

Rotation of magnetic islands regulated by self-generated zonal flow

S. Nishimura^{1,2}, S. Benkadda^{1,3}, M. Yagi^{1,4}, S.-I. Itoh^{1,4}, K. Itoh^{1,5}

¹ France-Japan Magnetic Fusion Laboratory LIA, CNRS-Provence Univ., Marseille, France

² Interdisciplinary Graduate School of Engineering Sciences, Kyushu Univ., Kasuga, Japan

³ Equipe DSC, Laboratoire PIIM, CNRS-Provence Univ., Marseille, France

⁴ Research Institute for Applied Mechanics, Kyushu Univ., Kasuga, Japan

⁵ National Institute of Fusion Science, Toki, Japan

1. Introduction

The control of magnetic islands is one of important issues for magnetically confined fusion plasmas. In tokamak plasmas, magnetic islands rearrange ideal equilibrium profiles, and contribute to the upper limit of β value. In addition, the locking of rotation of magnetic islands occasionally triggers the so-called 'disruptions' of plasmas discharges in tokamaks.

Much works have been done for the drive of magnetic islands, and it has been pointed out that many factors should be taken into account (free energy sources, parallel transport, ion sound wave, curvatures, external oscillations, turbulence, etc). Especially, the rotation of magnetic islands is an important factor for the excitation of magnetic islands via the polarization current[1, 2]. Moreover, once the rotation of magnetic islands is locked by the resistive wall, the rotation of plasmas is also slowed down, which causes the degradation of the confinement: for example, the breaking of the transport barrier and continuously the destabilization of the turbulence and the MHD activity. However, the theoretical prediction of the rotation frequency of magnetic islands in experimental devices has not been fully achieved. This is partially because equilibrium fields are strongly modified by magnetic islands, and this modification feeds back to the rotation of magnetic islands. In addition, the rotation frequency of magnetic islands is associated with the radial pattern of zonal flow (macroscopic electric field), which is crucial for the good confinement of fusion plasmas. Thus, the basic mechanism of the rotation of magnetic islands should be clarified from a theoretical point of view.

In this study, we simulate a four-field reduced set of two-fluid equations, and the detailed study of nonlinear dynamics of rotating magnetic islands is presented. In our model equations, the simple drift-tearing mode is considered, and zonal flow is included. Firstly, we focus on the radial pattern of zonal flow generated by magnetic islands. In the next step, it is clarified that self-generated zonal flow dominates the rotation of magnetic islands, and the direction of rotation strongly depends on the flow pattern. Our results might be a basis of understanding the more complex behaviour of the neoclassical tearing modes.

2. Model

In this study, large aspect ratio tokamak plasmas ($\varepsilon = a/R_0 \ll 1$, where ε , a and R_0 are the inverse aspect ratio, the minor and the major radii, respectively) in the cylindrical coordinate (r, θ, z) are considered. We introduce a reduced set of two-fluid equations derived from Braginskii's two-fluid equations, which describes the drift-tearing mode (the linearly unstable classical tearing mode combined with pressure gradient). The model equations are

$$\frac{D}{Dt} \nabla_{\perp}^2 \phi = \nabla_{\parallel} j_{\parallel} + \mu \nabla_{\perp}^4 \phi, \quad (1)$$

$$\frac{\partial}{\partial t} A = -\nabla_{\parallel} (\phi - \delta p) - \eta_{\parallel} j_{\parallel} + \alpha_T \delta \nabla_{\parallel} T, \quad (2)$$

$$\frac{D}{Dt} n + \beta_s \frac{D}{Dt} p = \delta \beta \nabla_{\parallel} j_{\parallel} + \eta_{\perp} \beta \nabla_{\perp}^2 p, \quad (3)$$

$$\frac{3}{2} \frac{D}{Dt} T - \frac{D}{Dt} n = \alpha_T \delta \beta \nabla_{\parallel} j_{\parallel} + \varepsilon^2 \chi_{\parallel} \nabla_{\parallel}^2 T + \chi_{\perp} \nabla_{\perp}^2 T, \quad (4)$$

