*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-5.056 (2008)*

Self-consistent energetic particle nonlinear dynamics due to shear Alfvén wave excitations*

Fulvio Zonca[1] and Liu Chen[2]

[1]*Associazione Euratom-ENEA sulla Fusione, C.P. 65 - I-00044 - Frascati, Rome, Italy*

[2]*Dept. of Physics and Astronomy, University of California, Irvine CA 92697-4575, U.S.A.*

Introduction.

We adopt the 4-wave modulation interaction model, introduced by Chen et al [1] for analyzing modulational instabilities of the radial envelope of Ion Temperature Gradient (ITG) driven modes in toroidal geometry, extending it to the modulations on the fast particle distribution function due to nonlinear Alfvénic mode dynamics, as proposed in Ref. [2]. In the case where the wave-particle interactions are non-perturbative and strongly influence the mode evolution, as in the case of Energetic Particle Modes (EPM) [3], radial distortions (redistributions) of the fast ion source dominate the mode nonlinear dynamics. In this work, we show that the resonant particle motion is secular with a time-scale inversely proportional to the mode amplitude [4] and that the time evolution of the EPM radial envelope can be cast into the form of a nonlinear Schrödinger equation a la Ginzburg-Landau [5, 6].

Long time-scale fast ion nonlinear behaviors.

Three dimensional gyrokinetic simulations show evidence of radial fragmentation of a single n (toroidal mode number) coherent EPM [2, 5, 6]. This radial fragmentation suggests the excitation of low frequency axisymmetric perturbations, for which a nonlinear mechanism is necessary, in analogy with the modulational instability of a single toroidal drift wave [1], where toroidal geometry plays a crucial role [4, 5, 6].

The generalization to e.m. fluctuations [7] of the 4-wave modulation interaction model, introduced by Chen et al [1], is based on the description of the nonlinear dynamics of a single n Alfvén Eigenmode (AE) or EPM given by a “pump” (0) mode, which interacts with a “zonal” toroidally and poloidally symmetric mode generating $(\pm)$ sidebands: i.e.,

$$\delta\phi_0(\text{pump}) = e^{i\int n\theta_k dq + in\varphi} \sum_m e^{-im\vartheta} \delta\phi_m + c.c$$

$$\delta\phi_z(\text{zonal mode}) = e^{i\int K_z dr} \delta\phi_z + c.c.$$

*Work supported by the Euratom Communities under the contract of Association between EURATOM/ENEA, by U.S. DOE Contract DE-AC02-CHO-3073, by NSF Grant ATM-0335279 and by the Guangbiao Foundation of Zhejiang University.

$$\delta\phi_{\pm}(\text{sidebands}) = \begin{pmatrix} e^{i\int n\theta_k dq} \\ e^{-i\int n\theta_k^* dq} \end{pmatrix} e^{\pm in\varphi + i\int K_z dr} \sum_m e^{\mp im\vartheta} \delta\phi_m^{(\pm)} + c.c. \tag{1}$$

with $\delta\phi_{\text{EPM}} = \delta\phi_0 + \delta\phi_+ + \delta\phi_- + c.c.$ and similarly for the parallel vector potential. Here, we have used (r,ϑ,φ) straight field line toroidal flux coordinates, q is the safety factor, $\theta_k = k_r/(nq')$ is the normalized $\mathbf{k}$ of the EPM radial envelope, m is the poloidal mode number and other symbols are standard. Given the strong zonal current screening by electrons on the collisionless skin depth, $\delta_e = c/\omega_{pe}$, the nonlinear equation for $\delta A_{\|,z}$ is $\delta j_{\|i,z} + \delta j_{\|e,z} \simeq \delta j_{\|e,z} \simeq 0$. This readily yields $\omega/(k_\| c)(\delta A_{\|,z}/\delta\phi_z) \approx k_\perp^2 \delta_e^2 \ll$, i.e. $\delta A_{\|,z}$ can be considered a passive scalar, which weakly influences the mode dynamics [7]. Introducing $\mathbf{b}\cdot\nabla\delta\psi \equiv -(1/c)\partial_t \delta A_\|$, pump EPM and sidebands obey the following nonlinear quasineutrality and vorticity equations [7]

$$\frac{ne^2}{T_i}\left(1+\frac{T_i}{T_e}\right)\delta\phi_k = \left\langle eJ_0\overline{\delta H}_i\right\rangle_k - \left\langle e\overline{\delta H}_e\right\rangle_k \tag{2}$$

