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FDTD algorithm for the propagation of EC waves in hot anisotropic plasma

C. Tsironis[1,2], T. Samaras[2], L. Vlahos[2]

[1]*National Technical University of Athens, GR - 157 73 Athens, Greece*

[2]*Aristotle University of Thessaloniki, GR - 54 124 Thessaloniki, Greece*

Introduction

The injection of EC waves is a standard method for achieving ECRH and ECCD in modern fusion devices. In the problem of EC wave propagation in plasmas, most popular are frequency domain methods like ray tracing, quasi-optics and beam tracing, where the determination of the plasma response presents less difficulty. However in many cases of interest, like e.g. mode conversion and off-axis heating, this approach breaks down and a full-wave analysis becomes necessary. The FDTD method is a robust tool in numerical electromagnetics which provides a full-wave solution of Maxwell's equations [1].

Here we present a scattered-field FDTD algorithm for anisotropic plasma. As an application, we study the perpendicular EC propagation in simplified geometry for different physics models of the plasma dielectric response. In general, since FDTD is a time-domain technique, conversion from the frequency domain is required for exploiting the existing knowhow on the dielectric response. However, in the case of constant-frequency wave and stationary plasma, FDTD can be applied using directly the frequency-domain dielectric tensor.

Scattered-field FDTD formalism

In the FDTD algorithm proposed by Yee [2], Maxwell's equations are discretized for the total fields in isotropic medium. The total-field formalism for anisotropic media has been derived in [3]. Alternatively, the total field may be expressed as the superposition of an incident and a scattered field, $\mathbf{E} = \mathbf{E_i} + \mathbf{E_s}$. The incident field is what would exist in the absence of the medium, while the scattered field is generated in response to the incident field. The rationale for such an approach is that the incident field can be specified (or solved) analytically. This simplifies the procedure, because boundary conditions are necessary only for the scattered field.

The fusion plasma, as a medium, is anisotropic due to the magnetic field. In anisotropic plasma, the dielectric displacement is related to the electric field by a permittivity tensor and the current density by a conductivity tensor, $\mathbf{D} = \bar{\bar{\varepsilon}}\mathbf{E}, \mathbf{j} = \bar{\bar{\sigma}}\mathbf{E}$. In this framework, Maxwell's curl equations for the scattered field take the form

$$(a)\ \nabla \times \mathbf{E_s} = -\mu_0 \frac{\partial \mathbf{H_s}}{\partial t}, \quad (b)\ \nabla \times \mathbf{H_s} = \bar{\bar{\sigma}}\mathbf{E_s} + \bar{\bar{\varepsilon}}\frac{\partial \mathbf{E_s}}{\partial t} + \bar{\bar{\sigma}}\mathbf{E_i} + \left(\bar{\bar{\varepsilon}} - \varepsilon_0 \bar{\bar{I}}\right)\frac{\partial \mathbf{E_i}}{\partial t}. \qquad (1)$$

The discretization of Eqs. (1) follows. For simplicity we use cubic cells, $\Delta(x,y,z)=\Delta r$, however the results are generalized to non-uniform grid geometries. By defining the "curl" vector

$$\Psi[\mathbf{A}|^n_{i,j,k}]=\sum_{m=-1}^{1} m\Big[A_z|^n_{i,j+m/2,k}-A_y|^n_{i,j,k+m/2}, A_x|^n_{i,j,k+m/2}-A_z|^n_{i+m/2,j,k}, A_y|^n_{i+m/2,j,k}-A_x|^n_{i,j+m/2,k}\Big],$$

Eqs. (1a) in each direction $q=x,y,z$ are simplified to

$$H_{qs}|^{n+1/2}_{i,j,k}=H_{qs}|^{n-1/2}_{i,j,k}-\frac{\Delta t}{\mu_0\Delta r}\Psi_q[\mathbf{E_s}|^n_{i,j,k}]. \tag{2}$$

