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Unstable drift-kinetic Alfven modes in the lowest part of the Alfvenic spectrum

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INTRODUCTION. A large number of the experimental data in facilities with weak shear indicates unstable modes in the lowest part of the Alfvenic spectrum. Such electromagnetic activity can lead to either deterioration of plasma confinement or be used in diagnostic purposes. It is therefore important to understand the whole variety of the electromagnetic modes which can possibly exist there. In a number of modern facilities the rotational transform of the magnetic field has weak shear to avoid low order resonant rational surfaces and thus suppress formation of magnetic islands and undesirable magnetohydrodynamic (MHD) activity. In such facilities the value of the rotational transform is close to a low order rational number in the entire radial domain. It a mode has helicity close to the rotational transform, its Alfven continuum lies in the range of very small frequencies and can be comparable to the local diamagnetic frequencies of the background species. If the local electron diamagnetic frequency curve crosses the Alfven continuum at some point, the shear Alfven waves and drift waves can strongly interact with each other through the parallel electric field at the locations where their local frequencies match, which generates drift-kinetic Alfven modes (DKAEs) . Such a mode, propagating in the direction of the electron diamagnetic drift, can become unstable when there is a significant number of the background electrons with diamagnetic frequencies exceeding that of the mode.
In the present study we perform a local analysis of a DKAE in a plasma with low beta~m_e/m_i, hot electrons, cold ions, and non-uniform density. The DKAE considered here is produced by coupling between the shear Alfven wave and the electron drift wave having the same poloidal numbers determined by the closeness of the mode helicities to the rotational transform.

BASIC EQUATIONS. To derive the equations governing the shear Alfven dynamics in a toroidal plasma with finite parallel electric field taken into account, we assume that the plasma has low beta, large aspect ratio, non-uniform density, but constant temperature. The background distribution function is assumed to be Maxwellian. For analytical treatment we use the magnetic Boozer coordinates to represent the magnetic field and neglect the ψ and θ covariant components of the field for the low-pressure plasma. Additionally, since we consider coupling only between the modes with same poloidal numbers, which is mediated by the parallel electric field, we neglect any dependence of the magnetic field strength on the angular coordinates and approximate the magnetic surfaces to have circular shape. It can be shown that after these assumptions normal modes corresponding to the DKAEs can be found from the two following equations,

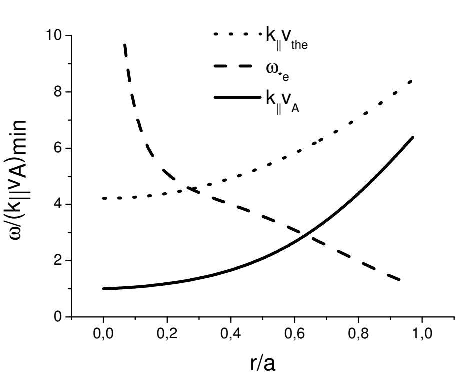
$$\frac{1}{r^2}\frac{d}{dr}r^3\left(\frac{\omega^2}{V_A^2}-k_\parallel^2\right)\frac{d}{dr}\frac{\phi_{mn}}{r}-\frac{m^2-1}{r^2}\left(\frac{\omega^2}{V_A^2}-k_\parallel^2\right)\phi_{mn}+G(r)\phi_{mn}$$
$$=\left(-\frac{k_\parallel m^2}{r^2}+\frac{k_\parallel}{r}\frac{d}{dr}r\frac{d}{dr}\right)k_\parallel\left(\psi_{mn}-\phi_{mn}\right)\quad(1)$$

with $A_\parallel \equiv k\psi/\omega$, which is basically MHD vorticity equation with the right hand side proportional to the parallel electric field , and the parallel electric field can be found from the following equation,

$$(\phi_{mn}-\psi_{mn})\left(1-\frac{\omega_{*e}}{\omega}\right)\frac{1+\varsigma_e Z(\varsigma_e)}{V_A^2}=\rho_s^2\left(-\frac{m^2}{r^2V_A^2}+\frac{1}{r}\frac{d}{dr}\frac{r}{V_A^2}\frac{d}{dr}\right)\phi_{mn}\quad(2)$$

with $\varsigma_e=\dfrac{\omega}{k_\parallel v_{the}}$.

DRIFT-KINETIC ALFVEN MODES. To see how the coupling between a shear Alfven wave and a drift wave gives rise to a family of discrete drift-Alfven modes we first note that such a mode should spatially reside close to the radial position where the local electron diamagnetic frequency is equal to the local frequency of the Alfven continuum. The mode frequency should also be close to this frequency. Consequently, we expand the equations given in the previous section in Taylor series to the linear order around the crossing point in the radial domain and around the corresponding frequency in the frequency domain. By performing this procedure followed by the Fourier transform, after an appropriate variable transformation the resulting equation can be shown to have the form of Schroedinger equation (this procedure is defined only



for $\omega > k_{\|} V_A$) :

