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Triangularity and Ellipticity Effects on Ware Pinch

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Abstract

Ware pinch has been analyzed for general plasma configurations including magnetic surfaces with ellipticity and triangularity. The analysis is performed using a recently published system of coordinates. The Ware-pinch width is discussed for different banana sizes and triangularities.

The inward transport, Ware Pinch, due to the inductive or toroidal electric field E_φ is usually described using the large aspect ratio approximation[1,2], and constant toroidal field E_φ. In a previous work this pinch was derived by using the ∇B drift and a conventional guiding center theory for circular toroidal geometries[2]. It was also shown that the banana orbit of the particles were not with up down symmetry, however it was assumed up down symmetry in the magnetic surface. The actual situation in tokamaks is rather different, since the magnetic cross section are not circular, and they usually include ellipticity and triangularity as well as other anomalies as quadrangularity, etc. Furthermore the divertors most of the time destroy the up-down symmetry in magnetic surfaces. This implies that a more general treatment of Ware Pinch is required, which now can be performed using a general kind of coordinates of recent publications[3,4]. In the general treatments here presented, toroidal axisymmetry will be assumed (i.e. ripple effects are not considered), but non-circular geometries will be analyzed. The effect of breaking the up/down symmetry in magnetic surfaces will be also studied. Though time independent inductive electric fields can be also treated, however this will perform in future works. The treatment here performed allows also the calculation of the pinch in different parts of the magnetic surface, though here the average calculation of perpendicular displacements will be only treated in detail.

Banana width calculations in terms of ∇B drift

For trapped particles with parallel velocity $u << v$ (total velocity), the curvature drift will be a small correction to the ∇B drifts. Considering also the low β approximation, the drift velocity will be

$$\vec{v}_d = -\frac{\mu}{e_i}\frac{\nabla B \times \vec{B}}{B^2} \quad , \tag{1}$$

where μ is the adiabatic invariant $\mu = \frac{mv_\perp^2}{2B}$, e_i is the charge of the particle and $\vec{B}$ is the total magnetic field. The magnetic field $\vec{B}$ has only two components toroidal and poloidal, that is,

$\vec{B} = B_\varphi\ \hat{\varphi} + B_p\ \hat{t}$ and $B = \sqrt{B_\varphi^2 + B_p^2}$, where $\hat{\varphi}$ is the unitary vector along the toroidal angle and $\hat{t}$ is the tangent unitary vector along the meridian magnetic curves, denoted as s-curves [3,4]. In this work the triortogonal frame of the curvilinear coordinates, will be denoted as $(\hat{N}, \hat{t}, \hat{\varphi})$, where $\hat{N}$ is the unit vector normal to the surface, this vector is equal to the to $-\hat{n}$, vector used in previous works [3,4]. The unit vector along the magnetic lines is denoted here with $\hat{l}$. The drift velocity $\vec{v}_d$ and the banana width $\triangle$, can be written as

$$\vec{v}_d = \frac{\mu}{e_i B^2}\left(B_\varphi \frac{\partial B}{\partial \sigma}\hat{t} - B_\varphi \frac{\partial B}{\partial s}\hat{N} - B_p \frac{\partial B}{\partial \sigma}\hat{\varphi}\right) \quad ,$$

$$\triangle = -2\int_0^{\tau/4} \vec{v}_d \cdot \hat{N} dt \ = \ 2\frac{\mu}{e_i}\int_0^{\tau/4} \frac{B_\varphi}{B^2}\frac{\partial B}{\partial s} dt \quad , \tag{2}$$

where τ is the period of the banana orbit, $\vec{v}_d \cdot \hat{N}$ measures the displacement perpendicular to the magnetic surface, and $\tau/4$ denotes the time for the trapped particle to move from the mid plane to the reflection point with major radius R_1 and R_b, respectively. The time zeroth correspond to the mid plane position. A long and cumbersome calculation leads to

