*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-5.048 (2008)*

Multi-Fluid MHD Equilibria for ST Plasmas with Near-Sonic Flow and Shift from Trapped to Co-Circulating Ions

Akio Ishida

Niigata University, Niigata, Niigata, Japan

Loren C. Steinhauer

University of Washington, Redmond Plasma Physics Laboratory, Redmond, WA, USA

Y.-K. Martin Peng

Oak Ridge National Laboratory, Oak Ridge, TN, USA

1. Introduction

Recent equilibrium study[1)] for uniform plasma density shows that two-fluid effects increase for smaller physical size, increased beta value, and flow speed approaching the ion diamagnetic drift speed. Additional studies[2,3)] reveal that two-fluid effects dominate in magnetic wave propagation properties when wave length is comparable to or smaller than the ion inertial length (c/ω_{pi}), the intrinsic scale of the two-fluid model. Further the flow-speed singularity of the two-fluid equilibrium equations is shown[4)] to differ markedly from the one-fluid model. These results suggest that a multi-fluid model may provide new insights in understanding the high-performance NSTX plasmas with high β (~ 15%), near-sonic rotation, and a flow-shear scale length of a few cm comparable to c/ω_{pi}[5)]. Two-fluid MHD equilibrium calculated to approximate the measured profiles of toroidal plasma flow, current, densities and temperatures indicates the likelihood of total ion current comparable or greater than the toroidal plasma current, where the trapped ions are strongly shifted to the co-circulating ions. The implications of this change due to near-sonic flow for the neoclassical model[6)], momentum transport, and steady state operations are noted.

2. Multi-fluid equilibrium model

We adopt here multi-fluid MHD models to describe axisymmetric equilibria with high beta and strong flows. Here we show equations which are used commonly in a model with high collisionality (Sec. 3) and a model with low collisionality (Sec. 4). We adopt the following equations:

$$\nabla\cdot(n\mathrm{u}_\alpha)=0 \quad (1) \qquad m_\alpha n(\mathrm{u}_\alpha\cdot\nabla)\mathrm{u}_\alpha=-\nabla p_\alpha+q_\alpha n(-\nabla V_E+\mathrm{u}_\alpha\times\mathrm{B}) \quad (2)$$

$$\nabla\times\mathrm{B}=\mu_0 e n(\mathrm{u}_i-\mathrm{u}_e) \quad (3) \qquad \nabla\cdot\mathrm{B}=0 \quad (4), \qquad p_\alpha=nT_\alpha \quad (5)$$

where $\alpha = e$ denotes the electron fluid (with $m_e = 0$, $q_e = -\mathrm{e}$) and $\alpha = i$ the ion fluid. We use the MHD (fast dynamics) ordering $u_e \sim u_i \sim v_{th,i}$ assuming $\rho_i / L \sim \delta << 1$ where $v_{th,i}$ is the ion thermal velocity, ρ_i the ion gyroradius and L the scale length[7,8]. Since the gyroviscous force is of higher order in δ than ∇p_α, we neglect it in eq.(2). From eqs. (1,4) express the magnetic field B and mass flux $n\mathrm{u}_\alpha$ using a flux function $\psi(R,Z)$ and $\Phi_\alpha(R,Z)$.

$$\mathrm{B} = \nabla\psi(R,Z)\times\nabla\phi + RB_\phi\nabla\phi \quad (6) \qquad n\mathrm{u}_\alpha = \nabla\Phi_\alpha(R,Z)\times\nabla\phi + nRu_{\alpha\phi}\nabla\phi \quad (7)$$

Here cylindrical coordinates (R,ϕ,Z) representing the major radius, the (ignorable) toroidal coordinate and the axial coordinate. Rewrite eq. (2) for multi-species

$$\nabla\left(T_\alpha + m_\alpha u_\alpha^2/2 + q_\alpha V_E\right) + T_\alpha\nabla\ln n = q_\alpha \mathrm{u}_\alpha \times \mathbf{\Omega}_\alpha \quad (8)$$

where $\mathbf{\Omega}_\alpha \equiv \mathrm{B} + (m_\alpha/q_\alpha)\nabla\times\mathrm{u}_\alpha$ is the vorticity of the species canonical momentum (divided by q_α) and is the *effective* magnetic field (exactly B for massless electrons). Since $\nabla\cdot\mathbf{\Omega}_\alpha = 0$, it can be expressed as

$$\mathbf{\Omega}_\alpha = \nabla Y_\alpha \times\nabla\phi + [B_\phi - (m_\alpha/q_\alpha)R\nabla\cdot(\nabla\Phi_\alpha/nR^2)]\hat{\phi} \quad (9) \qquad Y_\alpha \equiv \psi + (m_\alpha/q_\alpha)Ru_{\alpha\phi} \quad (10)$$

