

## **Bayesian approach to data validation of the oblique ECE diagnostic at JET**

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### **Introduction**

In this paper the validation of data produced by the Oblique Electron Cyclotron Emission diagnostic (ObECE) operating at JET [1] using a statistical treatment based on the Bayesian Probability Theory (BPT) is reported. The diagnostic is a six-channel Martin-Puplett interferometer. It allows a spectral analysis covering an extended bandwidth between 75 - 800 GHz, 10 ms/profile time resolution and 7.5 - 15 GHz spectral resolution. Observations can be carried on simultaneously at three lines of sight ( $0^\circ$ ,  $10^\circ$  and  $22^\circ$  with respect to the poloidal plane), and two linear polarizations (for the oblique views), corresponding to mostly ordinary and mostly extraordinary emissions. This setup enables the study of the electron distribution function in thermal and sub-thermal range.

In order to carry on a reliable measurement, a precise calibration is an essential requirement. The BPT approach [2, 3] allows a systematic error analysis of data sets sampled as Gaussian or Poisson distribution, and assigned in an appropriate parameter space. This technique has been applied in order to compare cross-calibrations performed using either absolute calibrated Michelson interferometer (ECM) or SPECE emission code [4]. The latter has been performed with plasmas where the magnetic field  $B(R)$  and plasma current were varied keeping the safety factor profile constant, in order to maintain the temperature profile as constant as possible while scanning the EC emission spectrum. Two of the pulses had Neutral Beam Injection heating to ensure high optical thickness in order to minimize spurious emission effects in higher harmonics (see tab.1 for the pulse list).

Bayes' theorem relates the conditional and marginal probability distributions of random variables, and it can be expressed by the following relation

$$P(\Theta | D) = \frac{P(D | \Theta)P(\Theta)}{P(D)}, \quad (1)$$

where  $\Theta$  are the parameters,  $D$  the ratios between the observed data representing ECM (or SPECE simulation) and ObECE at different timeslices,  $P(\Theta)$  represents the prior distribution of  $\Theta$  (i.e., what is known about  $\Theta$  before any measurements are taken),  $P(D|\Theta)$  is the conditional distribution of  $D$ , given a model with parameters  $\Theta$ . If  $H(\Theta)$  represents the model for the observed data, and the errors in the measurements are assumed to be Gaussian distributed, then the conditional distribution takes the form

$$P(D | \Theta) = \left( \frac{1}{\sqrt{2\pi}\sigma} \right)^n \exp \left( - \frac{\sum D_i - H(\Theta)}{2\sigma^2} \right). \quad (2)$$

In Eq. (1),  $P(D)$  is the total probability of  $D$  and acts as a normalizing constant.

### Calibration method

The ObECE perpendicular channel has been previously cross-calibrated “shot by shot” with the absolute calibrated ECM, i.e., the ratio ObECE/ECM is calculated within a time interval  $\Delta t$ , in which plasma parameters ( $T_e$ ,  $n_e$ ,  $I_{pl}$ ,  $B(R)$ ) are roughly constant. This ratio is then used as calibration function, later applied to each timeslice of the shot. This technique provides good calibrated spectra mostly within  $\Delta t$  and it is not applicable to a group of shots, due to the fact that ECM and ObECE lines of sight differ in the  $z$  coordinate (35.300 cm and 25.497 cm) and calibration is affected by systematic errors.

Therefore, an improvement was performed cross-calibrating ObECE over simulated spectra calculated using SPECE, once the necessary condition consisting in an almost perfect match between simulated ECM spectra and ECM data had been verified.

A Bayesian approach, based on a simple quadratic model, has been investigated to evaluate if the radiation incoming intensity  $I_C$  plays a role into the calibration procedure

$$UCSn = \theta_0 I_C + \theta_1 I_C^2. \quad (3)$$

Here UCSn represents the un-calibrated spectrum for a specific ObECE channel. Dividing by  $I_C$ , one obtains the following model, used in Eq. (2).

$$H(\Theta) = \theta_0 + \theta_1 I_C. \quad (4)$$

In Eq. (3),  $\theta_0$  is the cross-calibration coefficient, that takes into account the evolution of the ratio ObECE/ECM during the pulse, and  $\theta_1$  provides the weight of the contribution due to  $I_C$ . The ratio of  $\theta_1$  over  $\theta_0$  with their relative standard deviation, are shown in Fig. 1 for pulse 72315 during a  $B(R)$  ramp (3.2 T - 2.2 T) for ECM (left) and SPECE (right) cross-calibrations. The analysis shows that there is a weak dependency on the intensity over most of

the optically thick frequency range for both the calibration technique (values close to zero). Within frequency ranges between 73 – 130 GHz and 270 – 345 GHz large error bars on Fig. 1 (left) indicate presence of systematic errors.

