

Ion viscous heating in a wire-array Z-pinch

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When a wire array Z-pinch implodes and stagnates on the axis the kinetic energy that originally is concentrated in the inward radial motion of ions, is thermalised on a time scale less than 10^{-10} sec. By equipartition the electrons are heated from a temperature of some tens of electron-volts to hundreds of electron-volts or higher on a time scale typically of 1 to 5 ns. This process is one of the controlling factors affecting the rise time-time of the radiation pulse¹, the other being the assembly time of the incoming unstable plasma shell. The rise in electron temperature causes further ionisation and intense line and recombination radiation. In tungsten wire arrays the radiated energy from the 11 MJ Z-accelerator at Sandia has been as high as 1.8 MJ^{2,3} which represents a 16% wall-plug to X-ray energy conversion efficiency in a 5ns FWHM pulse.

In this paper we examine some intriguing experimental results in stainless steel wire arrays. This material is selected in order to obtain K_{α} radiation requiring a higher electron temperature. Here it was found that the radiated energy could be 3 to 4 times that of the ion kinetic energy⁴ from the implosion. Whilst in the 2-D simulations employed to model the radiation – magnetohydrodynamics some pdV work by the $\underline{J} \times \underline{B}$ force occurs to deposit some further energy mainly in the form of compressive rather than resistive heating, it has been impossible to account for such a discrepancy in energy. Furthermore it is difficult to see how a quasi-pressure balance can occur at stagnation because of insufficient plasma pressure, though framing camera images indicate that the pinch radius is approximately constant at about 2mm for at least 5ns.

The problem of pressure balance, though a secondary issue, will give a clue as to the likely resolution of the problem, and we consider this first. For shot Z1141 on the Z-accelerator at Sandia a nested stainless steel wire array of $317.8 \mu\text{g}/\text{cm}$ consisting of an outer array diameter of 55mm, inner of 27.5 mm and a 2:1 mass ratio stagnates on axis at a current of 18MA. The measured electron temperature, T_e , at stagnation is found to be 3keV from both line ratios and continuum slope. If no axial magnetic field is present, pressure balance requires that the Bennett⁵ relation

$$\mu_0 I^2 = 8\pi N_i e (T_i + ZT_e) \quad (1)$$

holds, where Z is the ionic charge, I the current, and N_i is the number of ions per unit length (or line density). To satisfy this equation for fully stripped iron, $Z=26$, requires a value of (T_i+ZT_e) of 297 keV while ZT_e only accounts for 78 keV. A caveat should be made that there is strong evidence now^{6,7} that in wire-array implosions generally a significant fraction of the current and mass does not arrive immediately on axis at stagnation but resides in the trailing mass arising from axially non-uniform erosion⁸ and ablation of the wire cores and subsequent magneto-Rayleigh-Taylor instabilities. However simultaneous reductions in both I and N_i will almost certainly still maintain difficulties in satisfying eq.(1) unless T_i is very high (219 keV for our example). If the Bennett relation were not applicable because of generation of an internal axial magnetic field through the development of a helical $m=1$ unstable kink mode. There is some evidence of such a helical instability at later times in the discharge; indeed at this time the pitch of the helix is about 45° in agreement with flux and current conservation with a corresponding radius of the helix ~ 1.6 times the original pinch radius. However during the main 5ns FWHM soft X-ray radiation pulse the dominant mode as observed in framing camera images is a long wavelength $m = 0$ mode with $ka \sim 1$ where k is the axial wave-number and a is the pinch radius. Another possibility, quite apparent in simulations, is radiative collapse^{9,10}, but this also is not observed experimentally.

Turning to the primary issue of energy conservation, the only obvious extra source of energy available is the magnetic energy which is much greater by a factor of about $(\frac{3}{4} + \frac{3}{2} \ln(r_0/a))$ than the kinetic energy of implosion, where r_0 is the initial radius of the wire array. The mean drift parameters associated with the current flow give $v_d/c_s \approx 0.03$ and $v_d/v_{Te} \approx 5 \times 10^{-4}$ and so any significant anomalous resistivity associated with ion acoustic, lower hybrid or two stream instabilities is ruled out except much later in time and located in the necks of the $m=0$ instabilities at disruption. Besides, these microinstabilities would primarily heat the electrons. Spitzer-like Joule heating is too low and has a characteristic time of several microseconds. Several models have been proposed which have some success in higher N_i/I^2 situations; Peterson¹¹ has current reconnection with anomalous resistivity in the trailing mass leading to a subsequent inward cascade of implosions. Chittenden¹² has a similar model with a much reduced current in the stagnated plasma, and more recently¹³ with 3-D simulations allows for

significant kink ($m=1$) modes. Rudakov and Sudan¹⁴ postulated reconnection of plasma and current around a $m=0$ instability, the resulting bubble drifting in radially converting magnetic energy by pdV work plus aerodynamic resistance (viscosity). Velikovich et al¹⁵ further developed this for large-scale bubbles but assumed that anomalous resistivity would be effective. But none of these can explain the low N_i/Γ^2 results discussed here.