$\frac{d^2}{dy^2}\Psi + (\varepsilon - U)\Psi = 0$, where y is normalized Fourier variable, energy is a function of the mode frequency and the potential can be represented in the following form, $U(y) = \sigma \frac{(1+y^2)^2}{(f+y^2)} + U_0(y)$ with $\sigma = \frac{1}{1+\varsigma_e Z(\varsigma_e)} \frac{k_{\|}^2}{F_0' a} \frac{\omega}{\omega_{*e}' a} \frac{\rho_s^2}{a^2}$. The first term in this expression dominates for large values of y and has the form of a harmonic oscillator potential. The second term describes a potential well centered at 0. In practice, owing to the fact that the potential well for small values of y is too shallow to contain a bounded state (this is a manifestation of the absence of discrete modes in the monotonous parts of the Alfven continuum, i.e. far from its maxima and minima, in the framework of ideal MHD with the parallel electric field set to zero), bounded states with small energies are also caused mainly by the harmonic oscillator part of the potential, so that the spectrum of the harmonic potential $\varepsilon_l = \sigma(2l+1)$ is generally applicable when estimating the spectrum of the whole problem.The largest error in using the harmonic oscillator spectrum is for the fundamental solution, which can be improved by use of a variational method. Then, dispersion relation of the DKAEs is determined from the harmonic oscillator spectrum by substituting the corresponding expressions for the energy and sigma in terms of the mode frequency. This procedure leads to a dispersion relation of the following form,

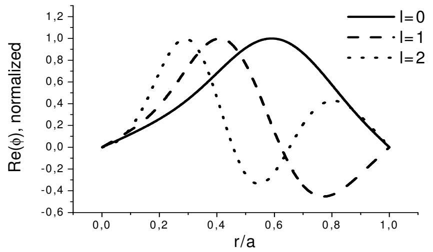
$h(\omega) = (2l+1)c$, where on the left hand side one has a real function of the complex argument and on the right hand side there is a complex constant. This form allows an estimate of the maximum growth rate for the family of DKAEs with different number of radial nodes,

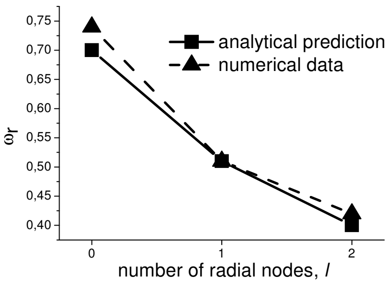
$$\gamma_{\max} = \frac{\mathrm{Im}(c)}{\mathrm{Re}(c)} \max_{\omega_r > k_{\|} V_A} \frac{h(\omega_r)}{\partial h(\omega_r)/\partial \omega_r} = \frac{\mathrm{Im}(c)}{\mathrm{Re}(c)} \max_{\omega_r > k_{\|} V_A} F(\omega_r),$$

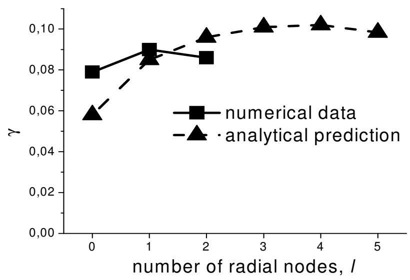
where real part of the frequency runs over all allowed values (greater than the minimum of the Alfven continuum).

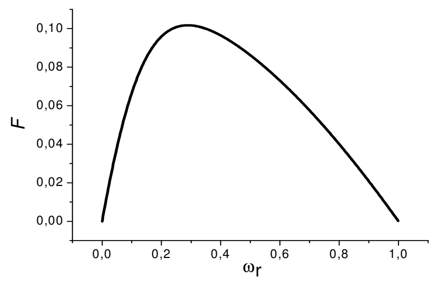
NUMERICAL RESULTS. In order to verify the outcomes of the analytical analysis we investigate Eqs. (1) and (2) by a Ritz-Galerkin method with B-splines as finite element basis.

As an example we use a plasma with parameters and profiles relevant to HSX, which is an operating optimized stellarator with low shear [2]. The background magnetic field strength is chosen to be equal to B=0.5 T, and the rotational transform $\iota = \iota_0 + (\iota_1 - \iota_0) r^2 / a^2$ with with $\iota_0 = 1.05$ and $\iota_1 = 1.1$. The appropriate mode has then n=1 and m=1. The particle density of the background plasma is taken to be $n = n_0 + (n_1 - n_0)\dfrac{(\tanh(r./a - s_*)/\Delta) - \tanh(-s_*)/\Delta))}{(\tanh(1 - s_*)/\Delta) - \tanh(-s_*)/\Delta))}$ with $n_0 = 2.e18$, $n_1 = 0.01 n_0$, $s_* = 0.3$, and $\Delta = 0.4$.The background temperature chosen is T_e(r)=1.5 keV, and T_i(r)=20 eV. For these parameters there are three DKAEs found above the Alfven minimum with different number of the radial nodes, their real frequencies are in good agreement with the analytical prediction. Although the growth rate predicted by the local analysis peaks at somewhat different radial number, this can be explained by the limitations of the local analysis. Otherwise, the maximal growth rate for the family of DKAEs agrees with the numerical results. (see Figs).









CONCLUSIONS. An electromagnetic mode in the lowest part of the Alfven spectrum possessing helicity close to the background magnetic field rotational transform arising of coupling between the modes with same poloidal numbers is treated analytically and numerically. A good agreement between the numerical data and the predicted values for the real part of the frequency and the growth rate was demonstrated.