Computation of radial electric field in the turbulent edge plasma of the T-10 tokamak

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It is widely recognized that the radial electric field \( E_r \) plays an important role in plasma confinement via \([E_r \times B]_x\) shearing stabilization mechanism. Theoretical description of the formation of \( E_r \) based on turbulent dynamics and, specifically, numerical modeling of the edge plasma electrostatic potential in the T-10 tokamak is the goal of this paper.

It has been shown experimentally by the direct measurements of Heavy Ion Beam Probe Diagnostics (HIBP), that electrostatic potential forms negative well \( E_r = -\frac{d\Phi}{dr} < 0 \) for Ohmic and ECRH regimes with moderate densities \( n_e = 1.5 - 2.5 \times 10^{13} \text{ cm}^{-3} \) in the T-10 tokamak [1, 2]. Transition from OH to ECRH regime was accompanied by the decrease of the absolute potential value, i.e. module of radial electric field decreased \( |E_r|_{\text{ECRH}} < |E_r|_{\text{OH}} \), while its sign still remains negative.

Numerical simulation of the T-10 regimes was based on solution of nonlinear MHD equations in the frame of reduced two-fluid Braginskii’ s hydrodynamics [3]. Approach of the plane geometry \((x,y,z)\) for magnetic field \( B = B_{0z}(x) e_z + B_{0r}(x) e_r \), \( B_{0z} = B_0(1 - \frac{x}{R}) \),

\[
B_{0r} = \frac{B_0 e}{q(r)},
\]

where \( R \) and \( a \) – major and minor radii of the torus, \( q \) is the safety factor,

\[ e = \frac{a}{R} \]

was used. The dimensionless system of MHD equations in the electrostatic approach has the form [3]:

\[
\frac{DW}{Dt} + H_{NL} = -\nabla || J - g_b \frac{\partial p_e}{\partial y} + v_\perp \cdot \Delta_n W, \tag{1}
\]

\[
\frac{Dn}{Dt} = -\nabla || J + g_b \frac{\partial (N_0 \phi - p_e)}{\partial y} + D_\perp \cdot \Delta_n n, \tag{2}
\]

\[
\frac{3}{2} \frac{Dp_e}{Dt} = -\frac{5}{2} T_{e0} \nabla || J + \frac{5}{2} K_e - w_e + \chi_{e\perp} \nabla_\perp^2 p_e + \chi_{e\perp} \cdot \nabla_\perp^2 p_e, \tag{3}
\]

\[
\frac{3}{2} \frac{Dp_i}{Dt} = -\frac{5}{2} T_{i0} \nabla || J + \frac{5}{2} K_i - w_i + \chi_{i\perp} \nabla_\perp^2 p_i + \chi_{i\perp} \cdot \nabla_\perp^2 p_i, \tag{4}
\]

\[
\eta_0 J = -\nabla || (\phi - p_e / N_0), \tag{5}
\]
Here

\[
K_x = p_{,\phi} \frac{\partial^n}{\partial y^n} - 2T \frac{\partial}{\partial y} T \frac{\partial}{\partial y} T + 3 \frac{\partial m}{\partial y} - T \frac{\partial}{\partial y} T - T \frac{\partial}{\partial y} T,
\]

\[
W = \rho^2 \cdot \nabla \cdot (\nabla \phi + \nabla \cdot p), \quad \rho = \rho_e + \rho_i.
\]  \hspace{1cm} (6)

To calculate the electrostatic potential we use the equation for the generalized vorticity, which includes both electric drift and the ion diamagnetic drift, as well as density fluctuation. It results in the appearance additional nonlinear terms \( H_{NL} \) in eq. (1):

\[
H_{NL} = \frac{\rho^2}{2} \left[ \nabla \times \left\{ \nabla \cdot (\nabla \phi + \nabla \cdot p) \right\} + \left\{ \nabla \cdot \left( \nabla \phi \right) \right\} + \left\{ \nabla \cdot \left( \nabla \phi \right) \right\} \right]
\]  \hspace{1cm} (7)

The following variable substitution was used to get eqs. (1)-(6) in presented dimensionless form:

\[
t \rightarrow \frac{t}{t_s}, \quad (x, y) \rightarrow (x / d, y / d), \quad \phi \rightarrow e \phi / T_s,
\]

\[
n \rightarrow \frac{n}{n_s}, \quad \rho \rightarrow \rho / n_s, \quad \omega \rightarrow \omega / \omega_{ci},
\]

\[
where \ t_s = \frac{1}{\omega_{ci}}, \quad \rho_s = \frac{V_e}{\omega_{ci}}, \quad V_i = \frac{T}{m_i}, \quad \omega_{ci} = \frac{e B}{m_i c}, \quad \eta = \frac{m_i}{2e^2 N_0}, \quad p_s = n_i T_i, \quad x = (r-r_0), \quad d = a-r_0 - \text{is the width of the computation region, } r_0 - \text{its inner boundary. The normalizing values for density and electron temperature were taken to be } n_s = 10^{13} \text{ cm}^{-3} \text{ and } T_s = 350 \text{ eV.}
\]

Developed numerical code used a quasispectral decomposition based on the Galerkin method in a single helicity mode approximation \( (m/n=q_{res} = \text{const}) \), which reduces 3D to 2D the problem, so leads to substantial decrease of the computation time.