We propose a new mechanism which will give rapid conversion of magnetic energy to radiation via fine-scale $m=1$ interchange instabilities, viscous heating of ions and equipartitions of energy to the electrons. Short wavelength (high k) modes have the fastest growth rate and, depending on the saturation level of these modes, the highest viscous dissipation.

The growth rate γ of $m=0$ instabilities has a value given approximately MHD by

$$\gamma_{MHD} \approx c_A \sqrt{\frac{k}{a}} \quad (2)$$

where the precise value depends on the current distribution. In general the ions will be magnetised, i.e. $\Omega_i \tau_i > 1$ (typically where Ω_i is the ion cyclotron frequency and τ_i is the ion-ion collision time, and a full ion stress tensor¹⁶ has to be employed. The shortest wavelength mode will be determined by the larger of the ion Larmor radius and the value of k^{-1} which leads to a viscous Lundqvist number L_μ , defined by

$$L_\mu = \frac{2\rho(c_A^2 + c_s^2)^{1/2}}{(\frac{1}{3}\mu_i + \nu_1)k} \quad (3)$$

to have a value of 12. At a value of 1 compressional Alfvén waves are critically damped. No stability analysis with the full ion stress tensor has been undertaken for large ka , Cox¹⁷ having considered values below 10. We assume that the fastest growing mode is that with $L_\mu=2$.

To estimate the amplitude of the MHD instability, the nonlinear saturation of the mode will occur when $(\tilde{v} \cdot \nabla) \tilde{v}$ (the convective derivative) is comparable with $\gamma_{MHD} \tilde{v}$. Then the eddy velocity magnitude \tilde{v} will be $c_A / \sqrt{(ka)}$. Since we are dealing with $(ka) \geq 10^2$ it follows that the eddy velocity is a small fraction of the ion thermal speed. The fractional magnetic field fluctuation level will be even smaller at $(\tilde{v}/c_A)^2$. The viscous heating rate for the $m=0$ mode has a component that is not reduced by ion magnetisation because this mode is an interchange mode and is compressible. Equating the ion viscous heating rate per unit volume to the equipartition rate leads to

$$\rho \frac{c_A^2}{a} (c_A^2 + c_s^2)^{1/2} = \frac{3n_e}{2\tau_{eq}} (T_i - T_e) \quad (4)$$

where the equipartition time τ_{eq} is given by

$$\frac{1}{\tau_{eq}} = \frac{8\sqrt{2\pi m_e} e^{3/2}}{3m_i (4\pi\epsilon_0)^2} \cdot \frac{Z^2 n_i \ln\Lambda_{ei}}{T_e^{3/2}} \quad (5)$$

where n_e , n_i , m_e and m_i are the electron and ion number densities and masses. In terms of global quantities and for a parabolic density profile Eqs. (4) and (5) combine to give

$$(T_i - T_e) = 2.1 \times 10^{36} \frac{a I^3 T_e^{3/2} A^{1/2}}{Z^3 N_i^{3/2} \ln\Lambda_{ei}} \quad (6)$$

where A is the atomic mass number (55.8 for iron). This and Eq.(1) are consistent with T_i of 219keV and a pinch radius of , 3.6 mm for the conditions described above.

Recent experimental data of time-resolved spectra give strong support to the validity of the model. The Doppler width of the relatively weak Fe He- δ line, $1s^2$ - $1s5p$ at 8.49 keV gives ion temperature measurements of 310, 230 and 250 keV at peak X-ray power and 3 and 6ns later, with an uncertainty of 60keV. From soft X-ray images of the pinch, the axially averaged FWHM radius of the pinch is roughly constant over this time at just over 2mm.

In conclusion it appears that the ubiquitous MHD instabilities of the Z-pinch can be utilised to advantage to provide fast, viscous heating of ions to record temperatures of over 200kev. Furthermore the broadened spectral lines will permit a greater radiative power to occur.

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