Here Y_α is the drift-surface variable for species α and $Y_e = \psi$. Since the toroidal component of the right hand side in eq. (8) must vanish because of axisymmetry, this leads to

$$\Phi_\alpha = \Phi_\alpha(Y_\alpha) \quad (11)$$

Note that the common form of eq. (8) for each species shows the natural symmetry emerging in the multi-fluid formulation unlike in single-fluid MHD. The toroidal and poloidal components of Ampere's law (3) yield respectively

$$-R\nabla\cdot\left(\nabla\psi/R^2\right) = \mu_0 en(u_{i\phi} - u_{e\phi}) \quad (12) \qquad RB_\phi = \mu_0 e\left(\Phi_i(Y_i) - \Phi_e(\psi)\right) \quad (13)$$

3. Equilibrium with high collisionality

We supplement eqs. (1-5) with the adiabatic relations, assuming negligible heat conduction compared with heat convection by each species.

$$\mathrm{u}_\alpha\cdot\nabla S_\alpha = 0 \quad \text{for} \quad \alpha = e,i \quad (14)$$

where $S_\alpha \equiv T_\alpha / n^{\gamma-1}$ represents the entropy. Using eqs.(7,11), eq. (14) yields

$$S_\alpha = S_\alpha(Y_\alpha) \quad (15)$$

Using $T_\alpha\nabla\ln n = \nabla T_\alpha/(\gamma-1) - n^{\gamma-1}\nabla S_\alpha/(\gamma-1)$ and taking the dot product of $\mathbf{\Omega}_\alpha$ with eq.(8) leads, after an integration to the Bernoulli equations:

$$H_\alpha \equiv \gamma T_\alpha/(\gamma-1) + m_\alpha u_\alpha^2/2 + q_\alpha V_E = H_\alpha(Y_\alpha) \quad (16)$$

Using six arbitrary functions defined by eqs. (11), (15) and (16), the multi-fluid equilibrium can be described by simultaneous equations for $Y_e = \psi$ and Y_i[9].

4. Equilibrium with low collisionality

Before addressing alternatives to the adiabatic relation eq. (15), consider when it is valid using Braginskii's formula for axisymmetric equilibria:

$$\left|\nabla\cdot(\kappa_{\|}^{\alpha}\nabla_{\|}T_{\alpha})/1.5n_{\alpha}\mathrm{u}_{\alpha}\cdot\nabla T_{\alpha}\right| \sim \left(v_{th,\alpha}/u_{\alpha p}\right)\left(B_p/B\right)^2\left(\lambda_{\alpha}/L_p\right) < 1 \qquad (17)$$

where λ_{α} is the mean-free path, $u_{\alpha p}$ the poloidal flow velocity, $v_{th,\alpha}$ the thermal velocity and L_p is the length scale in the poloidal direction. Since $u_{ep} \sim u_{ip}$, $\lambda_e/\lambda_i \sim (T_e/T_i)^2 Z^2$ and $\max L_p \approx \kappa a$ where Z is the ion charge state, κ the elongation of the separatrix and a the radius, the above conditions fail in NSTX and TCS. Hence it is useful to find a replacement for eq. (15) that is valid in low-collisionality plasmas, i.e. when heat conduction is comparable to heat convection. Allowing for anisotropic temperature, one could use the heat-flux evolution equations of eqs. (67,68) of Ref. 7. However, for well-confined plasmas the temperature anisotropy should be small, then the relations in Ref. 7 reduce to $\mathbf{\Omega}_{\alpha}\cdot\nabla T_{\alpha} = 0$ (where $\mathbf{\Omega}_{\alpha}$ is the proper generalization of B), i.e. isothermal surfaces. Therefore

$$T_{\alpha} = T_{\alpha}(Y_{\alpha}) \qquad (18)$$

This is the counterpart of eq.(15). Using eq.(18) in eq.(8) leads, after an integration, to the form of the Bernoulli equations for weak collisionality

$$F_{\alpha} \equiv T_{\alpha}(Y_{\alpha}) + m_{\alpha}u_{\alpha}^2/2 + q_{\alpha}V_E + T_{\alpha}(Y_{\alpha})\ln n = F_{\alpha}(Y_{\alpha}) \qquad (19)$$

Using six arbitrary functions defined by eqs. (11), (18) and (19) the equilibrium can be described by the simultaneous equations for $Y_e = \psi$ and Y_i.

