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BASIC EQUATIONS FOR RF-CURRENT DRIVE THEORY IN TURBULENT PLASMA

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Basic equations for the current drive by radio-frequency (rf) waves in a presence of electric and magnetic fluctuations are investigated using a renormalized perturbation technique of statistical dynamics.

We start with a relativistic Fokker-Planck equation for an electron distribution function $f(\boldsymbol{r},\boldsymbol{p},t)$ in the presence of rf waves and low-frequency fluctuations

$$\partial_t f + v_\parallel \boldsymbol{b}\cdot\nabla f + \delta\boldsymbol{v}_\perp\cdot\nabla_\perp f - C(f) = \frac{\partial}{\partial\boldsymbol{p}}\cdot\boldsymbol{S}_W, \tag{1}$$

where the subscripts $\parallel$ and $\perp$ refer to the parallel and the perpendicular to an averaged magnetic field $\boldsymbol{B}$; $\boldsymbol{b}=\boldsymbol{B}/B$; $v_\parallel=\boldsymbol{v}\cdot\boldsymbol{b}$; C is the Fokker-Planck collision operator; and $S_{\rm rf}=\partial/\partial\boldsymbol{p}\cdot\boldsymbol{S}_W$ is the momentum-space diffusion term due to rf waves. We write the velocity fluctuations as $\delta\boldsymbol{v}_\perp=\boldsymbol{b}\times\nabla\delta\phi(\boldsymbol{r}_\perp,t)/B$ for electrostatic fluctuations and $\delta_\perp\boldsymbol{v}=v_\parallel\delta\boldsymbol{B}_\perp/B\equiv v_\parallel\delta\boldsymbol{b}_\perp(\boldsymbol{r},t)$ for magnetic fluctuations, where $\delta\phi$ is the fluctuating electrostatic potential and $\delta\boldsymbol{B}_\perp$ is the fluctuating perpendicular magnetic field. Let us here assume the correlation functions of fluctuating electrostatic potential and fluctuating magnetic field in the form: $\langle\langle\delta\phi(\boldsymbol{r},t)\delta\phi(\boldsymbol{0},0)\rangle\rangle=\phi_0^2\exp\left[-|t|/\tau_c-x^2/2\lambda_x^2-y^2/2\lambda_y^2-z^2/2\lambda_\parallel^2\right]$, and $\langle\langle\delta\boldsymbol{b}_\perp(\boldsymbol{r},t)\delta\boldsymbol{b}_\perp(\boldsymbol{0},0)\rangle\rangle=-(\nabla\times\boldsymbol{b})(\nabla\times\boldsymbol{b})\langle\langle A(\boldsymbol{r},t)A(\boldsymbol{0},0)\rangle\rangle$ with $\langle\langle A(\boldsymbol{r},t)A(\boldsymbol{0},0)\rangle\rangle=\beta^2\lambda_x\lambda_y\exp\left[-|t|/\tau_c-x^2/2\lambda_x^2-y^2/2\lambda_y^2-z^2/2\lambda_\parallel^2\right]$.

Applying a renormalized perturbation theory to the Fokker-Planck equation (1), we can derive the following equation for the ensemble-averaged distribution function $\bar f(\boldsymbol{r},\boldsymbol{p})=\langle\langle f(\boldsymbol{r},\boldsymbol{p})\rangle\rangle$ in the steady-state current drive:

$$(v_\parallel\boldsymbol{b}\cdot\nabla-C)\bar f(\boldsymbol{r},\boldsymbol{p})-\nabla_\perp\cdot\int d\boldsymbol{p}'\mathcal{D}(\boldsymbol{p}',\boldsymbol{p})\cdot\nabla_\perp\bar f(\boldsymbol{r},\boldsymbol{p}')=\frac{\partial}{\partial\boldsymbol{p}}\cdot\boldsymbol{S}_W. \tag{2}$$

The momentum-dependent diffusion tensors due to electrostatic fluctuations in the weak and strong turbulent regimes are obtained :

$$\mathcal{D}(\boldsymbol{p}',\boldsymbol{p})=\frac{3}{4\pi\nu_c m^3c^3}\frac{\bar{\phi}_0^{\,2}}{\lambda_x^2\lambda_y^2}\mathrm{H}(\gamma'-\gamma)\frac{\zeta}{\gamma^2}\left(\frac{\gamma-1}{\gamma+1}\right)^{(1+Z)/2}\zeta'\left(\frac{\gamma'+1}{\gamma'-1}\right)^{(1+Z)/2}$$
$$\times\exp\left\{-\frac{1}{\nu_c\tau_c}\left[P(p')-P(p)\right]\right\}K(\gamma',\gamma',\gamma)\left(\lambda_x^2\boldsymbol{e}_x\boldsymbol{e}_x+\lambda_y^2\boldsymbol{e}_y\boldsymbol{e}_y\right) \tag{3}$$

