31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, O-2.07 (2004)

Probabilistic transport models for fusion

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Introduction – Transport in fusion plasmas is commonly described using Fickian diffusive transport models, in which the (particle and heat) flux is a function of the local parameters (gradients). A large body of literature testifies to the existence of phenomena that are difficult to fit into this framework. Such phenomena include: Bohm scaling of confinement (approximately linearly proportional to the system size L, whereas standard diffusion would predict $\propto L^2$); power degradation (i.e. the confinement time decreases with increasing heating, P); profile consistency or stiffness of profiles, making the plasma profiles relatively insensitive to the power deposition, even leading to profile peaking during off-axis heating; and rapid phenomena in which the core plasma seems to respond almost instantaneously to e.g. edge cooling (much faster than the diffusive timescale permits). The common approach to the problem of modelling these phenomena is to include additional convective terms in the diffusive model. However, the required convective terms are usually much larger than the available physical mechanisms (e.g. the Ware pinch), and the required convection varies from case to case, so that this approach does not permit making accurate predictions.

Diffusion revisited – In this paper, we reconsider the foundations of the standard diffusive model in the framework of the Continuous Time Random Walk formalism [1,2]. As is well known, diffusion is based on Brownian motion, in which individual particles remain a certain (random) time at their current position and then take a (random) step. These waiting times and steps are drawn from probability distributions. Let $\psi(t-t')$ be the waiting time distribution, giving the probability that a particle, having arrived at position x' at time t', takes a step at time $t \geq t'$, and $p(x-x',x',t)$ the step distribution, giving the probability at time t that a particle takes a step from x' to x. It can then be shown [3] that a collection of such particles obeys the Generalized Master Equation:

$$\frac{\partial n}{\partial t} = \int_0^1 dt'\phi(t-t')\left[\int_{-\infty}^{\infty} dx'\,p(x-x',x',t)n(x',t)\right] - \int_0^1 dt'\phi(t-t')n(x,t') \tag{1}$$

where $n(x,t)$ is the particle density. The function ϕ is related to the waiting time ψ above [3]. Naturally, if the waiting time is chosen exponential, $\psi(t-t') = \exp[-(t-t')/\tau_D]/\tau_D$, and the step distribution Gaussian, $p(x-x',x',t) = \exp[-(x-x')^2/4\sigma^2]/2\sigma\sqrt{\pi}$, normal Fickian diffusion is recovered and Eq. (1) is reduced to:

$$\frac{\partial n}{\partial t} \cong \frac{\sigma^2}{\tau_D} \frac{\partial^2 n}{\partial x^2} \quad (2)$$

However, the CTRW formalism allows other distributions ψ and p. This immediately raises the question of whether the standard choice of exponential waiting time distributions and Gaussian step size distributions is always justified. The Gaussian distribution is the limit distribution of the sum of independent identically distributed (i.i.d.) random variables whose *individual distribution has finite width.* Removing the latter restriction, the sums of general i.i.d. variables are distributed according to the Lévy distribution. Thus, the Lévy distribution is a natural generalization to the Gaussian distribution. It is characterized, among other things, by a parameter α. The Lévy distribution at $\alpha = 2$ is just the Gaussian distribution, with an exponential tail for large x, $p(x) \propto \exp[-x^2]$, but for $0 < \alpha < 2$ the tail is algebraic, $p(x) \propto x^{-\alpha-1}$. This tail has profound consequences, since it leads to long-range correlations and general unusual ("non-local") behaviour.

The question whether Lévy distributions are relevant for transport in fusion plasmas can only be partially answered at this moment. Almost no direct experimental evidence is available (such as the measurement of long-range correlations). However, indirect evidence is plentiful, namely: the observed system size scaling of the confinement time; the need for unrealistically large convective terms to model experiments; and on-axis profile peaking during off-axis heating experiments. Furthermore, Lévy distributions have been observed in numerical simulations of resistive pressure-gradient driven turbulence using particle tracking methods [4], and we suggest that other simulations should be checked for their presence.

