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Calculation of Poloidal Magnetic Field in Tokamak Code TOKES

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Abstract: A self-consistent numerical technique that provides stable configurations of poloidal magnetic field and confined plasma in two-dimensional MHD integrated tokamak code TOKES is described and applied to the future tokamak ITER.

For numerical modelling of tokamak operation the poloidal field (PF) in the entire vessel is essential, because it determines at each time step the shape of confined plasma and the distribution of plasma impact over the wall. The PF is a composition of the internal field generated by toroidal component of plasma currents and the applied field produced by the PF coils. PF is usually described with the poloidal magnetic flux $w(\mathrm{p})$, where the point $\mathrm{p} = (r,z)$, with r and z major cylindrical coordinates of tokamak.

In the code TOKES [1] the vessel's poloidal cross-section is covered by triangular meshes adjusted to the wall surface. The values of w at the mesh nodes P are updated after each time step, and $w(\mathrm{p})$ between the nodes are approximated by linear dependences. To calculate $w(\mathrm{P})$ the Green function G to the Maxwell's equation $\mathrm{rotB} = (4\pi/c)\mathrm{J}$ is applied [2], with $\mathrm{B} = (B_r,B_z)$ the poloidal field strength and J the current density. Due to the coils, $w(\mathrm{p})$ has generally a rather complex behaviour. In TOKES arbitrary PF field is allowed being constructed of dynamically calculated magnetic surfaces $w(\mathrm{p}) = \mathrm{constant}$. The volumes between the surfaces ('magnetic layers') are ordered as a graph and assembled of line segments ending at the triangle sides.

For each magnetic layer segment occupied by confined plasma a current I_p is calculated being driven by the loop voltage ϕ (the inductive current I_ϕ) and the derivative dp/dw of plasma pressure p (the bootstrap current I_{bs}): $I_p = I_\phi + I_{\mathrm{bs}}$. I_p is obtained based on the Ohm's law for the multi-species plasma model of TOKES. The plasma currents of all segments of a triangle are approximated by one current I_i flowing in a thin ring centred at $r = 0$ that crosses the triangle's centre $\mathrm{p}_i^{(c)}$.

The Green function $G(\mathrm{p,P})$, with $\mathrm{P} = (R,Z)$, provides the field of a ring of a radius $r = R$ and an elevation $z = Z$ that carries unit current. The whole flux $w(\mathrm{p})$ is given by

$$w(\mathrm{p}) = \sum_n I_n G(\mathrm{p},\mathrm{P}_n) + \sum_i I_i G\left(\mathrm{p},\mathrm{p}_i^{(c)}\right) \quad (1)$$

The coil currents I_n are obtained in 6 main PF coils approximated by the current rings at some fixed points $\mathrm{P}_n = (R_n,Z_n)$ and assuming the plasma currents I_i be given. It is to note that if I_n would be fixed but I_i not after each time step, the plasma gets unstable (despite the toroidal symmetry) and in a few steps the plasma boundary touches the wall, which is not desirable. Therefore to keep the plasma off the wall the updating of I_n is dynamically done.

To achieve the required feedback control a special technique is developed in TOKES. The following scheme is implemented: 1) fixed positions p_{x0} and p_{x1} for two x-points are chosen as expected in ITER; 2) the confined plasma is bounded by the separatrix magnetic surface $w(\mathrm{p}) = w_{x0} \equiv w(\mathrm{p}_{x0})$; 3) another separatrix $w(\mathrm{p}) = w_{x1} \equiv w(\mathrm{p}_{x1})$ locates outside the plasma and a small difference $\Delta w = w_{x0} - w_{x1}$ is fixed; 4) some fixed positions p_{cj} for several points near the vessel surface are chosen where the plasma can touch the wall most probably; 5) the point $\mathrm{p}_{cj\max}$ of maximum value of $w(\mathrm{p}_{cj})$ obtained with the available I_i and previous I_n is used for calculation of new I_n from the condition $w(\mathrm{p}_{i\max}) = w_{x1}$. Thus the code tries to keep the outer separatrix $w(\mathrm{p}) = w_{x1}$ near the wall and therefore the plasma at some distance from the wall. Small value of $\Delta w << (w_{\max} - w_{x0})$ follows from a desire to have a large plasma volume ($w_{\max}$ is the flux at the magnetic axis of confinement region).