in the weak turbulent regime, and

$$\mathcal{D}(\boldsymbol{p},\boldsymbol{p}') = \delta(\boldsymbol{p}-\boldsymbol{p}')\ \frac{\bar{\phi}_0}{\sqrt{2}\lambda_x\lambda_y} D(\bar{\tau}_{ce}^{-1};a)\left(\lambda_x^2 \boldsymbol{e}_x\boldsymbol{e}_x + \lambda_y^2 \boldsymbol{e}_y\boldsymbol{e}_y\right) \tag{4}$$

in the strong turbulent regime, where $\bar{\phi}_0 = \phi_0/B$; $\gamma = \sqrt{1+p^2/m^2c^2}$; $P(p) = p/mc - \tan^{-1}(p/mc)$; $\bar{\tau}_{ce} = (\sqrt{2}\bar{\phi}_0/\lambda_x\lambda_y)\tau_c$; $\zeta = p_\parallel/p$; $\nu_c = 4\pi ne^4\log\Lambda/m^2c^3$; H is the Heaviside step function; $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the unit vectors in the plain perpendicular to the averaged magnetic field; $K(\gamma,\gamma',\bar{\gamma}) = \exp\left[-c^2\zeta^2 G_2(\gamma,\gamma',\bar{\gamma})/2\lambda_\parallel^2\nu_c^2\right]$ with $R(\gamma,\gamma') = [(\gamma+1)(\gamma'-1)/(\gamma-1)(\gamma'+1)]^{(1+Z)/2}\ [(\gamma'^2-1)/\gamma'^2]$; and $G_2(\gamma,\gamma',\bar{\gamma}) = \left[\int_{\bar{\gamma}}^{\gamma'} d\tilde{\gamma}R(\gamma,\tilde{\gamma})\right]^2$ and Z is the effective ion charge; and $D(\bar{\tau}_{ce}^{-1};a) = D(\bar{\tau}_{ce}^{-1})/[1+\sqrt{2/\pi}a\ D(\bar{\tau}_{ce}^{-1})]$ with $a = (\lambda_x\lambda_y/\sqrt{2}\bar{\phi}_0)(|p_\parallel|/m\gamma\lambda_\parallel)$ and $D(\bar{\tau}_{ce}^{-1}) = \bar{\tau}_{ce}/(1+\bar{\tau}_{ce})$. The momentum-dependent diffusion tensor due to magnetic fluctuations in the collisionless regime is given by

$$\mathcal{D}(\boldsymbol{p}',\boldsymbol{p}) = \delta(\boldsymbol{p}-\boldsymbol{p}')\frac{|p_\parallel|}{\sqrt{2}m\gamma\lambda_\parallel} D(\bar{\tau}_{cm}^{-1})\left(\lambda_x^2 \boldsymbol{e}_x\boldsymbol{e}_x + \lambda_y^2 \boldsymbol{e}_y\boldsymbol{e}_y\right), \tag{5}$$

where $D(\bar{\tau}_{cm}^{-1}) = \alpha D_0/(\alpha + D_0)$ with $\alpha = \beta\lambda_\parallel/\sqrt{\lambda_x\lambda_y}$, $\bar{\tau}_{cm} = (\sqrt{2}|p_\parallel|/m\gamma\lambda_\parallel)\tau_c$ and $D_0 = \alpha^2\sqrt{\pi}e^{\bar{\tau}_{cm}^{-2}}\mathrm{erfc}(\bar{\tau}_{cm}^{-1})$.

A large number of Fokker-Planck codes for calculating the rf-driven current density are developed with and without taking account of fluctuations. The Fokker-Planck equation (2) for the ensemble-averaged distribution function accompanied with the momentum-dependent diffusion tensors (3), (4) and (5) can be used as a basic equation in developing the computational code for the rf current drive in the presence of fluctuations.

We next present an idea of decoupling the calculation of anomalous effects due to fluctuations from that of rf-driven current density. *In this method, the rf-driven current density in the presence of fluctuations can be calculated only by knowing the current density in the absence of fluctuations.* Applying the method discussed in ref.2 to the equation (2), we can derive a following radial diffusion equation for the rf-driven current density:

$$\frac{1}{r}\frac{\partial}{\partial r} r D_{\mathrm{rf}} \frac{\partial}{\partial r}\langle BJ\rangle - \langle BJ\rangle = -\langle BJ\rangle_0, \tag{6}$$

where $\langle\ \cdot\ \rangle$ denotes the flux-surface average and $\langle BJ\rangle_0$ is the rf-driven current density in the absence of fluctuations. The diffusion coefficient in this radial diffusion equation is expressed in the form:

