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The plasma filamentation instability in one dimension

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Abstract

The filamentation instability between counter-propagating beams of electrons is important for the magnetic field generation in astrophysical jets. Here this instability is considered for equally dense counterpropagating electron beams. It is demonstrated that the plasma evolves into a state, in which the electric fields driven by the magnetic pressure gradient balance the magnetic forces. This system is stationary in the 1D PIC simulation. The size distribution of the current filaments closely follows a Gumbel distribution within the statistical limitations.

Introduction

The filamentation instability (FI) generates magnetic fields through a spatial re-distribution of charged particles that move in the form of beams. Particle-in-cell (PIC) simulations have demonstrated that current flux tubes form in 1D/2D [1, 2] and in 3D [3], which are spatially confined by their self-generated magnetic fields. Here [4] we model with a 1D PIC simulation [5] the FI driven by two equally dense electron beams, which move in opposite directions at the same non-relativistic speed modulus. The velocity spread of the isotropic Maxwellian distribution of each beam is small compared to the beam flow speed.

We show that the FI develops an electrostatic component through the magnetic pressure gradient of the growing magnetic field during its quasilinear evolution. The interplay of the magnetic and electrostatic fields results in a k-spectrum of the magnetic field that is a power-law, which has been reported previously for multidimensional PIC simulations [6, 7]. The filaments develop into a stationary final state in 1D. This state is probably realistic also for higher dimensions for the instance, when the FI saturates and the filaments have not yet merged. Filament coalescence takes place in higher dimensions [1, 7]. The probability distribution of the filament size in the 1D simulation is well-approximated by a Gumbel distribution $\propto \exp\left[-u-\exp\left(u\right)\right]$. This implies that the exponential tail of this distribution may yield a seed of large current filaments in the large astrophysical domains. The filaments can then grow further by their mergers, which take place in more than one dimension [1, 7]. The coherent magnetic fields of large filaments might be necessary to explain the synchrotron emissions of gamma ray bursts.

Simulation

Two electron beams move along the **z**-direction. The mean velocity vectors of beam 1 and beam 2 are $v_b\mathbf{z}$ and $-v_b\mathbf{z}$ respectively, with $v_b = 0.3c$. The densities of both beams are equal. The current $J_z(x)$ in the simulation box thus vanishes at the simulation's start. The total electron plasma frequency is Ω_p. The beams have identical isotropic Maxwellian velocity distributions in their rest frames with the thermal speed $v_e = (k_b T/m_e)^{1/2}$ that gives $v_b = 18v_e$. The initial distribution is displayed in Fig. 1(a). We employ periodic boundary conditions and we resolve an x-interval with the length $L = 444\lambda_e$, where $\lambda_e = c/\Omega_p$. We use $N_g = 3\times 10^4$ cells and 200 computational electrons per cell per beam. The simulation duration is $T_t = 533/\Omega_p$.

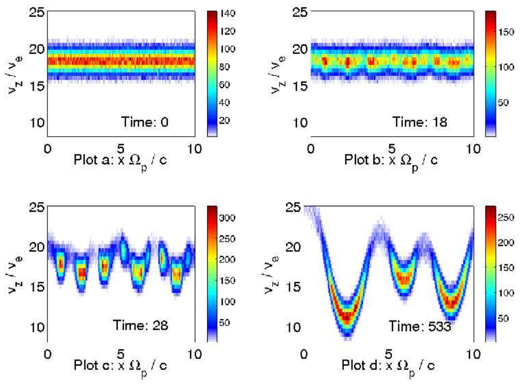

Figure 1: The electron phase space density in units of simulation electrons: (a) Initial distribution. (b) The B_y grows but not yet E_x. (c) The E_x component grows too. (d) B_y and E_x are in a steady state.

Initially the FI yields only the growth of B_y due to the chosen simulation x-direction and the beam flow z-direction. A representative electron distribution for this growth stage is depicted by Fig. 1(b). During the quasi-linear stage the electrostatic E_x field grows at twice the exponential rate of B_y [4] and the corresponding electron distribution is shown in Fig. 1(c). Both fields reach a steady state and the electron distribution is that in Fig. 1(d).

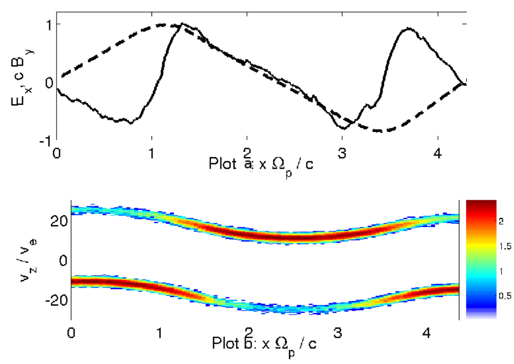

Figure 2: The fields at $t\Omega_p = 533$ are shown in (a), where B_y is dashed and normalized to 275 V/m and E_x (solid line) is normalized to 50.5 V/m. (b) is the electron phase space distribution.

