Relationship between Toroidal Momentum and Heat Transport in Tokamaks

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Toroidal rotation and associated $E \times B$ shear flow are believed to play important roles in controlling plasma turbulence and resistive wall modes. Since experiments found spontaneous or intrinsic rotation, there has been increased research interest in toroidal momentum transport focusing on searching for and understanding non-diffusive mechanisms. Global gyrokinetic simulations using the Gyrokinetic Tokamak Simulation (GTS) code [1] have been carried out to investigate turbulence driven momentum and energy transport and their relationship for realistic tokamak parameters. GTS simulation is based on a generalized gyrokinetic simulation model with a particle-in-cell approach, and incorporates the comprehensive influence of non-circular cross section, realistic plasma profiles, plasma rotation, neoclassical (equilibrium) electric fields, Coulomb collisions, and other features. Also studied are the neoclassical counterparts of the transport using the GTC-NEO code [2].

The gyrokinetic simulation model for rotating plasmas is briefly described first. The particle distribution is expressed as $f = f_0 + \delta f$. Here we separate the turbulence perturbation $\delta f$ from the equilibrium distribution $f_0$. The equilibrium $f_0$ is determined by the neoclassical dynamics and should be prescribed in the equation for the perturbed $\delta f$. The true neoclassical equilibrium distribution function is unknown analytically. Instead, we use the lowest order equilibrium solution which is a shifted Maxwellian consistent with (large) plasma rotation [2]:

$$f_0 = f_\text{SM} = n(r, \theta) \left( \frac{m_i}{2} \pi T_i \right)^{3/2} e^{- \left( \frac{m_i}{T_i} \right) \left( v_\parallel - U_i \right)^2 / 2 + \mu B}$$

where the parallel flow velocity $U_i$ is associated with the toroidal rotation by $U_i = I \omega_i / B$, with $I$ and $\omega_i$ the toroidal current and angular velocity, respectively, and $n(r, \theta)$ is the plasma density. In the electrostatic limit, the ion gyrokinetic equation for the turbulence perturbation $\delta f_i$, with magnetic moment $\mu$ and parallel velocity $v_\parallel$ as independent velocity variables, is

$$\frac{\partial \delta f_i}{\partial t} + (v_\parallel \hat{b} + v_\| E_0 + \vec{v}_E + \vec{v}_d) \cdot \nabla \delta f_i - \hat{b}^* \cdot \nabla (\mu B + \frac{e}{m_i} \Phi_0 + \frac{e}{m_i} \bar{\phi}) \frac{\partial \delta f_i}{\partial v_\parallel}$$

$$= \left[ \cdots \vec{v}_E \cdot \nabla \ln T - \vec{v}_E \cdot \nabla \ln n(r, \theta) - \frac{m(v_\parallel - U_i)}{T_i} \vec{v}_E \cdot \nabla U_i(r, \theta) \right.$$  

$$+ \frac{mU_i}{T_i v_\parallel} \mu \nabla B - \frac{1}{T_i} (v_\parallel \hat{b} + v_{\| d}) \cdot \nabla (e\bar{\phi})(1 - \frac{U_i}{v_\parallel}) \right] f_0.$$  

(1)
Here $\vec{v}_{E0}$ and $\vec{v}_E$ are the $\vec{E} \times \vec{B}$ velocities corresponding to the equilibrium and fluctuation potential $\Phi_0$ and $\bar{\phi}$, respectively. $\vec{v}_d$ is the $\nabla \vec{B}$ drift velocity, $\vec{b}^* = \vec{b} + \rho_\parallel \vec{b} \times (\vec{b} \cdot \nabla \vec{b})$ with $\vec{b} = \vec{B}/B$, and $C_1^f$ is the linearized Coulomb collision operator. On the right hand side, the third term proportional to $\nabla U_i$ is the Kelvin-Helmholtz type driver term. The other terms containing $U_i$ are also retained, which can be important when the Mach number of plasma flow is high.

Global gyrokinetic turbulence is characterized by distinguishable dynamical phases [1, 3], from linear, to nonlinear transient, to a well developed steady state. Ideally, the dynamics of gyrokinetic turbulence should not be sensitive to numerical techniques. This has been carefully examined for the GTS simulation. Indeed, it shows that the turbulence dynamics is not sensitive to i) how we solve the gyrokinetic Poisson equation, ii) how the simulation mesh is set up, and iii) the number of simulation particles. Therefore, the turbulence evolution process, appearing robustly, is not an artificial picture, but real physics. As will be presented later, a large inward toroidal momentum flux driven by ITG turbulence is closely related to the transient phase of global turbulence development.

