform PDF rescaling analysis which, for slow solar wind gives a good collapse of the curves. We also considered a possible model for the self-similar slow solar wind fluctuations based on the Fokker-Planck equation. We have compared functional forms for the PDF, predicted by this model, to an experimental PDF of the kinetic energy density fluctuation. It appears that the Fokker-Planck gives a good approximation of the slow solar wind fluctuations.

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