Toy model for transport – To test these ideas, we select an exponential waiting time distribution and a Lévy step distribution. Furthermore, we need a critical gradient mechanism to produce the required profile consistency. Thus, we design our 1-D, single-field toy model such that:

$|\nabla n| < [\nabla n]_{crit}$: "normal" transport, p = Gaussian

$|\nabla n| \geq [\nabla n]_{crit}$: "anomalous" transport, p = Lévy

A Generalized Master Equation can be derived for this model [3]:

$$\frac{\partial n}{\partial t} = \frac{1}{\tau_D} \left[\int_0^L dx' \, p[x - x', f(n(x',t))] n(x',t) \right] - \frac{n(x,t)}{\tau_D} + S(x) \quad (3)$$

including a local parameter dependence of the step distribution p and an external drive $S(x)$. This system will self-regulate its gradient around the critical value for a range of values of the drive, leading to stiff profiles. Fig. 1 shows the confinement time obtained from a power scan. It is seen how the critical gradient mechanism leads to power degradation. In addition, at low power the confinement time scales diffusively ($\tau \propto L^2$), whereas as high power the system scales anomalously ($\tau \propto L^\alpha$), which is reminiscent of Bohm/gyro-Bohm scaling behaviour. Fig. 2 schematically shows the same behaviour for various different choices of the system size L ($\propto 1/\sigma_2$). In the critical region between the two power extremes, the confinement time is seen to depend exclusively on the fueling rate S and the critical gradient, and not on either the step size or the waiting time. Since any (effective) diffusion coefficient must always depend on the step size and the waiting time, this means that the system behaviour cannot be described by an (effective) diffusion model in the critical region.

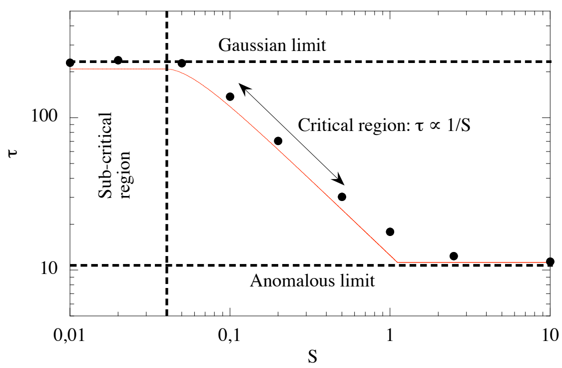

Fig. 1 – Power degradation.

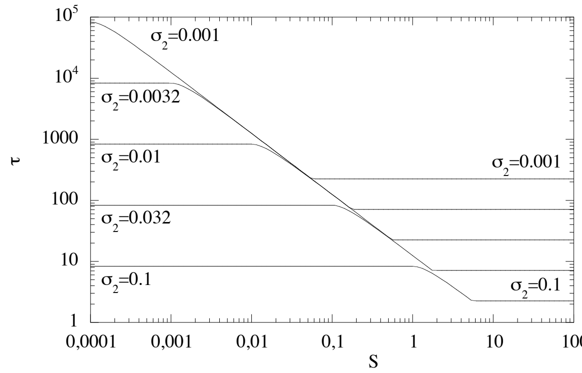

Fig. 2 – Scaling with system size.

The stiffness of the model also leads to the fast propagation of disturbances, e.g. a "cold pulse" (produced by a sudden reduction of n at the edge) was found to travel almost three orders of magnitude faster than what would be expected from a diffusive estimate [3].

A sensitive test of non-standard behaviour is also provided by an off-axis fueling experiment [5]. Figs. 3 and 4 show the response of the profile of n for two off-axis fueling amplitudes. At low S, the system is not very critical and responds almost diffusively; but at high values of S, the system is critical and the profile peaks, even though the source S is zero in the centre. The explanation for this behaviour is found in the existence of a significant fraction of particles in the region *at or outside* the heating zone that perform long-range steps due to the Lévy distributions, and end up in the central region.