$$B\mathbf{b}\cdot\nabla\left(-\nabla_\perp^2 \frac{\mathbf{b}\cdot\nabla\delta\psi}{B}\right) + \frac{\omega^2}{v_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right)\left(-\nabla_\perp^2\delta\phi\right) - \frac{4\pi}{c^2}\sum_s \langle e\omega\omega_d J_0 \overline{\delta H}_k\rangle =$$
$$= \frac{\mathbf{b}\cdot(\mathbf{k}_\perp''\times\mathbf{k}_\perp')}{cB}\partial_t\left(\delta A_{\|,k'}\nabla_\perp^2\delta A_{\|k''} - \frac{c^2}{v_A^2}\delta\phi_{k'}\nabla_\perp^2\delta\phi_{k''}\right) \tag{3}$$

where v_A is the Alfvén speed, ω_{*pi} the thermal ion diamagnetic frequency, ω_d the magnetic drift frequency, angular brackets indicate velocity space integration and $\mathbf{k} = \mathbf{k}' + \mathbf{k}''$. Furthermore, $\sum_s$ in Eq. (3) indicates summation on all thermal and energetic particle plasma species, $J_0 = J_0(\lambda)$ ($\lambda = k_\perp v_\perp/\omega_c$) is the Bessel function accounting for finite Larmor radius effects, and $\overline{\delta H}_k$ satisfies the nonlinear gyrokinetic equation [8]

$$\left(\partial_t + v_\|\mathbf{b}\cdot\nabla + i\omega_d\right)_k \overline{\delta H}_k = i\frac{e}{m}QF_0 J_0(\lambda)\delta L_k - \frac{c}{B}\mathbf{b}\cdot\left(\mathbf{k}_\perp''\times\mathbf{k}_\perp'\right)J_0(\lambda')\delta L_{k'}\overline{\delta H}_{k''} \ , \tag{4}$$

with $\delta L_k = \delta\phi_k - (v_\|/c)\delta A_{\|k}$, $QF_0 = 2\omega_k \partial F_0/\partial v^2 + \mathbf{k}\cdot\hat{\mathbf{b}}\times\nabla F_0/\omega_c$ and where, as usual, the perturbed particle distribution function has been decomposed as $\delta F = 2(e/m)\delta\phi\partial F_0/\partial v^2 + \sum_{\mathbf{k}_\perp}\exp\left(-i\mathbf{k}_\perp\cdot\mathbf{v}\times\hat{\mathbf{b}}/\omega_c\right)\overline{\delta H}_k$, F_0 being the equilibrium particle distribution function. Finally, the evolution of the zonal mode $\delta\phi_z$ is given by [7]

$$\begin{aligned}\partial_t\chi_{iz}\delta\phi_z &= (c/B)k_\vartheta K_z K_z^2\rho_{Li}^2\left[\left(\alpha_0 - \left|k_\| v_A/\omega_0\right|^2\right)\left\langle\left\langle|\Psi_0|^2\right\rangle\right\rangle + 2\alpha_0\mathbb{R}\text{e}\left\langle\left\langle(\Phi_0-\Psi_0)^*\Psi_0\right\rangle\right\rangle\right. \\ &\left. +\alpha_0\left\langle\left\langle|\Phi_0-\Psi_0|^2\right\rangle\right\rangle\right]\left(A_0^*A_+ - A_0A_-\right) \ .\end{aligned} \tag{5}$$

Here, $\chi_{iz} \simeq 1.6q^2K_z^2\rho_i^2/(r/R_0)^{1/2}$ [9], R_0 is the torus major radius, $\rho_i = (T_i/m_i)^{1/2}/\omega_{ci}$ is the ion Larmor radius, $\alpha_0 \equiv 1 + \delta P_{\perp i0}/(ne\delta\phi_0)$ [7], $(\delta\phi_k, \delta\psi_k) = A_k(\phi_0(n_0q-m), \psi_0(n_0q-m))$, with $k = (0),(\pm)$, $(\Phi_0(\theta),\Psi_0(\theta)) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\exp[i(n_0q-m)\theta](\phi_0(n_0q-m),\psi_0(n_0q-$

$m))d(n_0q-m)$ and $\langle\langle\ldots\rangle\rangle=\int_{-\infty}^{\infty}(\ldots)d\theta$. Thus, $\Phi_0(\theta)$ and $\Psi_0(\theta)$ represent $\delta\phi_k$ and $\delta A_{\parallel k}$ parallel mode structures, respectively, while A_0 and $A_\pm$ the amplitudes of pump and sideband fluctuations. Equation (5) demonstrates the exact cancellation of zonal flows and currents for a pure shear Alfvén wave, for which the r.h.s. of Eq. (3) also vanishes identically.

