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Spiral Modes in Astrophysical Plasma Disks and Twin Peak Quasi-Periodic Oscillations *

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Simplified General Relativity Effects

The tri-dimensional modes [1,2] that are likely to prevail in disk structures surrounding black holes are those whose growth rate is highest and are localized relatively close to the last stable orbit (a.k.a. ISCO). Therefore, we may consider the radius of the surface $R = R_0$ around which the prevailing mode is localized, as related to the radius of the last stable orbit, R_{Ls}. In particular, $R_0 = \alpha_0 R_{Ls}$ where $\alpha_0 \gtrsim 1$ is a finite parameter.

In order to formulate the theory of spiral modes in this region where general relativity effects are important, we have adopted the (pseudo-Newtonian) Paczynsky-Wiita gravitational potential to model the general relativistic effects for a non-rotating (Schwarzschild) black hole. This potential, which allows for the presence of a last stable orbit and the separation of the non-Newtonian expressions of the epicyclic frequencies and of the rotation frequency, is $\Phi_G^{eff} = GM_* / (R - R_c)$ where M_* is the mass of the central object and $R_c = 2M_* G/c^2$ is the Schwarzschild radius. Accordingly, the rotation frequency is $\Omega_c^2(R) \simeq GM_* \Big/ \left[R (R - R_c)^2 \right]$. Then $R_{sL} \simeq 3R_c$ and $\Omega_c^2(R_0) \simeq (GM_*/R_c^3) \Big/ \left[3\alpha_0 (3\alpha_0 - 1)^2 \right]$. We note that the observed High Frequency Quasi-Periodic Oscillations (QPO's) of X-ray radiation emission associated with black holes has been found [3] to scale as $1/M_*$. In addition to Ω^2, the other relevant quantity that enters the theory of unstable modes that are contained within the height of the disk [1,2] is $-R d\Omega^2/dR$. This becomes, $3\Omega_c(R_0)\left[1 - 1/(9\alpha_0)\right] \Big/ \left[1 - 1/(3\alpha_0)\right]$ for $R_0 \simeq \alpha_0 R_{Ls}$.

Elements for a Theory of High Frequency Quasi-Periodic Oscillations

Existing explanations of High Frequency QPO's from compact objects have shortcomings [4] that a theory based on the excitation of plasma modes co-rotating with the plasma [2] near a black hole can avoid. In particular, we propose that tri-dimensional spiral modes, whose excitation process involves the vertical structure of the disk and that can be fully described theoretically, be considered as the basis for a model of the origin of high frequency QPO's. This model has the following characteristics:

i. It gives a specific physical factor for the excitation of the collective plasma modes that result in a rotating source of enhanced radiation emission.

ii. It does not rely exclusively on considerations of single particle processes

iii. Plasma macroscopic parameters such as the vertical temperature and density profiles can have a substantial variability, and affect the excitation of the considered modes in addition to other factors that can justify the changes in the observed radiation emission over longer time scales than those of the mode frequency of oscillation.

iv. The frequency of the modes is tied to that of the rotation frequency of critical particle orbits around a black hole, as is reasonable to expect that the mode will correspond to the maximum realistic gradient of the rotation frequency and that this be strictly related to the radius of the last stable orbit.

In particular, we associate the local increase of radiation emission with the increase of the particle density resulting from localized tri-dimensional spirals that in the "linear" regime [2] can be represented by

$$\hat{n} \simeq \tilde{n}^0 \frac{z\Delta_z}{H_0^2} G_0^0(z) \exp\left[-\frac{z^2}{\Delta_z^2} - \frac{(R-R_0)^2}{\Delta_R^2} \right] \exp(\gamma_0 t)$$

$$\times \sin\left\{ k_R (R-R_0) - m_\phi \left[\Omega(R_0) t - \phi \right] \right\}. \qquad (1)$$

Here Δ_z represents the height over which the mode is localized vertically, Δ_R is the corresponding radial length, $k_R^2 \simeq k_0^2 \equiv -\left(d\Omega^2/dR\right) R \big/ \mathrm{v}_A^2$, $\mathrm{v}_A^2 = B_z^2 \big/ (4\pi\rho)$, $H_0^2 \equiv \left(-d \ln \rho / dz^2\right)^{-1}$, m_ϕ is the toroidal mode number, and γ_0 is the mode growth rate. In particular, $\Delta_R^2 \simeq -(\gamma_0/\Omega) R \big/ \left(k_R m_\phi\right)$ and $\Delta_z^2 \simeq H_0/k_0$. The function $G_0^0(z)\exp\left(-z^2/\Delta_z^2\right)$ represents the eigenfunction of the equation that gives the mode vertical profile [1,2].