Averaging of vorticity equation (1) over the poloidal coordinate \( y \) gives finally the following dimensionless equation for the evolution of poloidal momentum \( G_{by} = N_0 U_{by} \) [3]:

\[
\frac{\partial G}{\partial t} + \frac{\partial R}{\partial x} = \nu_0 \frac{\partial^2 G}{\partial x^2} - \nu_{neo} \left( G_{0,y} - G_{neo} \right) - \nu_{ci} G_{0,y},
\]  \hspace{1cm} (8)

where \( R = \rho < V_{Ex} \left( N_0 V_{Ey} + \frac{dp}{dx} \right) > -V_E \Gamma \) - turbulent Reynolds stress, \( \rho = \frac{\rho_i}{d} \), \( \Gamma = n_r \cdot V_{Ex} >, \ x \) corresponds to radial coordinate.

Note that taking into account new nonlinear terms \( H_{NL} \) in vorticity equation (1) results in the appearance of additional term \( <V_{Ex} \cdot dp/dx > \) in turbulent Reynolds stress tensor, which
plays an important role in numerical computation. Equation for electric field is following from ion radial force balance:

\[
\frac{cE_r}{BV} = V_k = -U_{0y} + V_{pr},
\]

\[
V_{pr} = \frac{\rho}{N_0} \frac{dp}{dx} < 0. \Phi(x) = \int E(x) dx, \quad \Phi(1) = 0
\]

Numerical simulations were carried out for the edge region \(r_0=26 \text{ cm} < r < 30 \text{ cm}=a\) of the T-10 tokamak for the following parameters: \(R=150 \text{ cm}, a=30 \text{ cm}, d=4 \text{ cm}, B=2.31 \text{ T}, q_{rel}=3, N_0(r_0)=1.6 \times 10^{13} \text{ cm}^{-3}, N_0(a)=0.2 \times 10^{13} \text{ cm}^{-3}, T_e(r_0)=80 \text{ eV}, T_e(a)=12 \text{ eV}.\) The choice of the resonant helical modes obeyed to rule \(m=3k, k=1, 2, \ldots, K_{max} ; K_{max} < 30.\) The radial extent of the calculation layer was limited due to the single helicity approach. Increase of electron temperature \(T_e(r_0)\) on the inner boundary of the calculation layer was used to simulate transition from OH regime to ECRH regime with various level of input EC-power.

Numerical results show the negative electric potential in the OH regime, \(T_e(r_0)^{OH} = 60 \text{ eV}.\) The gradual increase of the boundary electron temperature \(T_e(r_0)^{ECRH} = 90, 120, 150 \text{ eV}\) leads to corresponding decrease of the absolute value of the electric potential, as shown in Fig. 1. Poloidal velocity \(U_{0y}\) is shown in Fig. 2. The value of electric potential and mean \(E_r\) shows qualitative agreement with experimental data from Ohmic and ECR heating [4]. Dependence of \(E_r\) averaged over the calculation layer on the boundary electron temperature \(T_e(r_0)\) is shown in Fig.3. It shows that transition from low \(T_e(r_0)\) (OH) to higher \(T_e(r_0)\) (ECRH) results in decrease of the absolute value of electric field \(|E_r|^{ECRH} < |E_r|^{OH}\), remaining its negative sign.

![Fig. 1](image1.png)

![Fig. 2](image2.png)

Analysis of numerical calculations show, that heating of electrons in the edge region leads to some decrease of the mean amplitude of fluctuations due to the increase of longitudinal dissipation \(\sim T_e^{3/2}.\) On the other hand, it results in recovery of the phase relations
between different modes of electrostatic potential, which in turn leads to the increase of the turbulent Reynolds stress in spite of some decrease of fluctuations amplitude. As a result, the absolute value of poloidal velocity $U_{0y}$ increases, see Fig. 2. Since $U_{0y} < 0$ and $V_{pi} < 0$, then as follows from equation (9), absolute value of radial electric field decreases with growth $|U_{oy}|$, see Fig. 3. Thus, the computation results qualitatively agree with experimental data.

Dependence of mean electric field on plasma density was also modelled for the same plasma parameters. Plasma density at the inner boundary was chosen to be the only varied parameter for numerical simulations. Calculation was made for the realistic values of $N_0(r_0) = 1.2 - 2.1 \times 10^{13} \text{ cm}^{-3}$. Figure 4 shows the inverse dependence of modeled electric field on plasma density, which agrees with the tendency, observed experimentally [2].

In summary, the model for the $E_r$ calculation based on the Braginskii’s hydrodynamics was developed for the periphery of the tokamak plasma. Numerical results for T-10 conditions show qualitative agreement with direct experimental data, obtained by HIBP. Moreover, the experimental dependences of $E_r$ on plasma temperature and density were obtained by modeling. So, $E_r$ in the strong turbulent plasma of the tokamak periphery was satisfactorily described by developed model.

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References