As B = 0 at the x-points, the corresponding equations for new I_n read:

$$B_r(\mathrm{p}_{xm}) = \sum_n I_n G_r(\mathrm{p}_{xm},\mathrm{P}_n) + G_{rp}(\mathrm{p}_{xm}) = 0, \qquad m = 0,1 \tag{2}$$

$$B_z(\mathrm{p}_{xm}) = \sum_n I_n G_z(\mathrm{p}_{xm},\mathrm{P}_n) + G_{zp}(\mathrm{p}_{xm}) = 0 \quad , \tag{3}$$

$$\sum_n I_n G(\mathrm{p}_{x0},\mathrm{P}_n) + G_p(\mathrm{p}_{x0}) - \sum_n I_n G(\mathrm{p}_{x1},\mathrm{P}_n) - G_p(\mathrm{p}_{x1}) = \Delta w \tag{4}$$

$$\sum_n I_n G(\mathrm{p}_{x1},\mathrm{P}_n) + G_p(\mathrm{p}_{x1}) - \sum_n I_n G(\mathrm{p}_{cj\max},\mathrm{P}_n) - G_p(\mathrm{p}_{cj\max}) = 0 \tag{5}$$

Here $G_r = (1/r)\partial G((r,z),\mathrm{P})/\partial z$, $G_z = -(1/r)\partial G/\partial r$, and G_{rp}, G_{zp} and G_p are the corresponding sums on plasma currents, e.g. $G_p(\mathrm{p}) = \sum_i I_i G(\mathrm{p},\mathrm{p}_i^{(c)})$.

The linear (in respect to I_n) system Eqs.(2)-(5) is solved with a standard algorithm. The newly calculated I_n influence the magnetic layers and thus consequently the triangle's currents I_i. Therefore for a self-consistent calculation several iterations are required, which is not always possible because the iterations can diverge. Fortunately, this technique can provide the convergence on a wide range of plasma current profiles.

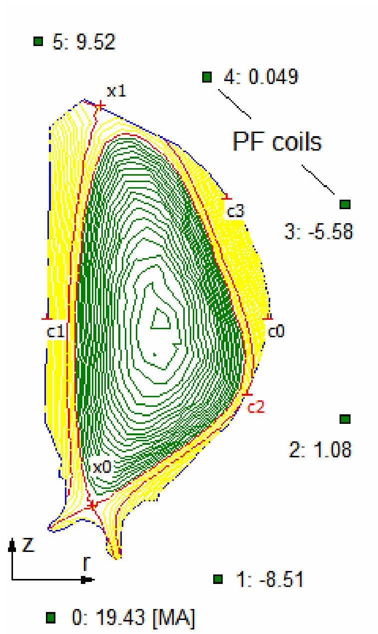

Fig. 1 The state with h = 1, P_0 = (4.186,-5.522) and P_1 = (7.479,-4.757) [m].

In this work we demonstrate the evaluation by TOKES of ITER coil positions P_n at which I_n acquire some moderate values, keeping total plasma current I_0 = 15 MA and assuming $I_{bs} = 0$. The segment inductive current of TOKES is given by

$$I_\phi = -\sigma_\parallel(w) s\phi / cr \qquad (6)$$

Here s is poloidal cross-section area prescribed to the segment, r segment centre radius, and ϕ is adjusted to I_0. Below the longitudinal plasma conductivity $\sigma_\parallel(w)$ is assumed be linearly dependent on w. With those assumptions and at fixed p_{xm}, p_{cj} and P_n, the ratio $h = \sigma_\parallel(w_{max})/\sigma_\parallel(w_{x0})$ determines the state completely.

To begin with, we consider the configuration Fig. 1 in which $\sigma_\parallel$ = constant (h = 1) and the PF coil 0 is located rather far from p_{x0}, which results in too large currents I_n at n = 0,1. The plasma locates inside the separatrix that crosses the x-point x0. Toroidal current density $J = I_\phi/s$ is proportional to $1/r$ and the control point c2 is active (j_{max} = 2). Notice that immediate iterations of Eqs.(2)-(6) are diverging. However, if the new plasma currents I_i weakly response to the values $I_i^{(0)}$ that follow from Eq.(6), namely as $I_i = (1-\varepsilon)I_i^{(pred)} + \varepsilon I_i^{(0)}$ with $\varepsilon \approx 0.2$, the iterations become stable. To ensure the stability below $\varepsilon = 0.1$ is used.