$$\triangle_p = \frac{m_i\ u_1}{e_i\ B_{p_1}}\frac{B_{\varphi_1}}{B_1}\left[\sqrt{\frac{R_b^2}{R_1\,(R_1 - R_b)}}\ \ ln\left(2\ \sqrt{\frac{R_1 (R_1 - R_b)}{R_b^2}} + 2\ \frac{R_1}{R_b} - 1\right)\right] \ , \tag{3}$$

where the term in parenthesis is about two, if $(R_1 - R_b)/R_1$ can be considered small and a linear expansion is performed, giving the usual formula with the Ware-pinch width. This result can be combined with the equation for R presented in previous works[4,5]

$$R_1 = R_m + \lambda a - \lambda^2 \triangle_s(a)\ \ ;\ \ R_b = R_m + \lambda a cos\theta_b - \lambda^2\left(\triangle(a) + aT(a)(1 - cos2\theta_b)\right)\ , \tag{4}$$

where R_m is the mayor radius of the minor axis, $E(a)$ and $T(a)$ are the elliptic and triangularity distortions, $\triangle_s(a)$ is the Shafranov shift, $2a$ is the width of the plasma along the mid-plane, λ is the parameter denoting the magnetic surface where the path of the particle is projected and θ_b and θ_b' are the poloidal angle corresponding to the largest and smallest reflection points, respectively. The coefficients of ellipticity and triangularity distortion, $E(a)$ and $T(a)$, are measured at the plasma on the 95% surface, and they are related to the ellipticity $\kappa(a)$ and triangularity $\delta(a)$ by the expressions given in in previous works [4,5]. In Figure 1, it is shown the correction factor $\tilde{\eta}$ as a function of triangularity for two different bananas, one of small angle amplitude $\theta_b = \pi/16$ and other of regular amplitude $\theta_b = \pi/4$. The correction factor $\tilde{\eta}$ is defined by

$$\tilde{\eta} = ln\left(2\ \sqrt{\frac{R_1\ (R_1 - R_b)}{R_b^2}} + 2\ \frac{R_1}{R_b} - 1\right) / \left(2\left(\ \frac{R_1\ (R_1 - R_b)}{R_b^2}\right)^{1/2}\right) \quad , \tag{5}$$

and it allows to compare the actual width of the banana with the classical Ware-pinch in the case of magnetic surfaces with up-down symmetry, where the correction for the curvature κ_σ is small and Δ_c can be neglected. As it is shown in the Figure, for small amplitudes, $\theta_b = \pi/16$ this value is about one, which means that the classical width is right. When the amplitude is not so small $\theta_b = \pi/4$, then the average correction factor is about 0.95, that is, only an small correction of about 5% is needed and the actual width is smaller than the usual one. However, that correction factor also depend on triangularity and that dependence is almost lineal, and its value decreases with increasing triangularity, which means that the differences between the actual and classical width increases with the triangularity. Furthermore that the correction discussed above, there is a second correction due to the curvature κ_σ, denoted by $\triangle_c$

$$\triangle_c = -\left(\frac{2\,\mu}{m_i\,B_1}\right)^{1/2} \frac{m_i}{e_i}\frac{B_{\varphi_1}}{B_1}\left(\frac{R_b}{R_1}\right)^{1/2} \int_0^{s_b} \frac{R^{1/2}}{(R-R_b)^{1/2}}\,\frac{B_p}{B}\,\kappa_\sigma\,ds \quad . \tag{6}$$

For magnetic surfaces with up-down symmetry κ_σ is zero at the mid-plane and very small nearby, therefore this term can be neglected for small bananas. For large bananas the contribution could be important, and it has to be calculated using the equations of the corresponding magnetic surface. For magnetic surfaces without up-down symmetry, the value of κ_σ could be important. On the other hand in this case the poloidal magnetic field at the midplanes is not perpendicular to that plane and therefore the usual treatment fails[1,2], since B_p is not in the z-direction, and the angle between $\vec{B}_p$ and $\hat{z}$ (perpendicular to the mid plane) must be considered.

