

Vortex Structures in Complex Plasmas

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Complex plasmas are formed when micrometer sized particles are introduced to the plasma environment. These macro-particles develop a charge, due to interactions with the ionized plasma species and the complex interplay of forces in the system can lead to the excitation of various dynamical phenomena. The excitation of vortex structures has been observed experimentally in strongly coupled, non-uniform environments [1] and this study outlines the numeric simulation of these structures based on a fluid model, treating the macro-particles as an electrical fluid. A fully Eulerian code was developed using the standard equations of fluid dynamics providing a 2D simulation of the plasma environment which incorporated the influence of external fields in addition to the parameters associated with a quasi-compressible electrical fluid.

An electro-magnetic fluid is subject to a combination of the Lorentz force and an additional force due to collisions with the plasma species, given by:

$$\mathbf{F} = \mathbf{F}_{\text{lorentz}} + \mathbf{F}_{\text{collisions}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \sum n_{\alpha} \nu_{\alpha, \beta} (\mathbf{v}_{\alpha} - \mathbf{v}_{\beta})$$

where α, β indicate the plasma species.

Due to the large mass of the dust particles $m_{\text{dust}} \gg m_{\alpha, \beta}$, the force due to collisions can be neglected. The additional force can be further simplified noting that no external magnetic fields were applied in the experimental apparatus and experimental observation has shown that the velocity of the dust is too low to generate significant magnetic fields. Thus, the motion of the dust can be approximated by that of an electrical fluid, and hence the vorticity-transport equation must be written in the form

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla) \omega = \nabla \times \rho (q_{\text{dust}} \mathbf{E} + \mathbf{F}_{\text{external}}) + \nu \nabla^2 \omega$$

where q_{dust} is the charge on dust particles and ρ is the dust number density

The term $\nabla \times \rho (q_{\text{dust}} \mathbf{E} + \mathbf{F}_{\text{external}})$ is a source term and can therefore result in the creation of vorticity given the right parameters. In order to study the excitation of vortex structures the plasma simulation was operated with an initial vorticity distribution of zero, requiring the appropriate plasma conditions to generate vortices. The specification of these additional parameters represents the interaction between the dust and the plasma and a suitable model for these quantities based on experimental data was developed.

The effect of dust charge gradients on vortex structures was initially investigated by imposing simple relations, including constant, linear and quadratic dust charge functions. To further simplify this initial study, only the probe potential was considered and an arbitrary value for the equilibrium dust charge was taken, allowing the qualitative investigation of dust charge function dependencies. Constant number density and temperature distributions were also assumed. It was shown that the application of a constant dust charge did not generate vortex structures irrespective of the value of charge. It was also shown that vortex structures were generated by both the linear and quadratic dust charge functions and that the characteristics of these vortices were related to the dust charge gradient irrespective of the value of charge (See Figure 1).

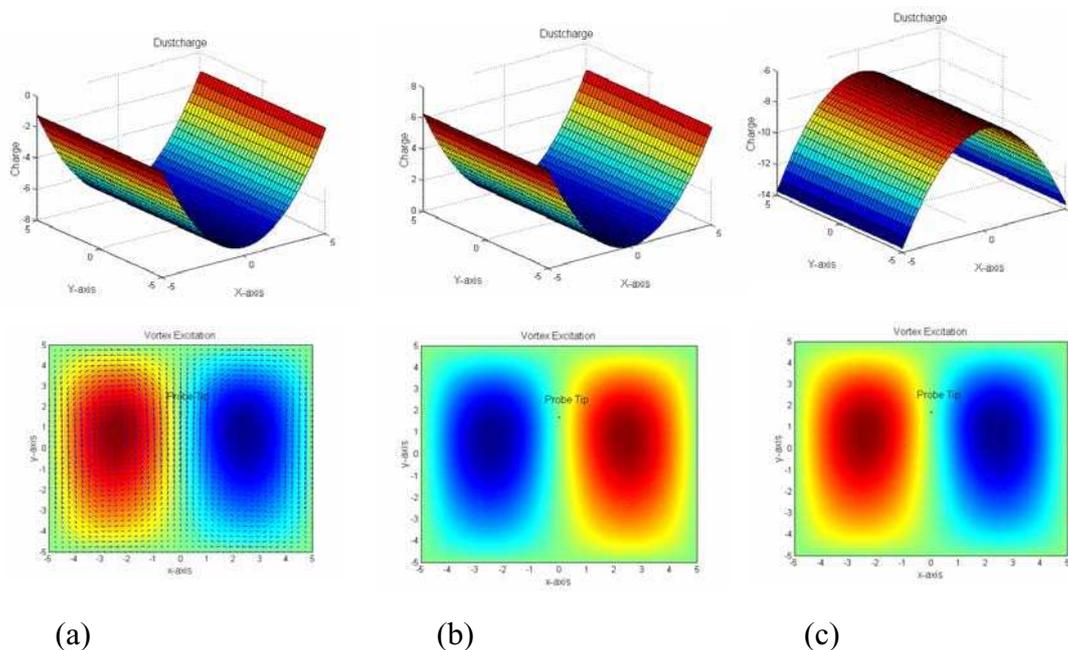


Figure 1. Vortex excitation with a quadratic dust charge function, showing that the rotation direction is determined by the dust charge gradient (velocity field shown). The dust charge function used in (b) differs from that in (a) only in the addition of a constant and shows that the vortex characteristics are independent of charge.

These results can be explained by an analysis of the source term in the vorticity -transport equation, $\nabla \times \rho(q_{dust} \mathbf{E} + \mathbf{F}_{external})$, which reduces to, $\nabla \times (-q_{dust} \nabla \phi)$, for these models. The case of constant charge reduces this term to zero, as $\nabla \times (\nabla \phi)$, is zero for all ϕ . Conversely, both the linear and quadratic dust charge dependencies result in a non-conservative field and hence non-zero curl.