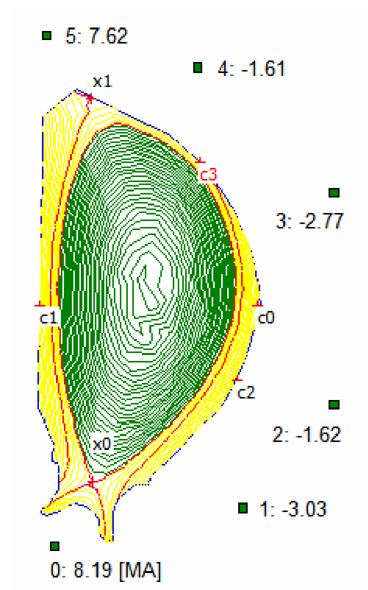

Fig. 2 The state with h = 1, P_0 = (4.391,-4.446) and p_{c3} = (7.289,3.258) [m]

A search for better configurations was then carried out. The points p_{xm}, p_{cj} and P_n, have been let to move slowly after each iteration, for example as

$$P_n = P_n^{(pred)} + (p_{c1} - P_n^{(pred)})\delta$$

at $n = 0$ and $\delta = 10^{-3}$ during 100 iterations. Visual examinations of slowly changing state when trying different moves allowed eventually the configuration Fig. 2. In the new state larger plasma volume compared to Fig. 1 is achieved, the coil currents acquired rather moderate values and the control point c3 became active. The vertical shift of P_0 for about 1 m and some minor displacements of p_{x0}, p_{x1}, p_{c3} and P_1 have been done.

It seems that further improvement by movements of coil positions is not reasonable, because the states depend also on the distribution of plasma currents, i.e. on h in our case. As to a dynamical optimisation driving p_{xm} and p_{cj} by plasma currents, it is not addressed here. Finally the positions remain the same as in Fig. 2 and h is varied. The configurations at $h = 4.13$ (a peaked plasma current profile) and $h = 0.39$ (a hollow current profile) and the respective profiles are shown in Figs. 3, 4 and 5.

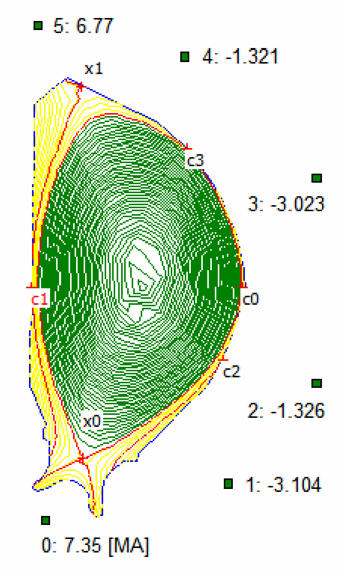

Fig. 3 The state at h = 4.13

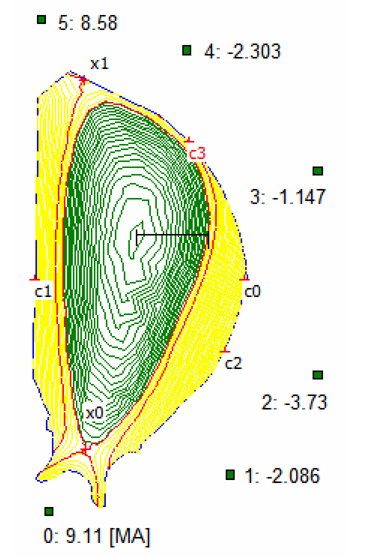

Fig. 4 The state at h = 0.39

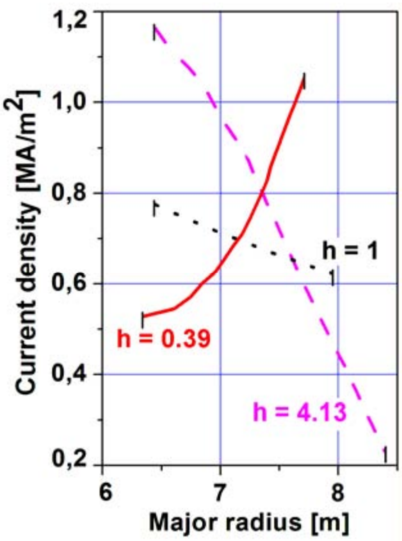

Fig.5 Plasma current profiles

When the plasma currents approach the boundary, some coil currents increase substantially. As we see, the separatrix that crosses the x-point x_0 always locates not appropriately in the outer divertor leg. While keeping moderate I_n, it is necessary but was impossible to shift the separatrix strike point (SSP) to the vertical divertor surface. Therefore additional coil near the divertor leg seems necessary for the required shift of SSP.

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References: 1. I.S. Landman, G. Janeschitz, "Contamination of ITER Core by Impurities of Tungsten and Carbon", 8th Intern. Symposium on Fus. Nucl. Techn., Heidelberg, Germany, Sept. 2007, paper #179, to be published., 2. L.D. Landau, E.M. Lifshitz, Course of theoretical physics, Moscow, "Nauka", 1982, Vol. 8, §30, Problem 2 (in Russian).