

Predictive Stability Analysis

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The success of a tokamak as a reactor depends on managed, safe operation at the highest possible beta. Reasonable predictions of the beta-limit are available in terms of gross plasma parameters, for example, the well-known Troyon limit, in various forms, gives a reliable estimate of an upper limit for beta. However, it does not predict the actual stability limit of a discharge, which depends on the details of the plasma profiles, which are evolving in time. Indeed, many discharges experience instability leading to disruptions well below the optimized beta-limit. The ability to determine the proximity of the stability threshold, would give the machine operators an opportunity to take corrective action to either modify the profiles to avoid the instability or trigger a controlled termination of the discharge.

Previous studies have mostly adopted the neural network approach, focusing on predictors of disruptions, a limited reference set is given below[1-3]. An attractive alternative would be to determine the stability of the plasma in near real time, using experimental data to produce equilibria and perform stability analysis, using specialized codes for the different instabilities. Ideal MHD stability codes have been used to model some disruption precursors, see References [4] and [5]. Note that this is also a limited reference set. However, stability analysis requires a highly accurate equilibrium, which requires several iterations to determine accurate plasma profiles needed for the solution of the Grad-Shafranov equation. A further complication is that intrinsic errors in experimental measurement admit sufficient latitude of subtle variations, which can modify the stability analysis. Even if experimental techniques are refined to produce detailed profile information, stability analysis requires computing numerical equilibria and mapping to magnetic coordinates before running the stability codes, which often take significant time to obtain a converged result. These considerations impel us to seek alternate methods of monitoring the stability of the plasma.

We have investigated several approaches, including; predetermining the stability for a range of relevant profiles and tracking the evolution of the discharge in that profile

parameters space; identifying a set of model eigenfunctions, which can be used to rapidly determine variations in δW , the potential energy, and hence the approach to marginal stability; as well as the use of empirical rules. We conclude that each of these methods has a limited range of validity, and the best approach is to generate a 'stability dashboard', which uses different techniques to assess stability for different modes, and present the information as multiple stability indices.

Developing a stability data base for a wide range of profiles is time consuming, but possible. Using this data base to look up the stability of the plasma requires detailed profile information, including subtle local features, which brings us back to the diagnostic issues, previously mentioned. Even if this were overcome, there is another significant issue to consider, namely that the result of such analysis is binary, stable or unstable, whereas to be useful it needs to also indicate how close the plasma is to the stability boundary and how the profiles are evolving, in relation to the stability boundary.

Ideal MHD stability is a strong indicator of the overall MHD stability, even for non-ideal modes. This suggests that it would be desirable to determine the δW of the stable mode and follow it as a function of time. The drawback of this approach is that it is very difficult to identify the specific eigenvector, which will eventually go unstable, within the sea of Alfvénic modes characteristic of the full MHD spectrum. An alternate approach is to estimate variations of δW due to changes in the plasma profiles, as was considered in Reference [6], where δW was expanded as a function of perturbations in the equilibrium profiles, and the unstable eigenfunction of the base equilibrium was used to determine the modified potential energy. Their results were not promising and this approach languished. Here, we introduce a variant, in which, we select a set of eigenvectors, $\{\xi\}$, obtained from analyzing similar equilibria, and use this set to determine a corresponding set, $\{\delta W\}$, at each time slice, we then track the minimum of this set as a function of time, to determine the approach to the stability threshold. Surprisingly, this method seems to provide a good indicator, when applied to the same class of discharges. Figure 1 shows the results for an NSTX shot, 124849, in which giant ELMs were observed, followed by a beta-collapse. The test function method shows that δW continuously decreases, becomes negative and reaches a peak-minimum approximately 30 milliseconds before the beta collapse. Note that the set of test-vectors does not include a vector from this discharge.