To calculate $\mathbf{E_s}$, one must solve Eqs. (1b) as an algebraic system. By defining the tensors $\bar{\bar{s}}=\bar{\bar{\varepsilon}}/\Delta t-\bar{\bar{\sigma}}/2$, $\bar{\bar{a}}=\left(\bar{\bar{\varepsilon}}/\Delta t+\bar{\bar{\sigma}}/2\right)^{-1}$, the components of $\mathbf{E_s}|^{n+1}_{i,j,k}$ reach a compact form

$$E_{qs}|^{n+1}_{i,j,k}=\sum_{l,m=1}^{3} a_{ql}|_{i,j,k}\{s_{lm}|_{i,j,k}E_{ms}|^n_{i,j,k}+\Psi_q[\mathbf{H_s}|^{n+1/2}_{i,j,k}]-\sigma_{lm}|_{i,j,k}E_{mi}|^{n+1/2}_{i,j,k}-(\varepsilon_{lm}|_{i,j,k}-\varepsilon_0\delta_{lm})\left.\frac{\partial E_{mi}}{\partial t}\right|^{n+1/2}_{i,j,k}\}. \tag{3}$$

Plasma dielectric tensor

The plasma response to EC waves is stimulated by the physics of electron motions. Therefore, in principle, a kinetic treatment is called for: (a) Linearize the kinetic equation assuming a homogeneous equilibrium plus a small-amplitude perturbation, (b) solve for the distribution function by integrating along the unperturbed orbits in velocity space, (c) calculate the current density as the 1st-order moment of the distribution, (d) determine the response tensors by fitting the result for the current density to Ohm's law. It is customary to express the result in terms of a complex dielectric tensor. The relation to the real tensors $\bar{\bar{\varepsilon}}$, $\bar{\bar{\sigma}}$ is $\varepsilon_0\bar{\bar{\epsilon}}=\bar{\bar{\varepsilon}}+i\bar{\bar{\sigma}}/\omega$, or inversely $\bar{\bar{\varepsilon}}=\varepsilon_0 Re(\bar{\bar{\epsilon}})$ and $\bar{\bar{\sigma}}=\omega\varepsilon_0 Im(\bar{\bar{\epsilon}})$. For the fully relativistic case the evaluation of the dielectric tensor requires numerics, so for obtaining analytic expressions one resorts to approximations.

In the non-relativistic approximation one sets $\gamma=1$ and uses the non-relativistic Maxwellian. This approach is valid when $\left|N_{||}\right|\gg max(v_{th}/c,|1-l\omega_c/\omega|)$, which guarantees that resonant electrons remain sub-relativistic and that the Doppler effect dominates the frequency downshift. When this condition breaks down, one should adopt the weakly-relativistic approach, where a Taylor expansion is employed in the Lorentz factor. The velocity space integration yields

$$\varepsilon_{xx}=1-\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega^2}\sum_{l=-\infty}^{\infty}\frac{l^2}{\lambda}\Gamma_{|l|}\mathscr{F}_{|l|+3/2},\qquad \varepsilon_{xz}=\varepsilon_{zx}=\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega\omega_c}N_{||}N_{\perp}\sum_{l=-\infty}^{\infty}\frac{l}{\lambda}\Gamma_{|l|}\left(\mathscr{F}_{|l|+3/2}-\mathscr{F}_{|l|+5/2}\right),$$

$$\varepsilon_{xy}=-\varepsilon_{yx}=-i\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega^2}\sum_{l=-\infty}^{\infty}l\Gamma'_{|l|}\mathscr{F}_{|l|+3/2},\qquad \varepsilon_{yz}=-\varepsilon_{zy}=i\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega\omega_c}N_{||}N_{\perp}\sum_{l=-\infty}^{\infty}\Gamma'_{|l|}\mathscr{F}_{|l|+5/2},$$

$$\varepsilon_{yy}=1-\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega^2}\sum_{l=-\infty}^{\infty}\left(\frac{l^2}{\lambda}\Gamma_{|l|}\mathscr{F}_{|l|+3/2}+2\lambda\Gamma'_{|l|}\mathscr{F}_{|l|+5/2}\right),$$

$$\varepsilon_{zz}=1-\frac{c^2}{2v_{th}^2}\frac{\omega_p^2}{\omega^2}\sum_{l=-\infty}^{\infty}\Gamma_{|l|}\left[\frac{c^2}{2v_{th}^2}N_{||}^2\left(\mathscr{F}_{|l|+7/2}-2\mathscr{F}_{|l|+5/2}+\mathscr{F}_{|l|+3/2}\right)+\mathscr{F}_{|l|+5/2}\right],$$

where $\Gamma_l(\lambda)=e^{-\lambda}I_l(\lambda)$, $I_l(\lambda)$ the modified Bessel function of argument $\lambda=k_\perp^2 v_{th}^2/\omega_c^2$, and $\mathscr{F}_q(\alpha,\zeta_l)$ the Shkarofsky functions of arguments $\alpha=c^2N_{||}^2/(2v_{th}^2)$, $\zeta_l=(c^2/v_{th}^2)(1-l\omega_c/\omega)$.

