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Mode conversion of non-axisymmetric modes in strongly non-uniform helicon-sustained plasma

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1. Introduction

The common theory of linear mode transformation is not applicable for helicon-produced plasmas with a steep radial density profile which was recently observed close to the axis of a helicon source operating in the *blue mode* [1]. In the present paper, we analyse mode conversion for non-axisymmetric ($m \neq 0$) modes taking account of *strong gradient density* effects on the wave propagation. We will show that the densities at which mode conversion occurs can be considerably lower then according to the common theory [2] if the density gradient is steep enough.

2. Theory

For simplicity, we treat the problem in plane geometry and describe the e.m. fields by the wave *ansatz* $\{\mathrm{E},\mathrm{B}\} = \{\mathrm{E}(x),\mathrm{B}(x)\}\exp[i(k_y y + k_z z - \omega t)]$. We then obtain from Maxwell's equations for helicon plasma conditions, $\varepsilon_{//} \gg g \gg N_z^2 \gg \varepsilon_\perp$

$$\begin{aligned} &\Delta_\perp E_z + \frac{\omega^2}{c^2}\frac{\varepsilon_{//}}{\varepsilon_\perp}\left(\varepsilon_\perp - N_z^2\right)E_z = i\frac{N_z}{\varepsilon_\perp}\left(\frac{\omega^2}{c^2}g + L_N^{-1}k_y\right)B_z, \\ &\Delta_\perp B_z + \left[\frac{\omega^2}{c^2}\left(\frac{\varepsilon_\perp^2 - g^2}{\varepsilon_\perp} - N_z^2\right) - \frac{k_y g}{\varepsilon_\perp}L_N^{-1}\right]B_z = -i\frac{N_z g}{\varepsilon_\perp}\frac{\omega^2}{c^2}\varepsilon_{//}E_z. \end{aligned} \tag{1}$$

where $\Delta_\perp = \frac{\partial^2}{\partial x^2} - k_y^2$, $L_N^{-1} \equiv \frac{\partial \ln|g|}{\partial x}$, $N_z = \frac{ck_z}{\omega}$, and $\varepsilon_\perp \equiv \varepsilon_{xx}$, $\varepsilon_\parallel \equiv \varepsilon_{zz}$ and $g \equiv -i\varepsilon_{xy}$ are the non-vanishing elements of the permittivity tensor. We present the solutions of the system of eqs.(6) to first order in the geometrical-optic approximation [6],

$$\{E_z(x), B_z(x)\} \propto \exp\left(i\int^x dx' k_x(x')\right). \tag{2}$$

We then get from (1) with the aid of (2) the dispersion relation as a bi-quadratic equation for the perpendicular wavenumber defined by $k_\perp^2 \equiv k_x^2(x) + k_y^2$ reading

$$k_\perp^4 - \alpha k_\perp^2 - \beta = 0\,, \tag{3}$$

where

$$\alpha = \frac{|\omega_{ce}|}{\omega}\left(k_z^2\frac{|\omega_{ce}|}{\omega} + k_y L_N^{-1}\right),\ \ \beta = -\frac{\omega_{pe}^2}{c^2}\left(\frac{\omega_{pe}^2}{c^2} - \frac{|\omega_{ce}|}{\omega}k_y L_N^{-1}\right).. \tag{4}$$

for the helicon frequency range, $|\omega_{ce}| \gg \omega \gg \sqrt{|\omega_{ce}|\omega_{ci}}$ ($\omega_{ci,ce} = \pm eB_0 / m_{i,e}$), with the assumption of a dense plasma ($\omega_{pe}^2 >> \omega_{ce}^2$, $\omega_{pe} = \sqrt{4\pi e^2 n_e / m_e}$). The general solution of eq.(3) reads

$$k_{\perp 1,2}^2 = \frac{1}{2}\left(\alpha \pm \sqrt{D}\right) \tag{5}$$

with the discriminant

$$D(x) = \alpha^2(x) + 4\beta(x)\,. \tag{6}$$

Let us first consider the plasma region, where $\alpha^2(x) \gg 4|\beta(x)|$. We then obtain from (5), (6) for *short-wavelength* electrostatic modes (Trivelpiece-Gould modes)

$$k_{\perp 1}^2 \approx \alpha \quad \text{or} \quad k_x^2(x) \approx \frac{|\omega_{ce}|}{\omega}\left(k_z^2\frac{|\omega_{ce}|}{\omega} + k_y L_N^{-1}\right) - k_y^2\,. \tag{7}$$

