Coupling of drift and ion cyclotron modes in solar atmosphere

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In the literature dealing with solar plasma, the ion cyclotron resonance and the ion cyclotron mode have been discussed in the context of problems related to the heating of the solar corona. This is due to the evidence obtained from in situ measurements in the solar wind and coronal holes of resonant ion cyclotron heating [1], and a preferential heating of coronal ions (with respect to electrons) which is most dominant in the direction perpendicular to the magnetic field lines. The damping of such IC waves is believed to be a good candidate for the consequent coronal heating and solar wind acceleration. Detailed studies performed in the past, which include direct observations, reveal the existence of ray-like structures that span from very large perpendicular cross sections, like in the case of polar plumes and streamers, down to very fine filamentary structures with a cross section of the order of a kilometer. Fine filaments are visible even from ground-based observations like those during the eclipse in 1991 showing a slow radial enlargement of the structures (i.e., in the direction out of the Sun) which is consistent with a low-beta plasma.

The presence of density gradients in the direction perpendicular to the magnetic field lines implies the possibility of the existence of a drift wave, a mode with the unique feature of being unstable both in the kinetic and the fluid domain (hence the term *universally unstable mode*). In our recent publications, some aspects of the drift wave instability in the solar plasma have been discussed. The collisional coupling between the drift and kinetic Alfvén waves in the upper solar atmosphere [3] reveals a strongly growing drift mode in the chromospheric plasma. The similar collisional instability of the mode, though with a much smaller increment, has been demonstrated also for the coronal plasma. In both cases, the kinetic Alfvén part of the mode is shown to be collisionally damped. In the presence of a plasma flow along the magnetic field lines the drift mode is subject to a reactive-type instability, provided that this background velocity has a gradient in the perpendicular direction. Such a problem has been discussed in our recent work [4] dealing with solar spicules. The drift mode has been shown to be unstable for typical spicule characteristic lengths of the density and the shear flow gradients, i.e. in the range of a few hundred meters up to a few kilometers, yielding wave frequencies of the order of a few Herz. One of the basic properties of the drift mode is the low frequency $|\partial/\partial t| \ll \Omega_i$. In the

case when this low-frequency limit is not well satisfied, there appears a coupling between the drift and ion-cyclotron modes. In the limit when $|\partial/\partial t| \sim \Omega_i$, this coupling is very effective and well known [5]. In the simplest case, it yields a coupled unstable drift-cyclotron mode with the instability driven by the plasma density gradient.

In the present work we describe the basic instability of the mode to focuss attention on the existence of such an instability that may be widespread and may contribute substantially to the problems discussed above. Following Ref. [5], the plasma dielectric function in the case of a negligible parallel wave vector and for the frequency limit $\omega \sim \Omega_i$, and $k_{\perp} = k_y = k$ within the kinetic theory for hot ions and electrons, is given by (in the local approximation)

$$\varepsilon(k,0,\omega) = 1 + \frac{k_e^2}{k^2} \left[\frac{\omega_{*e}}{\omega} + 1 - \Lambda_0(\beta_e) \right] + \frac{1}{k^2 \lambda_D^2} \left\{ 1 - (\omega - \omega_*) \left[\frac{\Lambda_0(\beta)}{\omega} + \frac{\Lambda_1(\beta)}{\omega - \Omega_i} \right] \right\}.$$
 (1)

Here,

$$\beta_e = k^2 \frac{T_e}{T_i} \frac{m_e}{m_i} \rho_L^2 < 1, \quad \beta = k^2 \rho_L^2 \gg 1,$$
(2)

$$\omega_* = \omega_{*e}T_i/T_e, \quad \omega_{*e} = \frac{n'_0}{n_0}\frac{T_e k_y}{m_e \Omega_e}, \quad k_e = \frac{\omega_{pe}}{v_{Te}}, \quad \Lambda_n(X) \equiv I_n(X)\exp(-X), \quad \rho_L = \frac{v_{Ti}}{\Omega_i},$$

 I_n denotes the modified Bessel function of the *n*-th order, and the prime denotes the derivative in the direction perpendicular to both the wave-vector and the magnetic field. Using $\Lambda_n(\beta) \rightarrow (2\pi\beta)^{-1/2} \exp(-n^2/2\beta)$ (for $\beta \rightarrow \infty$), we have $\Lambda_0(\beta) \simeq 1/[(2\pi)^{1/2}k\rho_L] \equiv \delta$. In view of the first expression in Eq. (2), we have $\Lambda_0(\beta_e) \simeq 1 - \beta_e$, while from the second one we have $\delta \ll 1$. The dispersion equation then becomes [5]

$$\left(1+k^2\lambda_d^2-\delta\right)\omega^2 - \left[\Omega_i\left(1+k^2\lambda_d^2\right)+\omega_*(1-\delta)\right]\omega+\omega_*\Omega_i=0, \quad \lambda_d^2=\rho_L^2\frac{m_e}{m_i}+\frac{1}{k_e^2}\frac{T_i}{T_e}.$$
 (3)

In the two limits ($\omega \ll \Omega_i$ and $\omega \sim \Omega_i$), the two modes are the drift wave and the IC wave, respectively, $\omega_1 = \omega_*/(1 + k^2 \lambda_d^2)$ and $\omega_2 \sim \Omega_i [1 + \delta/(1 + k^2 \lambda_d^2)]$. The instability may appear at the point of eventual intersection of the two dispersion curves, and the instability condition reads:

$$4\omega_*\Omega_i\left(1+k^2\lambda_d^2-\delta\right)>\left[\Omega_i\left(1+k^2\lambda_d^2\right)+\omega_*(1-\delta)\right]^2.$$
(4)

Below, we apply these expressions to the solar atmosphere in order to see if there is a window in the relevant parameter domain allowing for the instability.

