Lattice Kinetic Schemes in Fusion Plasmas

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Abstract

Lattice Boltzmann Methods (LBM) are an established alternative approach for numerical simulations of a large spectrum of physical processes. They are based on a mesoscopic analysis of the underlying physics through a velocity distribution function $f(x,\xi,t)$ which obeys the Boltzmann Equation (BE). Furthermore, it has been argued that a number of macroscopic processes can be modeled through a mesoscopic evolution equation similar to BE appropriately tuned to recover the desired macroscopic behavior while retaining the multi-scale characteristics of LBM. Such an approach can be utilized to analyze a magnetohydrodynamic (MHD) system via lattice kinetic schemes [1,2]. All macroscopic quantities are given as moments of f and the algorithm is seen to solve consistently the hydrodynamic and magnetic induction dissipative equations in generalized 3D geometry [3]. We examine the potential of such an algorithm for large-scale fusion simulations. Initial conditions may be provided by the Integrated Tokamak Modelling (ITM) mdsplus server in ENEA frascati, Rome, for ITER related scenarios [4]. The case considered is the evolution of continuous shear Alfvén waves in a plasma [5].

Formulation

The numerical implementation of the method is based on the discretized Boltzmann equation with a BGK formulation of the collision term (LBGK), which takes the form

$$\partial_t f_i + \xi_i \cdot \nabla f_i = -\frac{1}{\tau}(f_i - f_i^{(0)}) \tag{1}$$

where $f_i = f(\mathbf{x},\xi_i,t)$, with $\mathbf{x}$ the spatial vector, ξ_i the microscopic velocity set chosen, t the time and τ the relaxation time, all in dimensionless quantities. For the isothermal case, the equilibrium distribution function is given by a low Mach number series expansion of the Maxwellian as

$$f_i^{(0)} = \rho w_i \left[1 + \frac{\xi_i \cdot \mathbf{u}}{c_s^2} + \frac{(\xi_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}\cdot\mathbf{u}}{2c_s^2}\right], \tag{2}$$

with the weighting factors w_i depending on the lattice. The simulations are performed with the 19-velocity model incorporating $c_s = c/\sqrt{3}$ with $c = \delta x/\delta t$ being the lattice speed and w_i the corresponding weights [1,2].

The macroscopic quantities can be computed by moments of f, i.e. $\rho = \sum_i f_i$ and $\rho \mathbf{u} = \sum_i \xi_i f_i$, where ρ and $\mathbf{u}$ are the density and velocity vector respectively and the viscosity of the fluid is given by $\nu = \tau c_s^2$.

A corresponding formulation can be used for tracking in time the induction equation [1,2]. We use a vector distribution function with its zeroth moment providing the magnetic field vector $\mathbf{B} = \sum_{j=0}^{M} \mathbf{g}_j$.

The evolution of $\mathbf{g}_j$ obeys a BGK-type kinetic equation

$$\partial_t \mathbf{g}_j + \Xi \cdot \nabla \mathbf{g}_j = -\frac{1}{\tau_m}(\mathbf{g}_j - \mathbf{g}_j^{(0)}) \tag{3}$$

where $\mathbf{g}_j^{(0)}$ are the corresponding equilibrium distribution functions given by

$$\mathbf{g}_{j\beta}^{(0)} = W_j \left[B_\beta + \Theta^{-1} \Xi_{j\alpha} (u_\alpha B_\beta - B_\alpha u_\beta) \right] \tag{4}$$

with α, β denoting the spatial directions and Ξ the corresponding discrete velocity vector (not necessarily the same as ξ). The relaxation time τ_m allows us to set the magnetic resistivity as $\eta = \Theta \tau_m$, independently from the fluid's viscosity, which is related to τ.

The first moment of $\mathbf{g}$ gives the electric tensor as

$$\Lambda_{\alpha\beta}^{(0)} = \sum_{j=0}^{M} \Xi_{j\alpha} g_{j\beta}^{(0)} = u_\alpha B_\beta - B_\alpha u_\beta. \tag{5}$$

Note that consistent expressions for $\nabla \cdot \mathbf{B}$ and $\nabla \times \mathbf{B}$ can be obtained. Finally, the incorporation of the Lorenz force can be implemented in two ways either by an appropriate expansion of the $f^{(0)}$ [1] or by adding a forcing term in the Boltzmann equation [2].

Toroidal module

The, in house, LK3D code uses Cartesian coordinates, thus the surface of the torus can be described through the following equation

$$(R - \sqrt{x^2 + y^2})^2 + z^2 = a^2[\kappa^2 sin^2(\theta) + cos^2(\theta + \delta sin(\theta))] \tag{6}$$

where $-1 < x, y, z < 1$ are the independent variables of the domain, θ is the poloidal angle,α and R denoting the minor and major radius of the torus and κ and δ are identified as the ellipticity and triangularity of the plasma boundary respectively. The main advantage of the generalized Cartesian system is the ability to represent any geometry which coupled with the flexibility of the LBM in terms of enforcing the boundary conditions, allows the code to incorporate any given initial state.