For *long-wavelength* electromagnetic modes (helicon modes), we correspondingly have

$$k_{\perp 2}^2 \approx -\frac{\beta}{\alpha} \quad \text{or} \quad k_x^2(x) \approx \frac{\omega_{pe}^2}{c^2}\frac{\omega}{|\omega_{ce}|}\,\frac{\dfrac{\omega_{pe}^2}{c^2}\dfrac{\omega}{|\omega_{ce}|} - k_y L_N^{-1}}{k_z^2 + k_y L_N^{-1}\dfrac{\omega}{|\omega_{ce}|}} - k_y^2\,. \tag{8}$$

Note that the denominator in (8) must not be too small in order to satisfy $\alpha^2(x) \gg 4|\beta(x)|$. This holds for weakly non-uniform plasma, where $L_N^{-1} \ll k_z^2 \left|k_y^{-1}\omega_{ce}\right| / \omega$. The gradient term in the denominator of (8) can then be neglected, and we obtain the dispersion relation in the form

$$k_x^2(x) \;=\; \frac{\omega_{pe}^2}{c^2 k_z^2}\frac{\omega}{|\omega_{ce}|}\left(\frac{\omega_{pe}^2}{c^2}\frac{\omega}{|\omega_{ce}|} - k_y L_N^{-1}\right) - k_y^2\,. \tag{9}$$

For weak density gradient, $\dfrac{\omega_{pe}^2}{c^2}\dfrac{\omega}{|\omega_{ce}|} \gg \left|k_y L_N^{-1}\right|$, we get from (9) the common local dispersion relation

$$\omega = |\omega_{ce}| \frac{c^2 k_z k_\perp}{\omega_{pe}^2}, \tag{10}$$

whereas, in the opposite limit of strong gradient such that $\frac{\omega_{pe}^2}{c^2}\frac{\omega}{|\omega_{ce}|} \ll |k_y L_N^{-1}|$, we obtain

$$\omega = -|\omega_{ce}| \frac{c^2 k_z^2 k_\perp^2}{\omega_{pe}^2 k_y L_N^{-1}}. \tag{11}$$

We can easily see from (11) that the modes with $k_y L_N^{-1} > 0$ cannot propagate. Furthermore, the dispersion relations (10) and (11) reveal different, that is, respectively, linear and quadratic scalings with the axial wavenumber k_z. The effect of the radial density gradient on the dispersion relation (11) for helicon modes ($m = 1$) in a cylindrical plasma is demonstrated in Fig.1. For Gaussian profiles of constant width a we have $k_y L_N^{-1} = (m/r)(-2r/a^2) = -1/a^2$ $= const.$ and, thus, we expect the scaling $\omega = const. \times k_z^2/\omega_{pe}^2 \propto k_z^2/n_e$ which is confirmed by the diagram.