SHOT	B (T)	I <sub>p</sub> (MA)	n <sub>e</sub> *10 <sup>19</sup>	T <sub>e</sub> <sup>LIDAR</sup>	T <sub>e</sub> <sup>ECM</sup>	P <sub>LH</sub>	P <sub>ICRH</sub>	P <sub>NBI</sub>
72184	3.4/2.3	1.9/1.4	2.8	4	3.5	0	0	0
72314	3.45	2	3.5	5	5	0	0	10
72315	2.8/1.7	2	2	4	3	0	0	0
72828	2.7	1.7	5.2	4.5	4	1.8	1.1	18.3

TAB. 1: pulse list of the shots used for calibrating ObeECE diagnostic

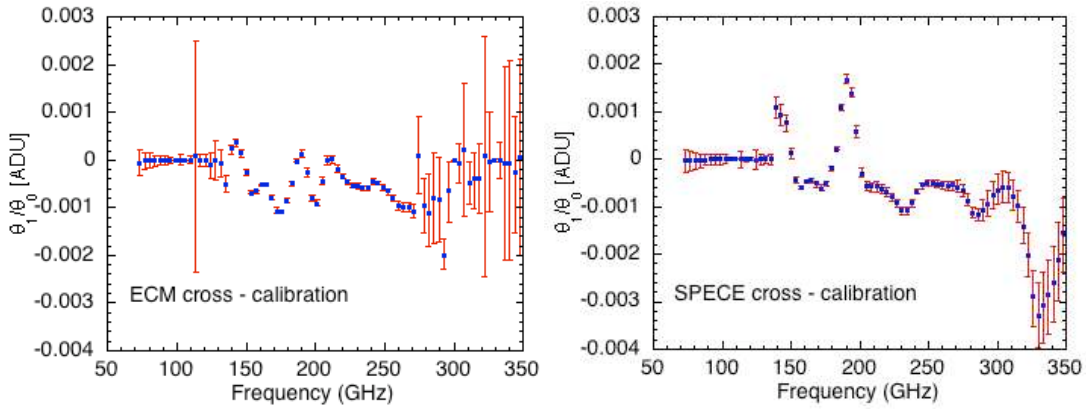


Fig. 1: Ratio  $\theta_1/\theta_0$  for ECM (left) and SPECE (right) cross - calibration for pulse 72315

The standard technique, that consists in keeping constant as much as possible the shape of  $T_e$  profile during a  $B(R)$  ramp, allows to distinguish between residual systematic frequency dependent errors and spectral features [5], and to determine a calibration correction coefficient

$$C(\omega) = \sum_{t=t_1}^{t_2} S_n(\omega, t) / (R(\omega)N(\omega)). \quad (5)$$

This coefficient varies up to 5% with an error bar up to 8%. In Eq. (5)  $S_n(\omega)$  and  $N(\omega)$  represent the spectra and the number of spectra during the ramp respectively, and  $R(\omega)$  is the normalized spectrum with respect to the temperature profile. In Fig. 2, the results of the new calibration are reported for the pulse 72624 @ 52.46 sec ( $B=3.0$  T,  $I_p=1.8$  MA,  $n_e=8*10^{19}$  m<sup>-2</sup>,  $T_e=6$  keV,  $P_{NBI}=18.5$  MW) taken during the session “Identify and document main fast ions loss mechanisms” for the three lines of sight, and vertical polarizations (mostly X- mode). The simulation calculated taking into account 5% error in ECM temperature profile match data on optically thick harmonics. Experimental error bars are calculated as standard deviation over 20 ms time average, corresponding to one ECM time profile.

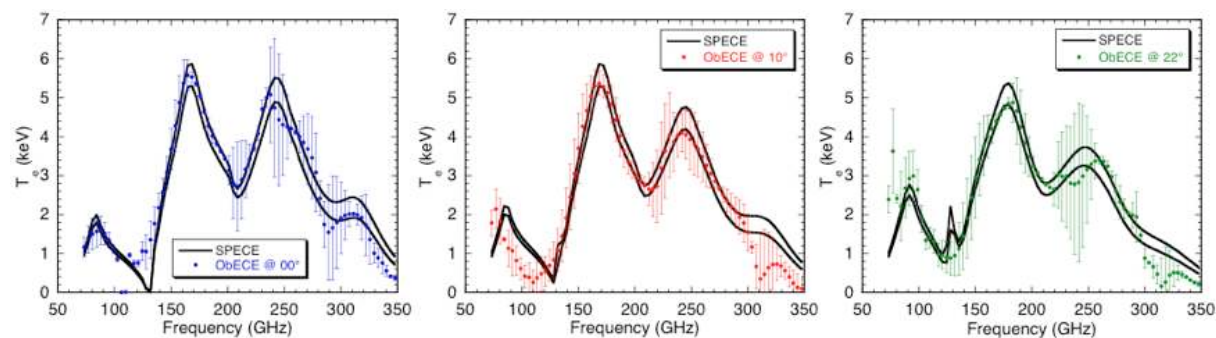


Fig. 2: Calibrated spectra for 72624 (dots) @ 52.46 sec left to right,  $0^\circ$ ,  $10^\circ$ ,  $22^\circ$ , compared with simulations (lines) calculated taking into account 5% error

## Conclusions

Data validation of ObECE diagnostic at JET is presently under development. In order to verify the quality of the calibration, a statistical treatment based on Bayes' theorem has been implemented. The simple quadratic model permitted to identify sources of systematic errors, mainly ascribed to the different ObECE and ECM lines of sight. In addition, it has been shown that the incoming radiation intensity plays a minor role into the calibration procedure over most of the spectral range. Spurious effects of the emission on optically thin harmonics require further analysis in order to give an affordable calibration function over the whole spectral region.

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