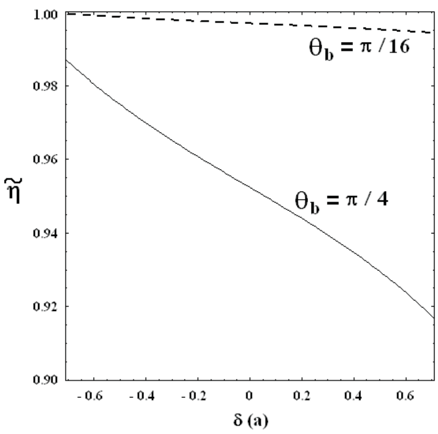

Figure 1: Correction factor to usual banana width as a function of triangularity, for two different angle amplitudes $\theta_b = \pi/16$ and $\pi/4$. The plasma characteristics are $R_0 = 3$m, $a = 0.95$ m, Shafranov shift $\Delta_s =$ a/10, ellipticity 1.2, $\tilde{\gamma}_1 = 0.3$, $B_{\varphi_1} =$ 5 Teslas and $\lambda = 1$

Up-down axysimmetry and the Ware pinch

Considering the motion along the magnetic field line, the main forces for this analysis will be the mirror force, and the projection of the electric force due to the toroidal electric field E_φ. The equation of motion along the magnetic field will be

$$\vec{F} = -\mu\nabla_{\|}B \quad ; \quad m\frac{d^2l}{dt^2} = -\mu\frac{\partial B}{\partial l} + eE_{\|}\ , \tag{7}$$

where $E_{\|}$ is the projection of the electric field along the magnetic line with unit vector $\hat{l}$. Introducing the variable Θ defined as the angle between $\hat{N}$ and $\hat{R}$, the following equation is obtained

$$\frac{d^2\Theta}{dt^2} = \omega_b^2\,(sin\alpha - sin\Theta) \quad , \tag{8}$$

where

$$\omega_b^2 = \frac{\mu}{m} \frac{B_p^2}{R\, B} \left(\frac{\partial s}{\partial \Theta} \right)^{-1} \quad , \quad sin\alpha = \frac{e_i}{\mu} \frac{R\, B_\varphi}{B_p\, B}\, E_\varphi - \frac{B_p^3}{B^2}\, R\, \kappa_\sigma\ . \tag{9}$$

A long calculation lead to the usual Ware pinch, using the period τ of the motion

$$\langle v_{w\sigma} \rangle = \frac{\Delta\sigma}{\tau} = -\frac{E_\varphi}{B_p} \frac{B_\varphi^2}{B^2} \cong -\frac{E_\varphi}{B_p} \quad , \tag{10}$$

more elaborated and precise results can also be obtained.

Conclusion

In this work, it has been shown that Ware pinch can be derived from the conventional guiding center theory for a general plasma configuration. The procedure allows also to treat more general case than the usual Ware pinch, in particular the width of the banana orbit can be determine in case where there is not up-down symmetry in magnetic surfaces and the procedure can be also extended to time dependent inductive electric field. The result here presented though in agreement with the usual Ware pinch results, however there are more precise formulas, and triangularity effects can be calculated. In the case of large bananas the calculation can be carried out if the magnetic surface shape is known. In this work, it is found that there some corrections to the usual formula for Ware-pinch width, however, this correction is usually small, i.e, about 5% for $\theta_b = \pi/4$. There is not corrections for small amplitude bananas and it appears only for regular amplitude bananas, where the actual width is smaller than the usual one. The correction factor decreases with triangularity, which means that the differences between the actual and classical width are larger for larger triangularities. These results are for magnetic surfaces with up-down symmetry. If this symmetry is not present there are additional corrections due to the curvature κ_σ, but for this calculation it is needed to know the form or the equation of the magnetic surfaces. The usual variables in the large aspect ratio approximation are (r, θ), but in the treatment here presented more general coordinates have to be used, and furthermore the most convenient angle variable now is the angle Θ between the normal at the magnetic surface and the major radius through the corresponding point.

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