The necessary instability condition (4) can be satisfied for a chosen set of plasma parameters $n_0, T, B_0, L_n = (n'_0/n_0)^{-1}$ and for a given wavelength. Because of the horizontal and vertical stratification, various values may be considered for the density, the temperature and the magnetic field. Yet, physically, in order to have an instability, the frequencies of the drift and IC

modes must become close to each other, and this is most easily controlled by the density inhomogeneity scale-length L_n and/or the wave-length. Here, L_n is a local, spatially dependent parameter that determines the local properties of the drift-cyclotron mode. Assuming a cylindric elongated density structure with a radius r_0 and with a Gaussian radial density distribution $n_0(r) = N_0 \exp(-r^2/a^2)$, where N_0 is the density at the axis of the cylinder, and *a* determines the radial decrease of the density, we have $L_n(r) = n_0(r)/n'_0(r) = a^2/(2r)$. Hence, we have a radially changing scale-length, which goes to infinity at the center and decreases towards the boundary. In the case of an e-folding decrease along *r*, we have $a = r_0$, $n_0(r_0)/N_0 = 37\%$, and $L_n/r_0 = 1/2$. For such a density profile L_n is minimum in the outer region of the plasma column.

Hence, Eq. (3) is solved numerically in terms of the wavelength and the density scale length, for parameter values applicable to the solar corona. As an example we take $B_0 = 10^{-3}$ T, $n_0 =$ 10^{13} m^{-3} , $T_e = T_i = 10^6 \text{ K}$, that may be used to describe the physical properties of the plasma at the altitude of around one solar radius, and we have chosen a very short density inhomogeneity scale-length, $L_n = 5$ m. In the case of the Gaussian profile discussed above and for the efolding decrease, this yields the characteristic radius of the structure $r_0 \simeq 10$ m. Such small values for L_n are necessary to obtain high values for the drift wave frequency because it is proportional to $1/L_n$. For these parameters, the plasma beta is 0.00035 and we have a proper electrostatic limit. The result is presented in Fig. 1. The instability develops in a narrow range of wave-lengths around 1 m. Note that the ion gyro radius for these parameters is equal to $\rho_L = 0.94$ m. For larger wavelengths, the frequencies of the two modes become well separated and the instability vanishes. The maximum increment ω_i is ≈ 5250 Hz at the real frequency $\omega_r = 102830$ Hz. This value of the increment is lower than the approximate theoretical value given by $\Omega_i (m_e/m_i)^{1/4}$. The small values for L_n imply a relatively short time for the existence of such structures, making them difficult to detect. As seen from Fig. 1, the frequencies of the corresponding modes are high, of the order of 10^5 Hz. On the other hand the ion collision frequency for the given parameters is about 10^{-2} Hz. The perpendicular ion diffusion coefficient is $D_{\perp} \approx \kappa T_i v_i / (m_i \Omega_i^2) = 0.01 \text{ m}^2/\text{s}$. The diffusion velocity in the direction of the given density gradient is $D_{\perp} \nabla n/n = 2$ mm/s only. So we have about 7 orders of magnitude difference for the two characteristic times, and this is enough time for the instability to develop before the equilibrium density structure disappears.

The instability discussed here implies very short scale lengths for the inhomogeneity of the equilibrium density and/or a very weak magnetic field. Only in such circumstances can the frequencies of the drift and IC modes become close to each other so that the two modes can effectively couple. The present analysis clearly demonstrates the instability of perpendicularly



Figure 1: The frequencies and the increment (multiplied by 10) of the coupled drift-cyclotron mode in terms of the wave-length.

propagating modes at frequencies in the range of the ion cyclotron frequency and at wavelengths of the order of the ion gyro radius. The tiny filaments are very elongated, they may extend to many solar radii, and the growth of the mode and the consequent dissipation and heating of ions may take place over large distances. The numbers used here are for the solar corona however, the large radial (from the Sun) length of the structures and the consequent decreasing of the magnetic field intensity implies larger density scale lengths at which the instability takes place. This can be easily shown by reducing the magnetic field to 10^{-4} T and the number density by one order of magnitude. As a result, the necessary density scale length L_n for the unstable modes becomes of the order of 60 meters. Therefore the development of unstable growing modes may take place at large distances along the same density filaments that pervade the corona and spread within the solar wind. The presence of hotter ions in and around such filamentary structures should be interpreted as an indication and a signature of the instability.

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