A key result of our simulations is the finding of an inward flux of toroidal momentum driven in the post saturation phase of ITG turbulence. Simulation results for a counter-rotating plasma are presented in Fig. 1, where the momentum diffusion is in the inward direction. It is observed that a remarkably large inward toroidal momentum flux occurs during the transient phase of turbulence development, which is after the nonlinear saturation of the ITG instability, but before a well developed steady state. As we discussed before, the appearance of this transient flux does not depend on the details of numerical techniques. This inward momentum flux pumps the toroidal momentum from the outer region to the core while maintaining approximately global momentum conservation, resulting in a change in the toroidal rotation with a magnitude of a few percent of the local thermal velocity. Other interesting observation is that the ITG driven momentum flux settles down to a
relatively low level in the long time steady state. Our simulations verify that there exists strong coupling between ITG driven ion momentum and heat transport, and that the ratio of effective momentum and heat diffusivities $\chi_\phi/\chi_i$ is on the order of unity, as seen in Fig. 1. This is in broad agreement with experimental observations in conventional tokamaks where low-k fluctuations are believed to be responsible for a high level of plasma transport [4].

More surprisingly, an inward momentum flux is driven for the case of positive rotation gradient, where the momentum diffusion is outward. This indicates its non-diffusive nature. As a consequence, core plasma rotation spins up, resulting in $\Delta u_\parallel$ a few percent of $v_{th}$ in the case of no momentum source at the edge. Roughly, there are two different channels which contribute to the non-diffusive momentum flux. One is a momentum "pinch" or convective flux which is proportional to the toroidal rotation velocity [5]; Another is the off-diagonal flux which is driven by residual stress with no dependence on rotation or rotation gradient. Identification of them and their significance are certainly highly interesting, but difficult in experiments. To this end, we have carried out a series of numerical experiments, with and without mean $E \times B$ shear flow, and with and without toroidal rotation as well as rotation shear. The inward momentum flux is robustly observed in various situations, and appears to be dominated by off-diagonal contributions. It is reasonably anticipated that this off-diagonal flux may lead to the buildup of an experimentally relevant rotation structure (i.e., spontaneous rotation) when there is a momentum source at the edge; but a true demonstration may require an edge momentum source to be implemented in future simulations on the transport time scale. Moreover, the fact that the long-time steady state toroidal momentum flows against the rotation gradient and the momentum flux vanishes in cases of rigid rotation (including zero rotation) indicates that it is mostly diffusive. The zonal flow shearing rate and turbulence $k_\parallel$ spectrum indicate that the
underlying physics for the inward flux is the generation of residual stress due to $k_{\parallel}$ symmetry breaking [6] induced by self-generated zonal flow shear, which is quasi-stationary in global simulations.

For neoclassical transport, our simulations also show that the ion temperature gradient can drive a significant inward nondiffusive momentum flux (Fig. 2). However, the overall neoclassical contribution to the momentum transport is negligibly small compared to experimental levels for NSTX and DIII-D plasmas. Moreover, the effective $\chi_\phi/\chi_i$ is $\sim 0.1 - 0.01$ due to the absence of banana orbit enhancement. Further, it is found that finite residual turbulence can survive strong mean $E \times B$ shear flow induced damping. As illustrated in Fig. 3, the ITG instability is shown to be linearly stable in the presence of the $E \times B$ shear. However, if we run simulations without the $E \times B$ shear first, and impose the $E \times B$ shear after the turbulence saturates nonlinearly, we observe that the turbulence, while significantly reduced (by a factor of 10 in intensity), is not totally quenched. Initial results indicate that applying the $E \times B$ shear later (rather than initially) produces results closer to the experimental trends, and the resulting ion heat flux (reduced by a factor of 10) corresponds reasonably closely to the neoclassical value, while the momentum flux remains anomalous, significantly higher than the neoclassical level. These findings may offer a possible explanation to recent experimental observations that the toroidal momentum transport remains highly anomalous, even while the ion heat flux is reduced to a neoclassical level.

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References