$$D_{\mathrm{rf}} = -\frac{\langle\int d\boldsymbol{p}\boldsymbol{S}_W\cdot\frac{\partial\chi_2}{\partial\boldsymbol{p}}\rangle}{\langle\int d\boldsymbol{p}\boldsymbol{S}_W\cdot\frac{\partial\chi_1}{\partial\boldsymbol{p}}\rangle} \simeq -\frac{\boldsymbol{s}\cdot\frac{\partial}{\partial\boldsymbol{p}}\chi_2}{\boldsymbol{s}\cdot\frac{\partial}{\partial\boldsymbol{p}}\chi_1}, \tag{7}$$

where $\boldsymbol{S}_W/|\boldsymbol{S}_W|$; $\boldsymbol{p}$ in the last equation represents the momentum at the position of resonance; and $\chi_1(\boldsymbol{p}) = -ecB\zeta G_1(\gamma)$ with $G_1(\gamma) = \int_1^\gamma d\gamma' R(\gamma,\gamma')$.

The diffusion coefficient due to electrostatic fluctuations is explicitly calculated in the form:

$$D_{\rm rf} = \frac{D_{\rm rf}({\rm weak};\ c/\lambda_\| \nu_c)}{1 - A(c/\lambda_\| \nu_c) + \frac{D_{\rm rf}({\rm weak};\ c/\lambda_\| \nu_c)}{D_{\rm rf}({\rm strong};\ c/\lambda_\| \nu_c)}}, \tag{8}$$

where $A(c/\lambda_\| \nu_c) = D_{\rm rf}({\rm weak};\ c/\lambda_\| \nu_c)/D_{\rm rf}({\rm strong};\ c/\lambda_\| \nu_c)\big|_{\xi_c=0.01}$ with $\xi_c = \bar{\phi}_0/\lambda_x\lambda_y\nu_c$, and

$$D_{\rm rf}\left({\rm weak};\ \frac{c}{\lambda_\| \nu_c}\right) = \frac{D_{\rm rf}({\rm weak};\ c/\lambda_\| \nu_c = 0)}{1 + \frac{D_{\rm rf}\left({\rm weak};\ c/\lambda_\| \nu_c = 0\right)}{D_{\rm rf}\left({\rm weak};\ c/\lambda_\| \nu_c >> 1\right)}}. \tag{9}$$

The diffusion coefficients $D_{\rm rf}\left({\rm weak};\ c/\lambda_\| \nu_c = 0\right)$ and $D_{\rm rf}\left({\rm weak};\ c/\lambda_\| \nu_c >> 1\right)$ in (9) are calculated using the following functions in (7):

$$\begin{aligned} \chi_2(\boldsymbol{p}) &= eBc\nu_c\tau_c\xi_c^2\lambda_x^2\zeta \int_1^\gamma d\gamma' R(\gamma,\gamma')\left[P(p) - P(p')\right] \\ &\quad \times \left\{1 - \frac{\nu_c\tau_c}{P(p) - P(p')}\left[1 - \exp\left(-\frac{P(p) - P(p')}{\nu_c\tau_c}\right)\right]\right\} \end{aligned} \tag{10}$$

for $c/\lambda_\| \nu_c = 0$, and

$$\chi_2(\boldsymbol{p}) = eBc\xi_c^2\lambda_x^2 \frac{\lambda_\| \nu_c}{c}\frac{\zeta}{|\zeta|}\sqrt{\frac{\pi}{2}}\int_1^\gamma d\gamma' G_1(\gamma') \tag{11}$$

for $c/\lambda_\| \nu_c >> 1$. The diffusion coefficient $D_{\rm rf}({\rm strong};\ c/\lambda_\| \nu_c)$ is obtained using

$$\chi_2(\boldsymbol{p}) = ecB\frac{\xi_c\lambda_x^2}{\sqrt{2}}\zeta\left(\frac{\gamma+1}{\gamma-1}\right)^{(1+Z)/2}\int_1^\gamma d\gamma' \frac{p'}{mc\gamma'}\left(\frac{\gamma'-1}{\gamma'+1}\right)^{(1+Z)/2} G_1(\gamma')D(\bar{\tau}_{ce}^{-1};a'), \tag{12}$$

where $a' = a\,(p'\gamma/p\gamma')[(\gamma+1)(\gamma'-1)/(\gamma-1)(\gamma'+1)]^{(Z+1)/2}$.

