*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-4.165 (2008)*

Localized excitations in Debye crystals: a survey of theoretical results

I. Kourakis[1], V. Koukouloyannis[2], P. K. Shukla[4]

[1] *Centre for Plasma Physics, Queen's University Belfast, BT7 1 NN Northern Ireland, UK*

[2] *School of Physics, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece*

[3] *Ruhr Universität Bochum, Fakultät für Physik und Astronomie, D-44780 Bochum, Germany*

Introduction. Dusty plasma crystals (DPCs) occur in dusty plasmas (DP), in low-temperature gas discharge experiments, wherein the charged dust particles are suspended under the combined action of gravity and electric forces [1]. DPC configurations typically consist of 2D – hexagonal in general – monolayers, but also be 1D chains, when appropriate trapping potentials are used for lateral confinement [2]. Our aim here is to revisit the nonlinear aspects of dust grain motion, from first principles, reviewing earlier [3, 4] and reporting recent results [5, 6].

Basic principles. The origin of nonlinearity in a plasma sheath is evident. First, electrostatic inter-grain interactions are generally associated with an interaction potential U_{int} (Debye or else). The interaction force acting on the $n-$th grain is $F_n = -\nabla U_{int}(|\mathbf{r}_n - \mathbf{r}_{n-1}|)$. For small displacements, U_{int} can be Taylor-expanded near the equilibrium position $\{x_n, y_n, z_n\} = \{nr_0, mr_0, 0\}$ ($n, m = 0, \pm 1, \pm 2, \ldots$; assuming gravity along $\hat{z}$), thus yielding a polynomial in the (small) displacements δx_n, δy_n and δz_n [3, 6, 8]. Furthermore, the sheath environment provides an on-site potential which may be strongly *anharmonic* (see Fig. 1) near equilibrium, i.e.,

$$\Phi(z) \approx \Phi(z_0) + \frac{1}{2}M\omega_g^2(\delta z_n)^2 + \frac{1}{3}M\alpha\,(\delta z_n)^3 + \frac{1}{4}M\beta\,(\delta z_n)^4 + \mathscr{O}[(\delta z_n)]^5\,. \qquad (1)$$

The coefficients α and β may be obtained from experiments [9] or from *ab initio* calculations.

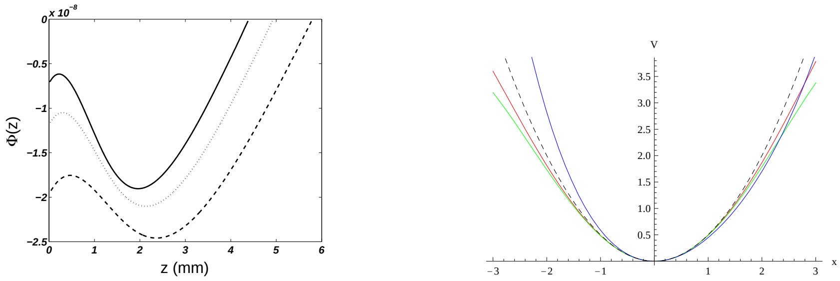


Figure 1: The (anharmonic) sheath potential $\Phi(z)$ is depicted vs. the vertical distance z from the negative electrode: (left) as results from *ab initio* numerical simulations (density n increasing from bottom to top) (data courtesy of G. Sorasio); (right) based on experimental data (A. Melzer, private communication).

Finally, dust-grain motion introduces a nonlinear transverse-to-longitudinal mode-coupling effect [8], as it combines 2 or 3 degrees of freedom, involving distinct modes.

Solitary waves. Localized excitations occur in DPCs, sustained via a mutual balance among nonlinearity and dispersion. The nonlinear horizontal (longitudinal, acoustic) and vertical (transverse, inverse-optic) dust grain motion in a 1D dust monolayer has been studied thoroughly [3, 6], so results need only be briefly summarized here.

Longitudinal solitons. DPCs support supersonic longitudinal solitary solitons. These are efficiently modelled Korteweg - de Vries and/or Boussinesq equation solutions [4, 6]. Experimentally they represent localized density perturbations, of stationary profile [10] and surviving collisions [11], as predicted by theory [3, 6]. Interestingly, rarefactive pulses have been observed experimentally only recently [7], although long predicted by theory [6].