Thermal particle nonlinearities enter both Eqs. (2) and (3). In Eq. (3), thermal particle nonlinearities are dominated by r.h.s., while fast particle nonlinearities enter via the ballooning-interchange term only (last on the l.h.s.), since they carry pressure but not inertia [2]. Similarly to e.s. case [1], Eqs. (2) to (5) allow the spontaneous excitation of zonal flows via modulational instability of the radial envelope of drift Alfvén waves and AE/EPM [7, 10]. This is the dominant nonlinear dynamics for weak instability drive [5, 6]. For strong unstable conditions, typical of EPM excitations above threshold, nonlinear evolutions are predominantly affected by spontaneous generation of radial modulations in the fast ion profiles [2, 5, 6]. It can be shown that these radial modulations are spontaneously produced with a nonlinear growth rate $\Gamma_z\propto\delta B_\vartheta/B$ and a corresponding frequency shift (line splitting) $\Delta_z\propto\delta B_\vartheta/B$, where [2]

$$\omega_z=\Delta_z+i\Gamma_z=\pm 3^{1/4}\mathrm{e}^{\pm i\pi/4}|\Delta_L\partial D_R/\partial\omega_0|^{-1/2}\gamma_M\ ,$$

$$\gamma_M^2=2\pi^2\left(\Delta'+\frac{r}{R_0}\right)\frac{n_ET_E}{B^2}\frac{v_E^2}{v_A^2}(k_\vartheta q\rho_E)^4K_z^2v_E^2\left|\frac{e_EA_0}{T_E}\right|^2\sum_\ell\sum_{\sigma=\pm}\left(\left|\frac{\omega_0^2}{\omega_A^2}-\frac{1}{4}\right|+\sigma|\Lambda_0|\right)$$

$$\left\langle\frac{QF_{0E}}{\omega_0}\frac{v_E^2}{n_E}\delta(\mathscr{L}_{\ell,\sigma}^{(lin)}/\omega_0)\left\langle\left\langle J_0^2(\lambda)\frac{\ell^2J_\ell^2(\lambda_d)}{\lambda_d^2}\right\rangle\right\rangle^2J_0^2(\lambda_z)\left(\frac{v_\perp^2}{2v_E^2}+\frac{v_\parallel^2}{v_E^2}\right)^4\right\rangle\ ,\qquad(6)$$

with D_R the real part of the EPM dispersion function, $\Delta_L=(nq')^2(\partial^2D_R/\partial k_r^2)(\partial D_R/\partial\omega_0)^{-1}$ $[1-\cos(K_z/nq')]$ is the linear frequency shift due to radial modulations, Δ' is the Shafranov shift, n_E, T_E and e_E the energetic particle density, temperature and electric charge, $v_E^2=T_E/m_E$, $\omega_A=v_A/qR_0$, Λ_0 is a measure of EPM continuum damping, $\lambda_z\equiv K_zq(v_\perp^2/2+v_\parallel^2)/(\omega_{ci}v_\parallel)$ and $\lambda_d\equiv k_\perp q(v_\perp^2/2+v_\parallel^2)/(\omega_{ci}v_\parallel)$. The (inverse) linear propagator, $\mathscr{L}_{\ell,\sigma}^{(lin)}$, is readily obtained from the (inverse) nonlinear propagator expression

$$\mathscr{L}_{\ell,\sigma}=(v_\parallel/qR_0)(\ell+\Lambda_0+\sigma/2)-\omega_0+(k_\vartheta^2c^2/B^2)K_z^2J_0^2(\lambda_z)(M_{\ell,\sigma}/\omega_z)\left(|A_+|^2+|A_-|^2\right)\ ,\qquad(7)$$

where $M_{\ell,\sigma}=(1/2)(|\omega_0^2/\omega_A^2-1/4|+\sigma|\Lambda_0|)\left\langle\left\langle J_0^2(\lambda_0)J_\ell^2(\lambda_d)(k_\vartheta^2/k_\perp^2)\ell^2v_\parallel^2/(q^2R_0^2\omega_0^2)\right\rangle\right\rangle$.

Nonlinear radial envelope equations.