The diffusion coefficient due to magnetic fluctuations is given by

$$D_{\rm rf} = \frac{1}{\sqrt{2}}\lambda_x^2\frac{c}{\nu_c\lambda_\|}|\zeta|\frac{\alpha D_0}{\alpha + D_0}G_1(\gamma). \tag{13}$$

We now explicitly calculate the rf-driven current density by solving both the Fokker-Planck equation and the radial diffusion equation in the uniform averaged-magnetic-field. Let us assume the following current profile in the absence of fluctuations: $J_0(x) = (J_0 /\sqrt{2\pi}\Delta)\exp\left[-(x-x_0)^2/2\Delta^2\right]$. Then the current profiles in the case of the Landau damping for $Z = 1, p/mc = 1$ and $\lambda_x/\Delta = 1$ are plotted in Figs. 1 and 2. The plots by solid curves show the current profiles obtained by the Fokker-Planck equation in the weak turbulent regime ($\xi_c = 10, \nu_c\tau_c = 0.01$) and the strong turbulent regime (

$\xi_c = 100, \nu_c\tau_c = 100$), respectively. The current profiles calculated from the radial diffusion equation are also depicted by dashed curves. The current profiles in the absence of fluctuations are plotted by dotted curves. These explicitly calculated results show that the current profiles obtained by the radial diffusion equation agree well with those by the Fokker-Planck equation.

In the formulation up to here, we have assumed that the ensemble-averaged distribution function changes little over a characteristic length of the correlation function (spatially local approximation). We finally consider the effect of spatially nonlocality on the rf-driven current density. Let us here concentrate on the strong turbulent regime for electrostatic fluctuations and the collisionless regime for magnetic fluctuations since the spatially nonlocality is important for these regimes. In consideration of the spatially nonlocality, the radial diffusion equation is found to be rewritten by an integro-differntial equation

$$\frac{\partial^2}{\partial x^2}\int_{-\infty}^{\infty} dx' \tilde{D}_{\rm rf}(x-x')J(x') - J(x) = -J_0(x), \tag{14}$$

where

$$\tilde{D}_{\rm rf}(x-x') \simeq -\frac{\boldsymbol{s}\cdot\frac{\partial}{\partial \boldsymbol{p}}\tilde{\chi}_2(\boldsymbol{p}, x-x')}{\boldsymbol{s}\cdot\frac{\partial}{\partial \boldsymbol{p}}\chi_1(\boldsymbol{p})}. \tag{15}$$

The function $\tilde{\chi}_2$ is calculated by solving the equation $\hat{C}(\tilde{\chi}_2(\boldsymbol{p}, x-x')) = \chi_1(\boldsymbol{p})K(x-x';\boldsymbol{p})$, where $\hat{C}(g) = C(gf_0)/f_0$ with the relativistic Maxwellian $f_0(p)$; $\boldsymbol{F}(\boldsymbol{r}-\boldsymbol{r}', t-t', \boldsymbol{p}, \boldsymbol{p}') = \langle\langle \delta\boldsymbol{v}_\perp(\boldsymbol{r}, \boldsymbol{p}, t)\delta\boldsymbol{v}_\perp(\boldsymbol{r}', \boldsymbol{p}', t')\rangle\rangle$; $K(x-x';\boldsymbol{p}) = \int dydz\boldsymbol{e}_x\cdot\boldsymbol{F}(\boldsymbol{r}-\boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p})\cdot\boldsymbol{e}_x G^\dagger(\boldsymbol{r}-\boldsymbol{r}', s=\tau_c^{-1})$; and $G^\dagger(\boldsymbol{r}-\boldsymbol{r}', s=\tau_c^{-1}, \boldsymbol{p}';\boldsymbol{p}) = \delta(\boldsymbol{p}-\boldsymbol{p}')G^\dagger(\boldsymbol{r}-\boldsymbol{r}', s=\tau_c^{-1})$ is the Laplace transform of the response function to an infinitesimal external perturbation.

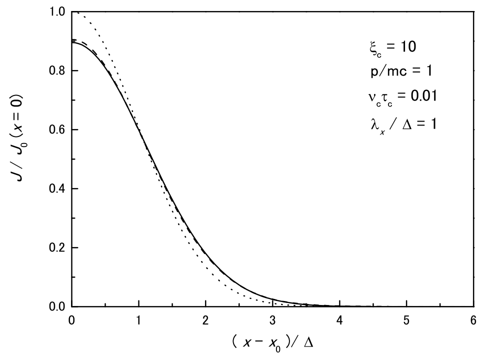

Figure 1: Rf-driven current density in the weak turbulent regime

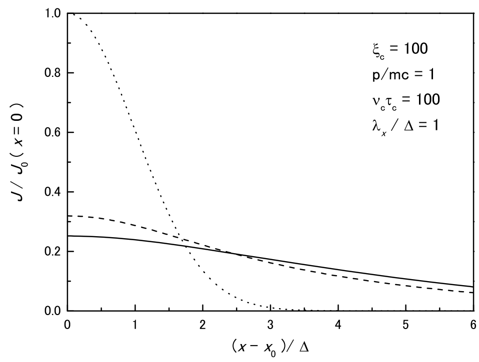

Figure 2: Rf-driven current density in the strong turbulent regime

References

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[2] M. Taguchi, Phys. Letts **A 178**, 165 (1993).