5. Example NSTX-like equilibrium and physics implications

An NSTX-like plasma with $\beta_{\text{t-thermal}} \sim 15\%$, $I_p \sim 700$ kA, $T_{i0} \sim 1.2$ keV, $T_{e0} \sim 0.8$ keV, $q_0 >$ 1, and maximum Mach number M ~ 0.7 were calculated in 2D with high collisionality. The calculated 2-fluid equilibrium shows that, among other features, the toroidal ion and electron flows are in the co-current direction (Fig.1), indicating that the total ion current exceeds substantially the total plasma current. This implies that the centroid of a Maxwellian ion distribution is shifted by about one ion thermal velocity V_{th} in the co-current direction (Fig. 2), substantially reducing the fraction of trapped ions in favor of increased co-circulating ions, when viewed in the in the $V_{\perp}$ and $V_{\|}$ space on the outboard mid-plane of the plasma.

Potentially important questions are raised by this change of the thermal ion distribution due to near-sonic flows. How much does this alter the neoclassical[6)] modeling and the associated plasma transport (energy, momentum, particles including impurities)? Does the large shift toward co-circulating ions introduce new free energy that drives instabilities and turbulences potentially leading to enhanced momentum transport? How does the large ion current change the Ohms law and alter the externally driven and bootstrap currents on the electrons needed for steady state operation? These and other related questions in view of the observed confinement properties on NSTX[10] will be discussed. Computed example equilibria will be discussed, including a three-fluid model with beam ion component. A numerical code for low-collisionality equilibrium is under construction. Helpful discussions with Steve Hirshman and Ron Bell, and USDOE funding for this work are acknowledged.

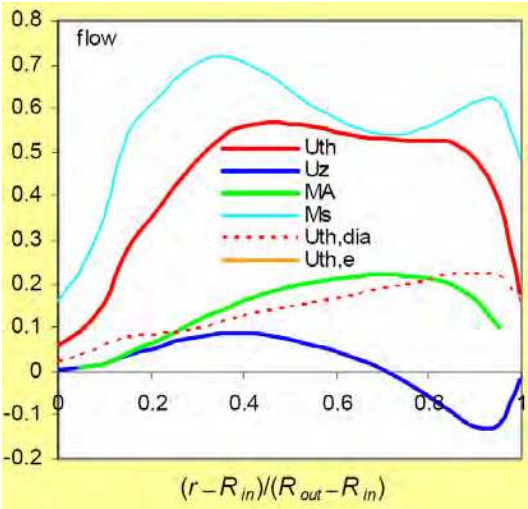

Figure 1. Mid-plane flow profiles of a NSTX-like 2-fluid equilibrium, Uth = toroidal ion flow, Ms = Mach number, Uth,e = toroidal electron flow, normalized to the ion sound velocity V_s.

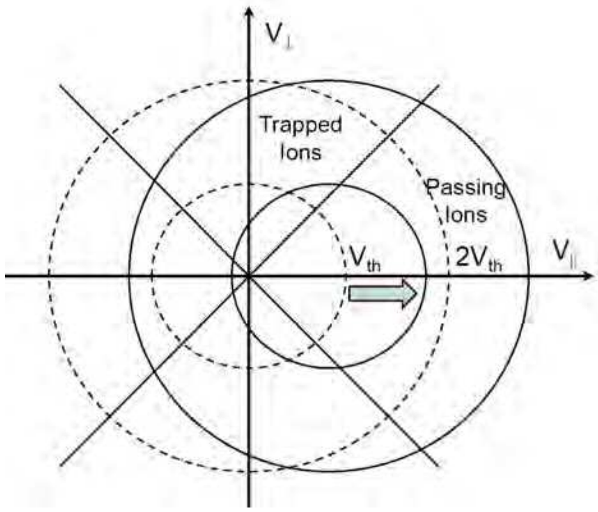

Figure 2. Maxwellian ion distribution on the outboard mid-plane shifted by ~V_{th} in the co-current direction, implying a factor of 2-3 reduction in the trapped fraction in case of an initial 50% fraction.

References

1) H.Yamada, T.Katano, K.Kanai, et al., Phys. Plasmas, 9 (2002), 4605.

2) A.Ishida, C.Z.Cheng and Y-K.M.Peng, Phys. Plasmas, 12 (2005), 052113.

3) E. Hameiri, A.Ishizawa and A. Ishida, Phys. Plasmas, 12 (2005), 072109.

4) A.Ishida, C.O.Harahap, L.C.Steinhauer et al., Phys. Plasmas, 11 (2004), 5297.

5) R. Bell, private communications; Fig. 2(b) of Y-K.M. Peng et al., Plasma Phys. Control. Fusion 47 (2005) B263.

6) F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. 42 (1976) 239.

7) J.J.Ramos, Phys. Plasmas 12 (2005), 052102.

8) J.J.Ramos, Phys. Plasmas 14 (2007), 052506.

9) L.C. Steinhauer, Phys. Plasmas, 6 (1999) 2734.

10) S.M. Kaye, et al., Nucl. Fusion 47 (2007) 499.