Off-plane (transverse) envelope solitons. Modulated envelope wavepackets associated with *backward*-propagating (negative group velocity) transverse (off-plane) oscillations are predicted by the nonlinear Schrödinger (NLS) theory in a quasi-continuum lattice approximation [12]. Such wavepackets are also observed in experiments [13].

In-plane modulated wavepackets. *Asymmetric* localized envelope solitons, involving a nonzero zeroth harmonic contribution, occur in the longitudinal (in-plane, acoustic mode) direction [14]. In 2D, hexagonal DP crystals may sustain modulated bell-shaped envelope structures, formed as a result of modulational instability of in-plane vibrations [15].

Discrete dust-lattice modes. Intrinsic localized modes (*Discrete Breathers, DBs*) are frontier research in present-time nonlinear science. They consist of highly localized (only few sites moving) periodic oscillatory lattice modes. Non-Anderson-type localization thus occurs, due to the crystal discreteness, in combination with nonlinearity. DB excitations may occur, related with transverse dust lattice vibrations, either in 1D [16, 17] or in 2D [5, 18] crystals. A discrete analysis of hexagonal crystals from first principles suggests the occurrence of ultra-localized multipole modes (discrete vortices; see Fig. 2) [5, 18]. The stability profile of DBs depends on the discreteness parameter $\varepsilon = \omega_{T,0}^2/\omega_g^2$, which is the (square) ratio of the transverse mode eigenfrequency by the transverse gap frequency (related to the sheath potential well as $\sim \Phi''(0)$). In general, the smaller the value of ε, the “more dicrete" a lattice system is. Detailed results will be reported soon [18].

Discrete dust lattice excitations, so far essentially unexplored, open new directions for Debye crystal applications, once experimentally confirmed and eventually manipulated. A brief overview of existing results on DBs in hexagonal crystals is presented in [5], and will soon appear in the form of a published article. We shall now dedicate the remaining part of this brief report on DBs in 1D dust crystals.

Discrete Breathers in 1D crystals. We have considered a 1D lattice with on-site potential

$V(x) = x^2/2 + ax^3/3 + bx^4/4$ with indicative parameter values taken from [9]. The conditions for multisite breathers are [17] $\phi_i = 0, \pi$, where ϕ_i are the phase differences between successive oscillators. The stability of these solutions is determined by the Floquet multipliers of the periodic orbit. If all the multipliers lie on the unit circle in the complex plane, the structure is stable. For every $\phi_i = \pi$ there is a pair of multipliers leaving the unit circle along the real axis for ε however small; see Fig. 2. So, the only stable configuration is the in-phase one, $\phi_i = 0$; see Fig. 3. Numerical investigations using values from [9] showed that it remains stable for $\varepsilon < 0.02$, yet it undergoes a "bubble" of instability for approximately $0.02 < \varepsilon < 0.03$ and becomes stable again until $\varepsilon < 0.04$, as shown in Fig. 4. This scenario suggests that, for $\varepsilon = 0.016$ (i.e., as drawn from the data from [2]), this kind of motion can be supported.

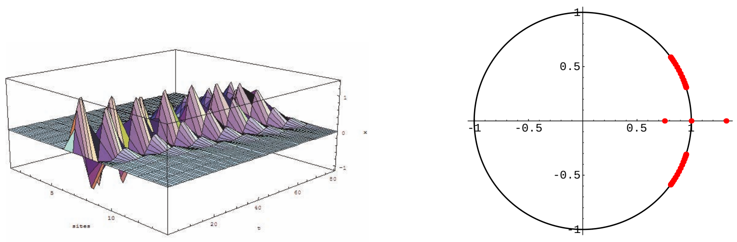


Figure 2: (From left to right) (a) Time evolution of an unstable 1D breather: see that the excitation is not localized anymore, after some time; (b) The corresponding linear stability profile: two eigenvalues have departed from the imaginary circle.