The modulation of the radiation emission that reaches the observer has to be evaluated by extending the general relativistic theory developed in Ref. [5] for a "hot spot" model of QPO's and in Ref. [6] for a different model. A relevant analysis, to the theoretical model presented here is being undertaken by M. Bursa.

It is reasonable to consider that both axisymmetric [1] and tri-dimensional modes [2], associated with the combined effects of the rotation frequency gradient and the vertical pressure gradient, will be excited. Given the structure of axisymmetric modes that do not have a frequency of oscillation, we do not expect them to be suitable for a theoretical model of QPO's. When referring to the class of tri-dimensional spiral modes it is necessary to attempt to identify, on the basis of qualitative but plausible physical considerations, those that can prevail following their non-linear evolution. In this connection we notice that the width Δ_R over which a modes is localized decreases as $|m_\phi|^{-1/2}$. Therefore, modes that have values of m_ϕ close to a relatively large toroidal mode number m_ϕ^0 have about the same Δ_R widths.

Another point to consider is that all modes of the analyzed class, that are contained vertically within the disk, have about the same value of $|k_R|$. Therefore, a coupling between two modes can involve either of the two decay conditions

$$k_R)_1 + k_R)_2 \simeq \delta k_R)_{1,2}$$

$$\text{with} \;\; m_\phi)_1 / m_\phi)_2 < 0 \quad \text{and} \quad |\delta k_R|)_{1,2} < |k_R|)_1 \sim |k_R|)_2 \;,$$

$$k_R)_1 + k_R)_2 \simeq 2k_0 \;\; \text{with} \;\; m_\phi)_1 / m_\phi)_2 > 0 \,.$$

The non-linear quasi-modes associated with the first condition that have a larger radial modulation period, may serve as an intermediate step for high m_ϕ modes to decay into lower m_ϕ modes.

We note that although the nature of the plasma modes observed in laboratory experiments is drastically different from that of the modes we have discussed, these experiments show frequently that that a decay from the higher harmonics to the lower harmonics takes place. In our case we assume that as a result of these decays the lowest harmonics $m_\phi = 2$ and $m_\phi = 3$ of the non- axisymmetric modes in the end acquire the largest amplitudes. This is the justification for the $3/2$ ratios of the double peak feature of the frequency spectra [3] of high frequency QPO's. A model for QPO's that would be consistent with the experimental observations should require also that $m_\phi = 1$ modes have relatively low amplitudes, a circumstance that we assume without elaborating further on the processes that can justify it. In galactic disks $m_\phi = 1$ spirals are not commonly observed.

On the other hand we note that in other kinds of plasmas such as cylindrical and toroidal magnetically confined plasmas the dynamics of $m^0 = 1$, $n^0 = 1$ modes (m^0 indicating the azimuthal or the poloidal mode number, and n^0 the longitudinal or toroidal mode number, respectively) is drastically different from that of modes with different values of m^0/n_0 (e.g., $m^0 = 2$, $n^0 = 1$).

Importance of Vertical Temperature Gradients

The structure and the stability of tri-dimensional spiral modes is strongly influenced by the values of the temperature gradient parameter $\eta_T = \left(d \ln T/dz^2\right)/\left(d \ln \rho/dz^2\right)$, ρ being the mass density and $2T/m_i \equiv p/\rho$. When $\eta_T > 2/3$ the lowest eigenfunction of the equation [1,2] that gives the vertical mode profile is unstable and leads to the expression of the perturbed density represented by Eq. (1). If $\eta_T < 2/3$ this eigenfunction is stable (purely oscillatory in time) while the next eigenfunction [1] remains unstable and its growth rate is given by

$$\gamma_0^2 = \frac{9}{7}\frac{\mathrm{v}_A^2}{H_0^2}\left\{\left(1-C_0^0\right)+\sqrt{\left(1-C_0^0\right)^2+\frac{1}{9}C_0^0\left(14-5C_0^0\right)}\right\} \quad (2)$$

where

$$C_0^0 = \frac{2}{5}\left(\eta_T - \frac{2}{3}\right)\left(H_0 k_0\right) > -\frac{14}{5}, \quad (3)$$

and $H_0 k_0 > 1$. The mode frequency is $m_\phi \Omega\left(R_0\right)$ in this case. On the other hand, when $\eta_T < 2/3$, the frequency of the lowest eigenfunction [1] is $\omega_0 = m_\phi \Omega\left(R_0\right) + \delta\omega_0$ where

$$\delta\omega_0 = \left(\Omega_k \frac{\mathrm{v}_A}{H_0}\right)^{1/2}\left[\frac{6}{7}\frac{\sqrt{3}}{5}\left(\frac{2}{3}-\eta_T\right)\right]^{1/2} \quad (4)$$

The latter kind of mode is not localized radially [2] like that represented by Eq. (1) but is oscillatory.

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