A more realistic model for the dust charge was obtained using the results of OML theory. Equating the orbit limited currents of the plasma species; the dust charge gradient using an argon backing gas can be approximated:

$$\Delta_{\rho} q_{dust} \approx -0.26 \langle q_{dust} \rangle \frac{\delta\rho}{\rho}$$

If the plasma species exhibit Maxwellian energy distributions, this equation can be further approximated using $\delta\rho \approx \phi$, giving $q_{dust}(x, y) \propto \phi(x, y)$. This approximation is often used in numeric simulations in the modified form $q_{dust} \propto \phi^{0.5-1.5}$, and various dust charge functions in this range were investigated. Constant number density and temperature distributions were also assumed for this model. It was shown that this dust charge function did not lead to the generation of vortices irrespective of the characteristics of the potential. This result can be explained by expanding the source term for this model:

$$\nabla \times \rho (q_{dust} \mathbf{E} + \mathbf{F}_{external}) = \nabla \times \phi^a \nabla \phi$$

where: $0.5 < a < 1.5$. Expanding this operator in 2-D we obtain:

$$\begin{aligned} \nabla \times \rho (q_{dust} \mathbf{E} + \mathbf{F}_{external}) &= \nabla \times \phi^a (x, y) \left(\frac{\partial \phi(x, y)}{\partial x} \hat{x} + \frac{\partial \phi(x, y)}{\partial y} \hat{y} \right) \\ \text{by the chain rule:} &= \nabla \times \frac{1}{a+1} \left(\frac{\partial \phi(x, y)^{a+1}}{\partial x} \hat{x} + \frac{\partial \phi(x, y)^{a+1}}{\partial y} \hat{y} \right) \\ \text{resulting in a conservative field:} &= \frac{1}{a+1} (\nabla \times \nabla \phi^{a+1}) = 0 \quad \forall \phi \end{aligned}$$

Using this result, it can be seen that within the bounds of this model, vortex structures cannot be generated when approximating the dust charge distribution to the plasma potential. Based on OML theory, this requires a dust charge gradient to be imposed which results in a non-conservative field either via a variation in number density or temperature gradient.

The variation in number density in the region surrounding the pin electrode was investigated as a possible mechanism for vortex generation. Taking:

$$\Delta_{\rho} q_{dust} \approx -0.26 \langle q_{dust} \rangle \frac{\delta\rho}{\rho}$$

The approximation, $\delta\rho \approx \phi$, can be improved considering the maximum variation for these quantities observed experimentally.

Let $\delta\rho \approx \beta\phi$ and considering the maximum variations, $\Delta\rho \approx 8 \times 10^{13}$, $\Delta\phi \approx -30\text{V}$

$$\begin{aligned} \Delta\rho &\approx \beta \Delta\phi \\ \beta &\approx -2.67 \times 10^{12} \end{aligned}$$

which gives $\delta\rho \approx -2.67 \times 10^{12} \phi$. Using this result, the dust charge gradient can be approximated as:

$$\Delta_{\rho} q_{dust} \approx -0.26 \langle q_{dust} \rangle \frac{2.67 \times 10^{12} \phi}{\rho}$$

Neglecting temperature-induced variations, this leads to a dust charge function:

$$q_{dust} \approx \langle q_{dust} \rangle \left(1 + 6.94 \times 10^{11} \frac{\phi}{\rho} \right)$$

Using this method, the following results were obtained for the maximum velocity and average angular frequency:

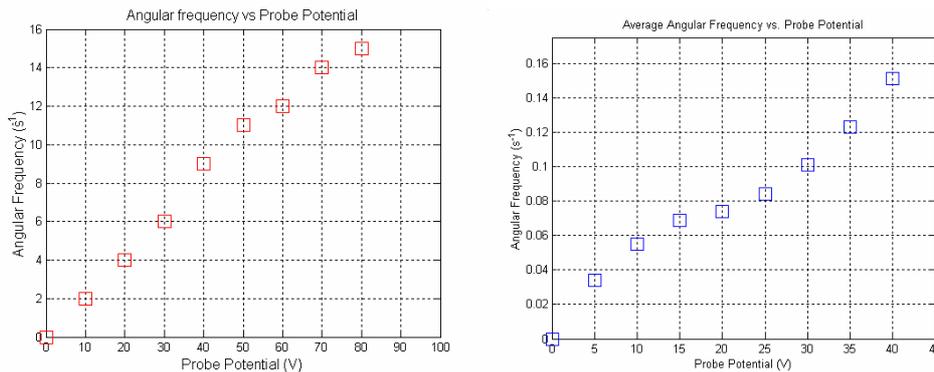


Figure 2. The experimental data (a) and simulated results (b) for the average angular frequency of the structure vs applied potential.

It has been shown within the limitations of this model that the excitation of vortex structures in a complex plasma environment requires the curl of the force distribution to be non-zero. It was shown that this condition could be satisfied by simple dust charge functions, but that those investigated did not lead to a stable equilibrium state. Using these simple functions, it was also shown that the characteristics of the vortex structure generated were determined by the dust charge gradient irrespective of the value of the dust charge. A model based on the variation in dust number density in the region of the probe was proposed and could provide a mechanism for vortex excitation as shown from our investigation. Structures produced by the proposed gradient were shown to establish a stable equilibrium state. Comparison with experimental results showed that the qualitative features can be reproduced, despite that some characteristics of the system were not predicted accurately.

References

1. A. Samarian *et al*, *Physica Scripta*, **T98**, 123 (2002).