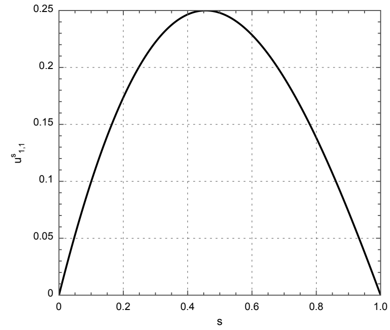

Figure 1: Profile of the radial velocity in terms of normalized magnetic surface -zero corresponding to the magnetic axis - of an {1,1} perturbation mode.

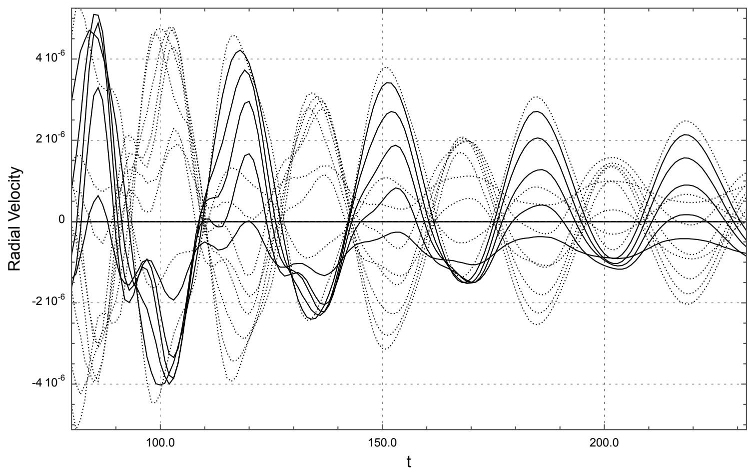

Figure 2: Radial component of the perturbed velocity vs. time for a given poloidal and toroidal angle at a number of radial positions.

Results and concluding remarks

As part of a benchmarking procedure, we test the response of the system to perturbations of the equilibrium. The case considered is the evolution of continuous shear Alfvén waves in a plasma with cylindrical symmetry. It is seen [5] that the nature of these waves is of an oscillatory spectrum in a screw pinch configuration that is described by two numbers {m, n}, the poloidal and toroidal mode respectively.

Since initially the velocity u is zero, we choose to initiate the deviation from equilibrium by introducing a velocity field with a specific radial component of the velocity. The profile given in Figure 1 corresponds to a {1,1} mode perturbation. This perturbation will decay exponentially in time. This exercise is a significant challenge for the LBM approach. The generalized 3D formulation and the dissipative nature of our algorithm pose severe constraints in dealing with

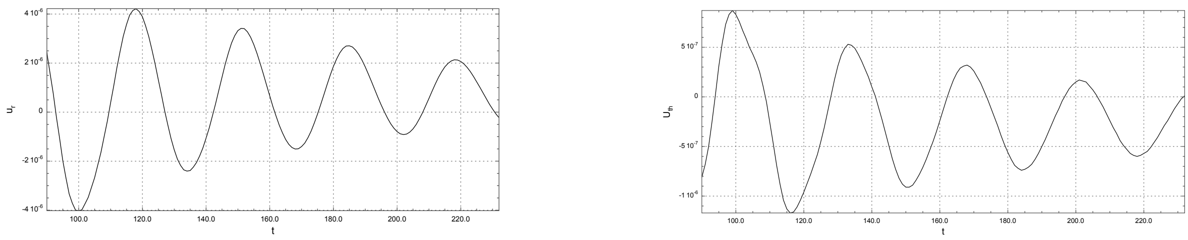

Figure 3: Evolution of the radial (left) and poloidal (right) velocity component at a given radial position

high Reynolds flows (in essence a linear response system).

Following the time evolution of the system at a specific location we get the patterns given in Figure 2 at a number of radial positions. The decay of the disturbance does indeed, as expected, follows an oscillatory pattern as it can be seen in more detail in Figure 3. However there are some discrepancies present.

The fluctuations should average to zero and this not the case for all the curves in Figure 2. Furthermore the relevant scaling of the oscillations with the equilibrium B field needs to be explored (i.e. sorting out the alfvénic character of the oscillations). These relate to various aspects of the code such as the treatment of the equilibrium, the initialization procedure, etc. The benchmark also implies a steady-state equilibrium. However, in our case, the magnetic field changes in time due to dissipation, so one needs to develop ways to ameliorate this effect. Testing with higher discretization and a dimensional analysis (scheme's limits) should provide the necessary insight. These tasks will be addressed in the future in an effort to explore the advantages of the mesoscopic formulation.

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