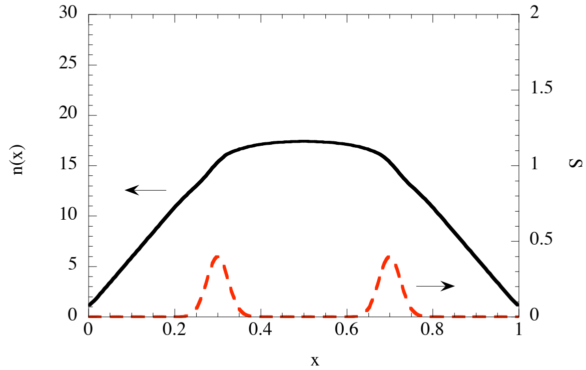

Fig. 3 – Off-axis fueling, low power.

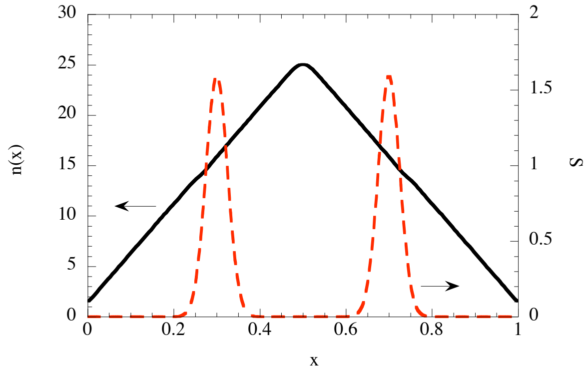

Fig. 4 – Off-axis fueling, high power

The amount of profile peaking depends on the inverse of the size of the central non-critical region, as shown in Fig. 5; or in other words, the peaking is reduced when the diffusive transport channel is enhanced. This is possibly in agreement with actual experiments in which the collisionality was varied [6].

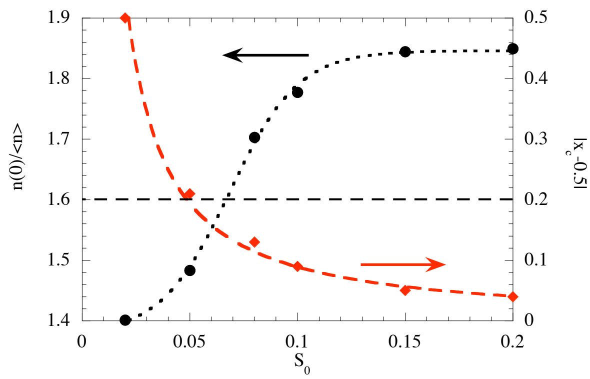

Fig. 5 – Profile peaking factor and width of central non-critical region as a function of heating.

Summary – In this paper, we have presented a straight-forward generalization of the common diffusive transport model. The generalization consists of allowing the particle step distributions to include Lévy distributions. Based on this idea, we have designed a numerical 1-D, 1-field toy model which incorporates a critical gradient mechanism to switch between normal (Gaussian) transport and anomalous (Lévy) transport. The model was shown to produce interesting behaviour: (1) stiff profiles, (2) power degradation, (3) system size scaling reminiscent of Bohm/gyro-Bohm behaviour, (4) rapid transport events, and (5) on-axis profile peaking during off-axis heating. This behaviour is in qualitative agreement with observations in fusion plasmas.

The model needs to be extended to at least two fields (*n* and *T*) before any modelling of actual experiments can be attempted; this work is in progress. An important message from this work is the following. Even assuming that Lévy distributions do indeed play an important role in the description of transport in fusion plasmas, it is still possible to model any individual experiment using the conventional description (based on effective diffusion and convection coefficients etc.) [3]. However, the Lévy distributions introduce a fundamental dependence of these effective parameters on the system size *L*. Therefore, the effective diffusion and convection coefficients *cannot be used* to predict the outcome of an experiment in a system of size *L'* ≠ *L*! The only correct way to generalize the results of the transport analysis to systems of any size is then to determine the probability distributions directly.

Due to the possibly important consequences of the presence of Lévy distributions for transport analysis, we would like to suggest that more experimental effort be dedicated to their detection.

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