The steady state B_y, E_x fields are related to the electron distribution in Fig. 2. The steady state gives rise to $E_x(x)$ and $B_y(x)$, which have constant slopes over broad x-intervals. For values $x = x_0$ with $B_y(x_0) = 0$ or $dB_y(x_0)/dx = 0$ we get $E_x(x_0) = 0$, evidencing $E_x(x) \propto B_y(x)[dB_y(x)/dx]$. The constant slope gives $dB_y(x)/dx = const$ and the slopes of $E_x(x)$ and $B_y(x)$ differ only by a constant factor. The spatial power spectra of both $E_x(x)$ and $B_y(x)$ follows a power-law, as it is discussed in more detail by the Ref. [4]. The power-law is also observed for more realistic 2D simulations [7] and for 3D simulations of colliding plasmas [6].

The steady state reached in the 1D simulation [4] corresponds to the time in Ref. [7], when the filamentation instability has just saturated but when the filaments have not yet started to coalesce.

We calculate $J_z(x) = \int_{v_z} f(x, v_z) dv_z$ from the electron phase space distribution $f(x, v_z)$ when the system is time-stationary, as in Fig. 1(d). A representative electron distribution is shown in Fig. 3(a). The current distribution $J_z(x)$ plotted in Fig. 3(b) jumps between two almost constant levels, which are the current maxima and minima. The positions where $J_z(x) = 0$ are sharp domain boundaries and we define the distance between two neighboring boundaries to be the domain size.

The domain sizes in the simulation are now determined. A second simulation uses the box length $\tilde{L} = 6600\lambda_e$, resolving many domains.

These have a typical size of a few λ_e, as the Fig. 3(b) demonstrates. A steady state is reached within the chosen simulation time of $390/\Omega_p$. The sizes of all current domains in the simulation box are measured at the final simulation time step. The number of domains with sizes that fall into a given interval are counted. The size intervals have a uniform width that yields a statistically significant count rate, while providing a good resolution.

The size distribution is revealed in Fig. 4 on a linear and on a 10-logarithmic scale. The Gumbel distribution $P_D = 210 \times \exp[-u - \exp(u)]$ is overplotted, with $u = 1.45 \times (x/\lambda_e - 1.65)$. The measured size distribution follows the Gumbel distribution on both the linear and 10-logarithmic scale.

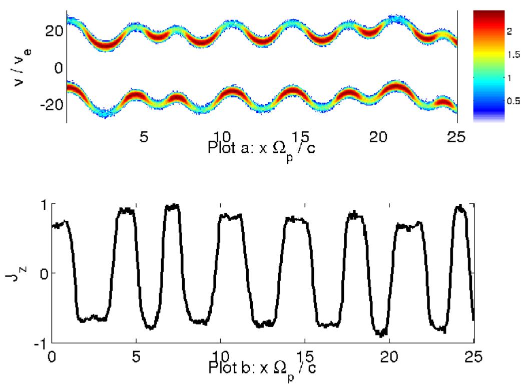

Figure 3: (a) Electron phase space distribution at the simulations end $t\Omega_p = 533$. The color is the number of simulation electrons. (b) corresponds to the J_z computed from the $f(x, v_z)$ shown in (a) that is normalized to its peak value in (b).

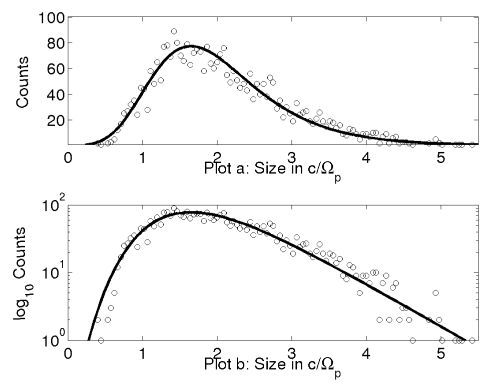

Figure 4: The size distribution of current filaments at the final time $t\Omega_p = 390$: (a) Displays the number of domains with a given size on a linear scale and (b) shows the same on a 10-logarithmic scale.

Summary

Some statistical properties of the filamentation instability (FI) have been examined here and in more detail by the Ref. [4]. The FI yields initially the aperiodic growth of magnetic fields. The unstable wavevectors are orthogonal to the beam flow direction. A steady state distribution is reached by the FI, when the electrostatic fields, which are driven by the pressure gradients of the magnetic field, cancel the instability by locking in the electrons [2]. The steady state fields show a power-law distribution of their electric and magnetic fields.

This steady state distribution spatially separates the electrons moving in opposite directions, yielding current filaments (domains). The current oscillates between two extrema. We have used here the zero crossing of the current to define the size of the current domains. A large simulation box has revealed a size distribution, which followed closely a Gumbel distribution. The largest current domains are also located next to each other [4]. The probability of finding large current filaments is not negligible if the FI develops in a large domain. The filament coalescence can further increase their size. We may thus find large domains with a significant magnetic field strength in the vast astrophysical jets, which could help explaining their synchrotron emissions.

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