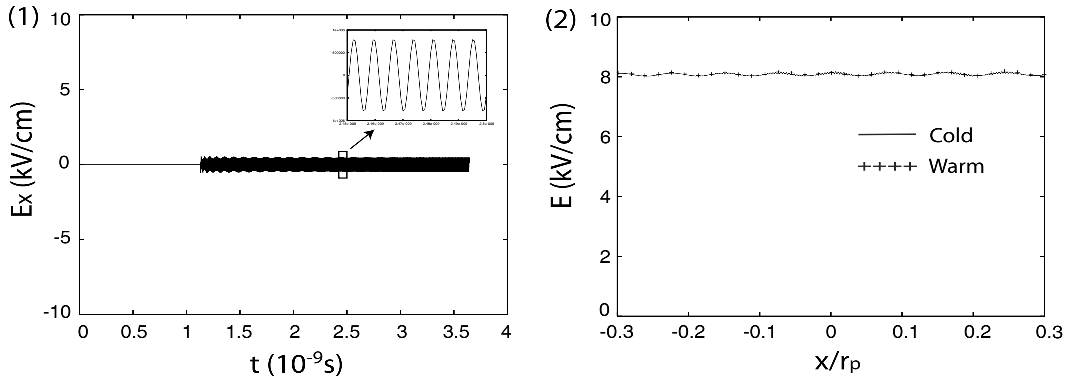
A finite temperature introduces a dependence of the dielectric tensor on the refraction index. $\mathbf{N}$ is determined by the dispersion relation, $|-N^2\bar{\bar{I}}+\mathbf{NN}+\bar{\bar{\varepsilon}}|=0.$ where one has to keep the solution corresponding to the specific mode of propagation. The parameter λ measures the importance of Finite Larmor Radius (FLR) effects. In the limit of very small Larmor radius, only the zero-order correction is retained and $\bar{\bar{\varepsilon}}$ reaches a form known as the warm plasma tensor, while in the limit of zero Larmor radius it reduces to the cold plasma tensor.

Numerical application

We study the perpendicular EC propagation in simplified tokamak geometry. The simplification lies in adopting the large aspect ratio approximation, $r_p \ll r_t$, which corresponds to a flat magnetic geometry with negligent poloidal field. In this framework, the magnetic field, density and temperature may be assumed to vary only along the $x-$axis, in the region $[-r_p, r_p]$, and the magnetic field lies just along z. The variation of ω_c is the typical $(1+x/r_t)^{-1}$ of the toroidal field, while the profiles of ω_p, T_e are parabolic curves. The incident field is a sinusoidal wave, launched from the low-field side in the negative $x-$direction.

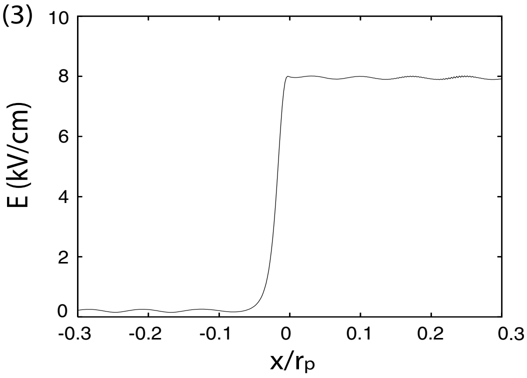
Since the wave starts along the plasma inhomogeneity, the propagation stays perpendicular. Regarding FDTD, this allows a simpler description based on the 1D algorithm, obtained by setting $1/\Delta y = 1/\Delta z = 0$ in (2), (3). The grid size is 5% the vacuum wavelength, the time step is 10% the Courant stepsize, and the parameters are those of AUG and ITER. In AUG it is $r_t = 1.65$m, $r_p = 0.6$m, $B_0(0) = 2.5$T, $n_e \in [1.4, 1.6] \cdot 10^{13}\text{cm}^{-3}$, $T_e \in [0.2, 2]$KeV, $\omega/2\pi =$ 140GHz (X2), $P_0 = 1$MW and $w = 2$cm. In ITER: $r_t = 6.2$m, $r_p = 1.9$m, $B_0(0) = 5.51$T, $n_e \in [1, 10] \cdot 10^{13}\text{cm}^{-3}$,$T_e \in [1, 10]$KeV, $\omega/2\pi = 160$GHz (O1), $P_0 = 10$MW and $w = 3$cm.