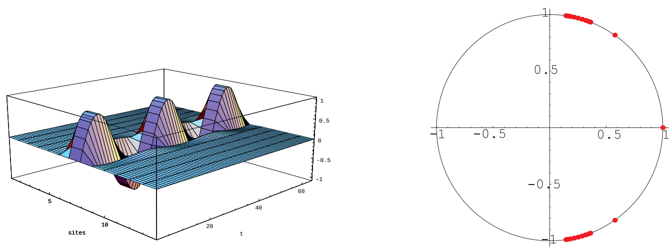
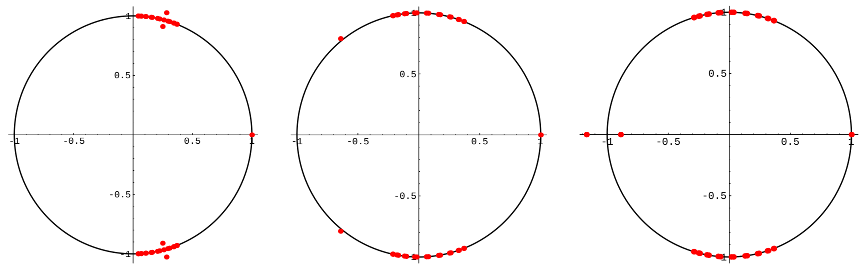


Figure 3: (From left to right) (a) Time evolution of a stable (1D, 1:1 here) 2-breather for $\varepsilon = -0.016$; (b) The linear stability profile: all eigenvalues lie on the imaginary unit circle.

Conclusions. DPCs provide an excellent test-bed for continuum and discrete nonlinear theories. Apart from density solitons [10, 7] and transverse wavepackets [13], these theoretical predictions have still not rigorously been tested in the laboratory. This provides a challenging direction for experimental investigations, which will hopefully confirm these results.

Acknowledgments. Prof André Melzer and Dr Gianfranco Sorasio are warmly acknowledged for providing access to experimental and numerical, respectively, data.



$\varepsilon = 0.022$ $\varepsilon = 0.037$ $\varepsilon = 0.041$

Figure 4: The stability scenario of an in phase 2-breather for increasing ε

References

[1] G .E. Morfill *et al*, *Plasma Phys. Cont. Fusion* **44**, B263 (2002).

[2] T. Misawa *et al*, *Phys. Rev. Lett.* **8**6, 1219 (2001).

[3] I Kourakis and P. K. Shukla, *Int. J. Bif. Chaos* **16**, 1711 (2006).

[4] I. Kourakis and P. K. Shukla, *Eur. Phys. J. D* **29**, 247 (2004).

[5] V. Koukouloyannis *et al.*, paper P4.162, these Proceedings.

[6] I. Kourakis, Proc. *5th Int. Conf. Phys. Dusty Plasmas (ICPDP5)*, Azores Portugal (2008).

[7] R. Heidemann *et al.*, Proc. *5th Int. Conf. Phys. Dusty Plasmas*, Azores Portugal (2008).

[8] I. Kourakis, P.K. Shukla, *Phys. Scripta* T**113**, 97 (2004).

[9] C. Zafiu, A. Melzer and A. Piel, Phys. Rev. E **63**, 066403 (2001).

[10] D. Samsonov *et al.*, *PRL* **88**, 95004 (2002); V. Nosenko *et al.*, *PRL* **88**, 215002 (2002).

[11] D. Samsonov, experimental work in progress; private communication.

[12] I. Kourakis, P.K. Shukla, *Phys. Plasmas*, **11**, 2322 (2004); *ibid*, **11**, 3665 (2004).

[13] D. Samsonov, S. Zhdanov and G.E. Morfill, *Phys. Rev. E* **71**, 026410 (2005).

[14] I. Kourakis, P.K. Shukla, *Phys. Plasmas*, **11**, 1384 (2004).

[15] B. Farokhi, I. Kourakis and P. K. Shukla, *Physics of Plasmas* **13**, 122304 (2006).

[16] I. Kourakis, P.K. Shukla, *Phys. Plasmas*, **12**, 014502 (2005).

[17] V. Koukouloyannis and I. Kourakis, *Phys. Rev. E* **76**, 016402 (2007).

[18] V. Koukouloyannis and I. Kourakis, *in preparation*.