The spontaneous generation of radial modulation in the fast ion profiles are due to "zonal" fast ion responses $\overline{\delta H_{Ez}}$ and are accompanied by frequency splitting of EPM spectral lines $\propto\delta B_\vartheta/B$. For strong modulations, the scaling becomes $\omega_z\propto(\delta B_\vartheta/B)^{2/3}$ [2]. Once the "zonal" response $\overline{\delta H_{Ez}}$ is computed, fast ion transport equations are readily derived [2, 5, 6]. Fast particle nonlinearities enter in Eq. (3) via the last term on the l.h.s., through $\overline{\delta H_{kE}}\propto\delta L_{k'}\overline{\delta H_{Ez}}$

computed from Eq. (4). This term describes "hole-clump" dynamics in phase space [11], when separation of nonlinear time scales applies, but, more generally, it accounts for phase space structure formation where equilibrium geometry and wave-particle resonances play a major role [4, 5, 6, 12]. Resonant particles are more efficiently scattered (than nonresonant particles) out of the resonance region and dictate the characteristic non-linear time scaling $\tau_{NL} \propto |A_0|^{-1}$ [4, 5]. This can be seen from Eq. (7), where $\mathscr{L}_{\ell,\sigma}^{(lin)} \simeq -i\partial_t$ for resonant particles; meanwhile, $\omega_z \simeq -i\partial_t$. Thus, the nonlinear propagator structure demonstrates radial particle motion with characteristic velocity $\propto |A_0|$, consistent with the Mode Particle Pumping process [13], introduced originally for energetic particle transport due to fishbones.

Nonlinear partial differential equations (PDE) for the slow radial and time evolution of the EPM and sideband radial envelopes can be derived within this approach. The EPM radial envelope equation can be put in the form of a complex Ginzburg-Landau equation [5]

$$\left(\partial_t + i\omega_0 + \frac{i}{2}\frac{\partial^2 D_R/\partial k_r^2}{\partial D_R/\partial\omega_0}\partial_\xi^2\right) A_0 = -iL_{NL}^2 \partial_\xi^2 \left(\gamma_L |A_0|^2\right) A_0 \ , \tag{8}$$

where $\xi = r - v_{gr}t$, v_{gr} is the EPM radial group velocity, L_{NL} is a nonlinear characteristic scale-length and γ_L the energetic particle drive; while nonlinear sideband envelope equations are

$$\begin{aligned} D_+\partial_t^2 (A_0^* A_+) &= -i\gamma_M^2 (2A_0^* A_+ + A_0 A_-) \ , \\ D_-\partial_t^2 (A_0 A_-) &= i\gamma_M^2 (A_0^* A_+ + 2A_0 A_-) \ , \end{aligned} \tag{9}$$

where $D_\pm$ are the linear EPM sideband dispersion functions, $D_\pm \simeq [\partial(D_R \pm iD_I)/\partial\omega_0](\mp i\partial_t) - (\partial D_R/\partial\omega_0)\Delta_L$ and D_I is the imaginary part of the EPM dispersion function.

Acknowledgments

Useful and stimulating discussions with S. Briguglio, G. Fogaccia, G. Vlad and A.V. Milovanov are kindly acknowledged.

References

[1] L. Chen et al, Phys. Plasmas **7** 3129 (2000)
[2] F. Zonca et al, Theory of Fusion Plasmas (Bologna: SIF) 17 (2000)
[3] L. Chen, Phys. Plasmas **1** 1519 (1994)
[4] F. Zonca et al, Nucl. Fusion **45** 477 (2005)
[5] F. Zonca et al, Plasma Phys. Contr. Fusion **48** B15 (2006)
[6] L. Chen and F. Zonca, Nucl. Fusion **47** S727 (2007)
[7] L. Chen, Z. Lin, R.B. White and F. Zonca, Nucl. Fusion **41** 747 (2001)
[8] E.A. Frieman and L. Chen, Phys. Fluids **25** 502 (1982)
[9] M.N. Rosenbluth and F.L. Hinton, Phys. Rev. Lett. **80** 724 (1998)
[10] P.N. Guzdar, R.G. Kleva, A. Das and P.K. Kaw, Phys. Rev. Lett. **87** 015001 (2001)
[11] H.L. Berk et al, Phys. Lett. A **234** 213 (1997)
[12] Y. Elskens and D.F. Escande, *Microsc. dynam. of plasmas and chaos* (Bristol: IOP) (2003)
[13] R.B. White et al, Phys. Fluids **26** 2958, (1983)