First we present results for cold/warm plasma propagation in AUG. In Fig. 1 we show $E_x(t)$ at $x = -0.3r_p$ in cold plasma. The field is zero up to $\approx 1.2 \cdot 10^{-9}$s and reaches a steady-state after $\approx 3.5 \cdot 10^{-9}$s. The fields vary periodically in time (see top-right subfigure). In Fig. 2 we give the steady-state profile of the amplitude E for cold and warm plasma. E remains constant, as in the absence of thermal motions from the modeling there is no mechanism for wave absorption.

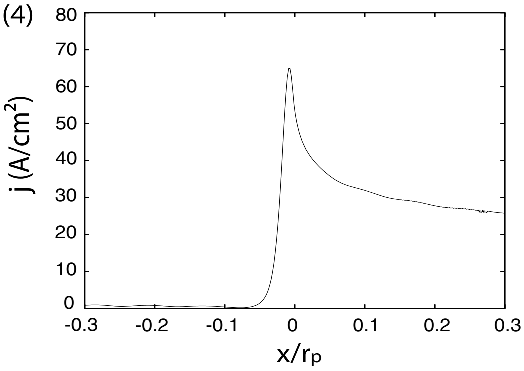


We studied the case of hot, weakly-relativistic plasma for both AUG and ITER. For AUG, in Figs. 3, 4 the profiles of E, j are presented. The absorption of the wave occurs in the narrow resonance layer near the plasma center, and the damping profile is very steep. The current density increases towards the resonance layer, due to the resonant interaction, and then it is nullified past the resonance, since the the wave has been completely damped.

(3)

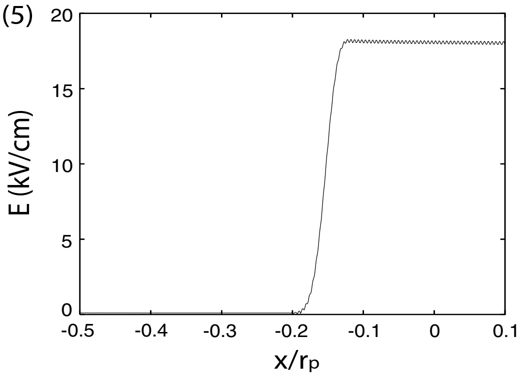


(4)

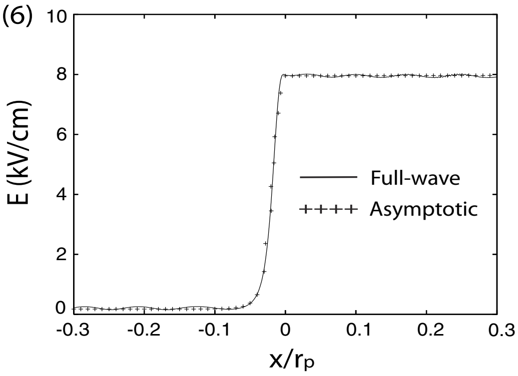


In Fig. 5 we give the profile of E for parameters relevant to ITER. In this case, due to the specific value of the magnetic field on the axis, the resonance layer is shifted towards the high-field side. In Fig. 6 we provide the result of a successful benchmark of our model with the beam tracing routine NGBT [4]. Unlike ECFW, which models the plasma response in terms of the fully kinetic tensor, NGBT considers cold plasma propagation with kinetic effects reflected only in the reduction of the amplitude according to the hot plasma optical depth.

(5)



(6)



A limitation of our model is that in the 1D description it is not possible to model effects like mode conversion, ECCD or diffraction. Such issues can be studied only under a 2D/3D implementation of FDTD, which is the subject of current work.

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