where $j_{\parallel} = -\nabla_{\perp}^2 A$, $\alpha_T = 0.71$, $\frac{D}{Dt} = \frac{\partial}{\partial t} + [\phi,]$, $\nabla_{\parallel} = \frac{\partial}{\partial z} - [A,]$, $\nabla_{\perp} = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial}{\partial \theta}$, $[f, g] = \hat{\mathbf{z}} \cdot \nabla f \times \nabla g$. ($\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\mathbf{z}}$) indicate unit vectors. $\{\phi, A, n, T, p\}$ indicate electrostatic potential, vector potential parallel to the ambient magnetic field, electron density, electron temperature and electron pressure defined by $p = n + T$, respectively. $\{\mu, \eta_{\parallel}, \eta_{\perp}, \chi_{\parallel}, \chi_{\perp}\}$ are ion viscosity, parallel resistivity, perpendicular resistivity, parallel and perpendicular thermal conductivity, respectively. In the dissipationless limit, the energy is conserved. $\{\delta, \beta\}$ indicate ion skin depth normalized by minor radius and plasma beta value at the plasma center, respectively. β_s is the beta value at the rational surface. The normalization is $\{\varepsilon v_A t / a \rightarrow t, r/a \rightarrow r, z/R_0 \rightarrow z\}$, where v_A is Alfvén velocity. The perturbed quantity $f(r, \theta, z, t)$ is assumed to vary as $f_0(r) + \sum_{m,n} \tilde{f}_{m,n}(r, t) \exp\{i(m\theta - nz)\}$, where m and n are the poloidal and toroidal mode number. The perturbation $\tilde{f}_{m,n}(r)$ satisfies boundary conditions: $\tilde{f}_{m,n}(0) = \tilde{f}_{m,n}(1) = 0$ for $m, n \neq 0$ and $\partial \tilde{f}_{0,0} / \partial r|_{r=0} = \tilde{f}_{0,0}(1) = 0$ (the center and edge of plasma correspond to $r = 0$ and $r = 1$). The equilibrium quantities are chosen such that $q(r) = 1.5 + 0.5 \left(\frac{r}{r_s}\right)^3$ ($q(r)$ stands for the safety factor defined by $\frac{1}{q(r)} = -\frac{1}{r} \frac{d}{dr} A_0(r)$), $n'_0(r) = -\frac{2\beta}{\varepsilon} r$, $T'_0(r) = -\frac{2\beta}{\varepsilon} r$, where the prime indicates the radial derivative. The equilibrium current $j_0(r)$ is evaluated by the cylindrical MHD equilibrium. The stability parameter for the tearing mode is given by $\Delta' = 17.78 > 0$. In the numerical simulation, we set $r_s = 0.6, \varepsilon = 0.2, \beta = 0.01$ and $\delta = 0.01$ and default values of the transport coefficients are chosen as $\mu = 10^{-5}, \eta_{\parallel} = 10^{-5}, \eta_{\perp} = 2 \times 10^{-5}, \chi_{\parallel} = 1, \chi_{\perp} = 10^{-5}$. In this study, we adopt a predictor-corrector scheme to calculate time evolutions, and a spectral method to evaluate nonlinear terms, where only modes which satisfy $m/n = 2$ (resonant on the $q = 2$ surface) is considered, for simplicity. The radial grid has 512 meshes, and the time step is 0.01. For the spectral resolution, $-4 \leq n \leq 4$ Fourier modes are considered.

3. Analysis

In our parameter regime, only $(m, n) = (2, 1)$ mode is unstable and other higher modes (the drift wave) are stable. In the nonlinear simulation, magnetic island is excited at the rational surface $r = 0.6$, and the so-called Rutherford growth regime and the nonlinear saturation are observed. Figure 1 shows the time evolution of magnetic island and zonal flow. The growth rate of zonal flow is double of that of magnetic island, i.e. zonal flow is quasi-linearly excited by magnetic island.

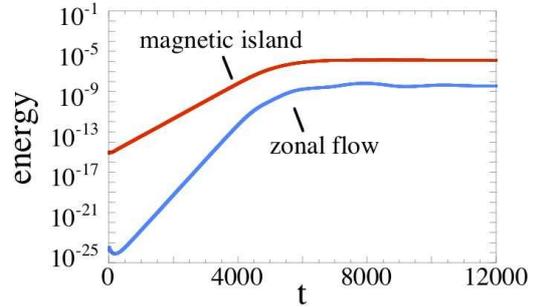


Figure 1: The time evolution of energy.

Figure 2 shows the contour plot of the real part of helical flux function at $z = 0$ and $t = 12000$, where the helical flux function is defined by $A_*(r, \theta, z, t) = A_0(r) + \tilde{A}_{0,0}(r) + \frac{r^2-1}{4} + \sum_{m/n=2} \tilde{A}_{m,n}(r) \exp[i(m\theta - nz)]$ in our normalization and parameters. The gradient of the helical flux is perpendicular to the magnetic field, then its contour indicates the magnetic flux surface.