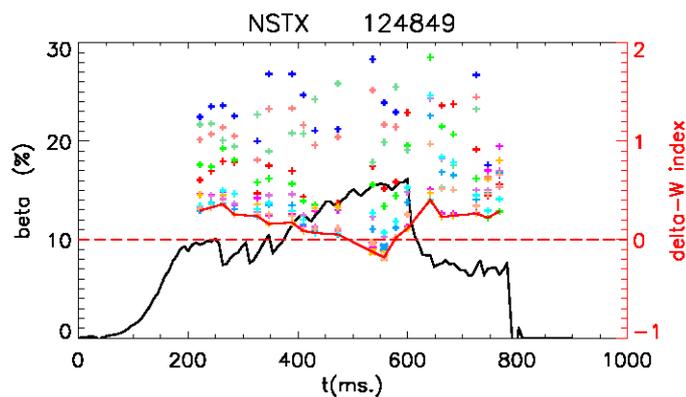


Figure 1. Evolution of β and the minimum of the potential energy, based on a pre-determined set of eigenvectors. The crosses, represent $\delta W(\xi_i)$, for each time slice. The red line tracks the minimum. Note that it steadily decreases reaching its minimum, shortly before the beta collapsed.

We have examined several discharges, with similar results; however it should be noted that the time between the peaking of negative δW and the beta-collapse varies from zero to about 50 msec. Note also that the normalization of δW , used here, is arbitrary, so one should not read too much into the numerical value, instead the focus should be on the slope of the curve and the sign. More specifically, as δW changes from positive to negative, and continues to increase in amplitude, the likelihood of an imminent beta-collapse approaches unity. The success rate, using this set of vectors, is 70%. We believe this can be improved by refining the set, $\{\xi\}$, through additions and subtractions.

We have also investigated an approach to predicting smaller drops in beta, for instance due to giant ELMs. In this approach we use an empirical function of key variables, which could play a role in determining the stability of the system. The main parameters are the beta, pressure gradient and current density near the plasma edge. Additional variables are the shear, s , ($r/q \, dq/dr$), at the plasma edge and at the location of q_{\min} ; the pressure gradient at q_{\min} , and at the low q surfaces, $q=1,2$ and 3. This function can be evaluated from standard EFIT data. The results for the same discharge, 124849, are shown in Figure 2. In this instance the stability index varies periodically in coincidence with the beta collapses. Note that there are no units and the threshold, indicated by the broken line, is set arbitrarily to match the experiment. Keeping the functional form and constants fixed, we have examined several other discharges. The results are very promising, as at least 85 per cent of the minor beta collapses are correctly predicted. The advantage of this latter approach is that it requires data, which in principle are available in near real-time, and could be used for real time prediction. The coefficients were chosen through trial and

error for expediency, A more rigorous approach would be to use SVD analysis with a larger data set.

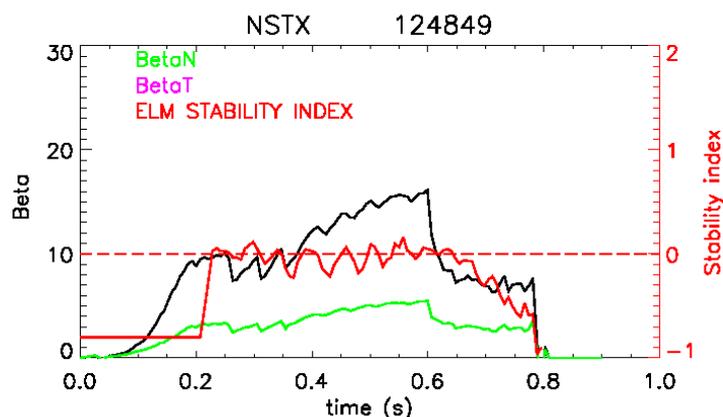


Figure 2. Evolution of β and a stability index, defined as an empirical formula, dependent on plasma equilibrium parameters. Notably, this includes; the current density and pressure gradient near the plasma edge, the location of the q -min, $q=1$, 2, and 3 surfaces; as well as the pressure gradient at those locations. The normalization was chosen such that a positive value for the index means that the plasma is unstable. Note that the index oscillates about zero, synchronously with the beta collapses.

The results reported here provide two indices for monitoring the stability of the plasma, these are based on a limited set, of twenty-five NSTX discharges. It is necessary and straightforward to supplement these with monitors for the 1/1 mode, NTMs, RWMs and locked modes. These several indices could constitute a MHD stability dashboard. This is the subject of future work.

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