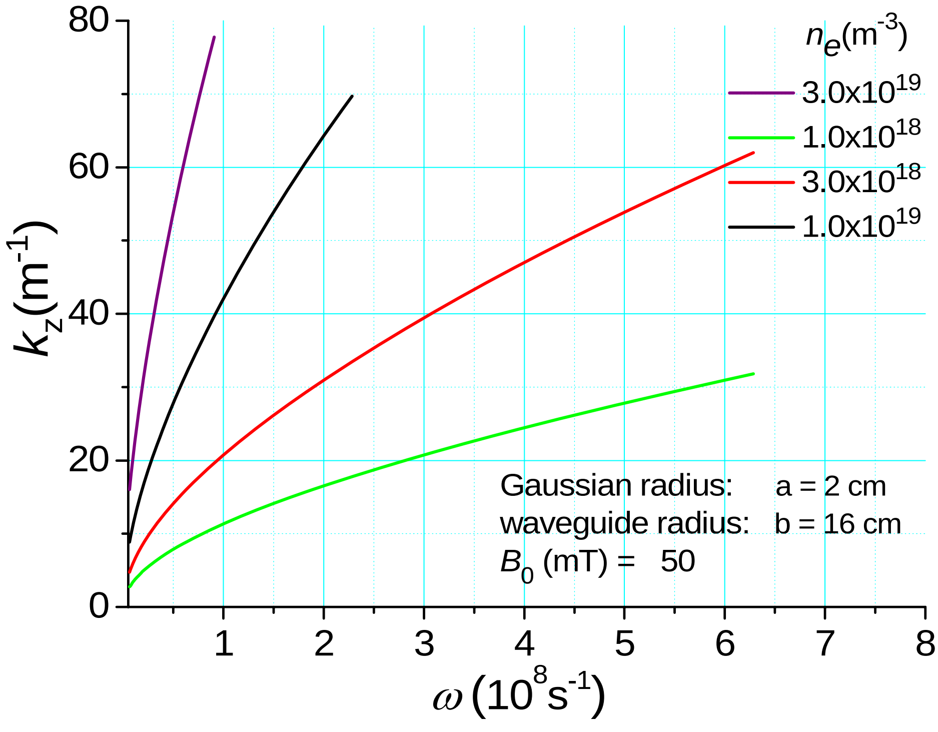

Fig.1 Helicon dispersion for Gaussian density profiles

Let us now discuss mode conversion of helicon modes into electrostatic modes in strongly non-uniform plasma. For the condition that the denominator in (8) becomes small, $k_x(x)$ grows and this equation is not valid anymore. As a result, the branches of (5) merge and mode conversion takes place. At the position x_0, where $D(x_0)=0$ and the both modes have the same wavenumber we have

$$\frac{4\omega^2}{c^2k_z^2}\frac{\omega_{pe}^2(x_0)}{\omega_{ce}^2}=\left[1+\frac{\omega k_y}{|\omega_{ce}|k_z^2}L_N^{-1}(x_0)\right]^2 . \qquad (12)$$

Unlike the common theory predicting mode conversion, if [2]

$$\varepsilon_\perp \approx \frac{\omega_{pe}^2}{\omega_{ce}^2}=\frac{1}{4}N_z^2 , \qquad (13)$$

we infer from (12) that mode conversion can occur at much lower density,

$$\varepsilon_\perp \ll N_z^2 , \qquad (14)$$

if the gradient scale-length tends to a critical value defined by the relation

$$L_N(x_0)\approx -\frac{\omega k_y}{|\omega_{ce}|k_z^2} . \qquad (15)$$

From (15) follows further that this takes place only for modes propagating in one azimuthal direction, $-k_y L_N^{-1}\geq 0$. The geometrical-optic solutions presented above are not valid in the vicinity of the branch point x_0 at which mode conversion takes place. As in the common theory, the fields in this region should be described by four-order differential equation. Following the well-known approach [4] one can get the connection formulas showing *total* transformation of the helicon mode into a strongly damped electrostatic mode.

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References

1. K. Niemi and M. Krämer, Phys. Plasmas, in press (2008).
2. P. Shamrai and V.B. Taranov, Plasma Sources Sci. Technol.5, 474 (1996).
3. V.L. Ginsburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon Pr., Oxford, 1964), p. 251.
4. V. Kopecky and J. Preinhaelter, *Linear Mode Conversion in an Inhomogeneous Magnetized Plasma*, Vol. 93, Number 5 (Rozpravy CSAV, Praha, 1983);