Figure 3 shows the ion viscosity dependence of radial pattern of zonal flow velocity in the nonlinear saturation level. Three cases are plotted: $\mu = 10^{-6}$, $\mu = 10^{-5}$ and $\mu = 10^{-4}$. It is noted that the modification of saturation width of magnetic islands by ion viscosity is negligible. It is found that, the amplitude of zonal flow is monotonically decreasing function of ion viscosity. In addition, the radial pattern of zonal flow inside magnetic islands is changed, according to the increase in ion viscosity. Zonal flow is generated by Reynolds stress and Maxwell stress, associated with electrostatic and electromagnetic fluctuations of magnetic islands, respectively. In our analysis, it is found that the strength ratio of Reynolds stress to Maxwell stress is a monotonic increasing function of ion viscosity; Reynolds stress is stronger in small ion viscosity regime, on the other hand, Maxwell stress is dominant in large ion viscosity regime. Therefore, the radial pattern of zonal flow is controlled by ion viscosity through changing the balance between Reynolds stress and Maxwell stress[4].

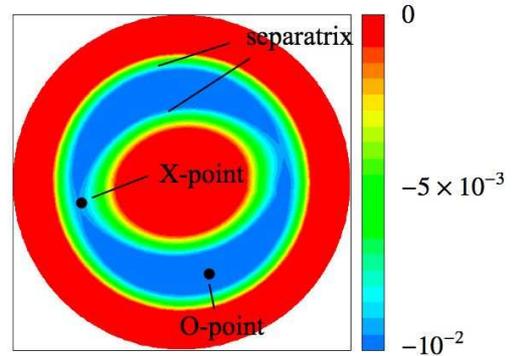


Figure 2: The contour of helical flux ($t = 12000$).

Figure 4 shows the rotation frequency of magnetic islands with different ion viscosity at the non-linear saturation level. The positive and negative signs correspond to the direction of the electron and the ion diamagnetic drift, respectively. It is found that the rotation frequency exhibits a monotonic increasing function of ion viscosity. Especially, depending on ion viscosity, the direction of the magnetic islands rotation is changed from negative to

positive. We also examine the resistivity dependence, and it is found that the dependence of the rotation frequency on the magnetic Prandtl number μ/η_{\parallel} shows a monotonic increase.

From the (2, 1) component of Ohm's law, the rotation frequency of magnetic islands ω_r is approximately given by the summation of diamagnetic drift and zonal flow as $\omega_r \approx \langle \tilde{\omega}_* \rangle + \langle \tilde{\omega}_{E \times B} \rangle$, where $\tilde{\omega}_* = -\delta k_{\theta} \left\{ n'_0 + \tilde{n}'_0 + (1 + \alpha_T)(T'_0 + \tilde{T}'_0) \right\}$, $\omega_{E \times B} = k_{\theta} \tilde{\Phi}'_{0,0}$, $k_{\theta} = 2/r$ and $\langle \rangle$ indicates the radial average inside magnetic islands[3]. The diamagnetic drift frequency has the positive contribution, and zonal flow give the main mechanism to change the direction of rotation.

4. Conclusion

The rotation frequency of the drift-tearing mode strongly depends on the self-generated zonal flow, where the competition between Reynolds stress and Maxwell stress forms the energy sink of zonal flow, and the amplitude of zonal flow is controlled by the magnetic Prandtl number. In future works, such effects of self-generated zonal flow on the rotation of magnetic islands should be examined in more generalized cases, especially for the neoclassical tearing mode.

References

- [1] A. I. Smolyakov, Plasma Phys. Control. Fusion **35**, 657 (1993).
- [2] J. W. Connor, F. L. Waelbroeck, H. R. Wilson, Phys. Plasmas **8**, 2835 (2001).
- [3] M. Ottaviani, F. Porcelli, and D. Grasso, Phys. Rev. Lett. **93**, 075001 (2004).
- [4] S. Nishimura, M. Yagi, S.-I. Itoh, and K. Itoh, J. Phys. Soc. Jpn. **77**, 014501 (2008).

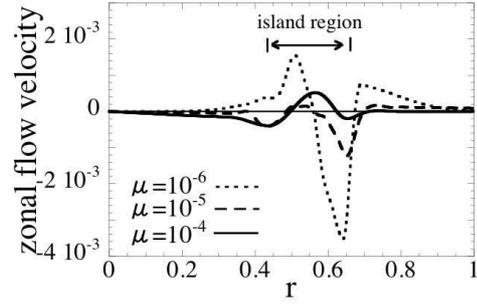


Figure 3: The radial pattern of zonal flow.

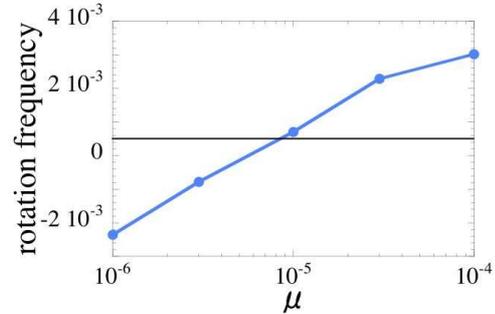


Figure 4: The rotation frequency of magnetic islands ($\langle \tilde{\omega}_* \rangle_{t=0} = 5.4 \times 10^{-3}$).