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Effect of Recombination and Ionization on the Deduction of Mach Numbers in Flowing Magnetized Plasmas

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Along the magnetized presheath formed by the probes, not only the diffusive ion source in the magnetized pre-sheath with Boltzmann electrons[1], but also other sources such as charge exchange[2], ionization[3,4], and recombination[5] are important in the scrape-off layer of divertor-type tokamaks. Recombination and charge exchange processes in the fusion edge plasmas are important for the reduction of particle and heat fluxes onto the divertor targets. Molecular activated recombination (MAR) processes induced by the hydrogen or hydrocarbon puffing recently have shown the capability of contributing to the volume recombination[5,6], along with electron-ion recombination processes characterized by radiative and three-body recombinations.

Using the Mach probe, the ratio (R) of ion saturation currents are measured in the upstream and downstream directions, then the relevant Mach numbers (M_∞) is deduced using appropriate theories of recombining and ionizing plasmas in the magnetized plasma. For this, consider a Boltzmann transport equation of ions without volumetric sources such as ionization and recombination, assuming $\underline{E} = E_z\underline{k}$, $\underline{B} = B\underline{k}$

$$v_z \frac{\partial f_i}{\partial z} + \frac{q}{m} E_z \frac{\partial f_i}{\partial v_z} = -v_x \frac{\partial f_i}{\partial x}, \tag{1}$$

where the cross-field transport source can be approximated as [7]

$$S_t \equiv -v_x \frac{\partial f_i}{\partial x} \approx W\left[\alpha(f_{i\infty} - f_i) + (1-\alpha)(1-\frac{n}{n_\infty}) f_{i\infty}\right], \tag{2}$$

where $n_i,\ n_\infty,\ f_i,\ f_{i\infty},\ v_z,\ W$ and α are ion density along (or within) the pre-sheath, ion density outside the pre-sheath, atomic ion distribution along the pre-sheath, atomic ion distribution outside the pre-sheath with the drift velocity v_d, flow velocity, cross-field frequency ($\equiv D_\perp/a^2$, $D_\perp$ = anomalous cross-field diffusivity, a = probe radius) and normalized shear viscosity ($\equiv \eta_\perp/n_i m D_\perp$), respectively. One can add ionization, recombination and charge exchange to the above equations for the general cases. The ionization source, which is dominant for $T_e < 2\ eV$ in hydrogen plasma, can be either (a) $S_i = <\sigma v>_{ion}\ n_e(z) f_{N\infty}(v)$ from Bissel and Johnson (BJ)[8], or (b) $S_i = <\sigma v>_{ion}\ n_{eo} f_{N\infty}(v)|v|/C_s$ from Emmert, *et al.* (EE)[9], where $f_{N\infty}(v)$ is the distribution function of atomic neutrals. BJ's case seems to be reasonable, but their model cannot recover the Maxwellian distribution as $z \to \infty$, while EE's case can become Maxwellian although their ionization term does not seem to reasonable, because there is no source particle with zero velocity. We adopt BJ's case along with cross-field transport source, then we can recover $f_{i\infty}$ as $z \to \infty$, and obtain the similar sheath values as those by a kinetic model[7].

As for the recombination, there are two cases: $S_r = - < \sigma v >_{rec} n_e f_i(v)$ for electron-ion recombination (EIR), which is dominant for low temperature ($T_e < 2\ eV$) in hydrogen plasma, and $S_r = - < \sigma v >_{rec} n_e f_M(v)$ for molecular-activated recombination (MAR), which exists over the wide range of electron temperature in hydrogen plasma. Here $f_M(v)$ is the distribution function of relevant molecular ions. Dominant hydrogen-MAR processes are (a) dissociative attachment (DA) followed by mutual neutralization (MN), (b) ion conversion (IC) followed by dissociative recombination (DR), and (c) charge exchange ionization (CX) followed by dissociative recombination (DR). The reaction rate of hydrogen MAR can be approximated as

$$< \sigma v >_{MAR} \approx < \sigma v >_{MN \leftarrow DA} + < \sigma v >_{DR \leftarrow IC} + < \sigma v >_{DR \leftarrow CX} \approx < \sigma v >_{DR \leftarrow IC},$$

where A is a neutral atom such as H, He or Ar, since some portion of $(AH)^+$ are dissociated into A and H^* after recombined with electron. Since molecular ion density ($n_M \equiv n_i((AH)^+)$) is increased by IC, while it is decreased by DR, the contribution of DR to MAR can further be approximated as

$$< \sigma v >_{DR \leftarrow IC} n_e f_M \approx (1-\delta) < \sigma v >_{DR} ((AH)^+ + e \to A + H^*) n_e f_M$$

where δ ($\equiv < \sigma v >_{non-IC} / < \sigma v >_{IC}$) is the ratio of non-IC process among IC, so that $1-\delta$ is the probability that DR occurs after IC. So plasma volume source of atomic processes becomes

$$S_a = -K_M n_e f_M(v) - K_E n_e f_i(v) + K_I n_e f_N(v), \tag{3}$$

where f_M, f_i, f_N, K_M, K_E, and K_I are the distributions and reaction rates of molecular ions, atomic ions, atomic neutrals, the reaction rates of MAR ($\equiv < \sigma v >_{MAR}$), EIR ($\equiv < \sigma v >_{EIR}$), and ionization ($\equiv < \sigma v >_{ION}$), respectively. Then the Boltzmann equation with cross-field transport, recombination and ionization sources becomes

$$\begin{aligned} v_z \frac{\partial f_i}{\partial z} &+ \frac{q}{m} E_z \frac{\partial f_i}{\partial v} = S_t + S_a \\ &\approx W[\alpha(f_{i\infty} - f_i) + (1-\alpha)(1-\frac{n}{n_\infty}) f_{i\infty}] + n_e[-K_M f_M - K_E f_i + K_I f_N]. \end{aligned} \tag{4}$$

Assuming $T_N = T_M \equiv \tau T_i$, and $m_N = m_i$, the unperturbed distribution functions of ions ($f_{i\infty}$) and atomic neutrals ($f_N \equiv f_{N\infty}$) are given as

$$f_{i\infty}(v) = n_\infty \sqrt{\frac{m_i}{2\pi T_i}} exp\left[-\frac{m_i(v-V_d)^2}{2T_i}\right],\ f_N(v) = v_1 n_\infty \sqrt{\frac{m_i}{2\pi\tau T_i}} exp\left[-\frac{m_i v^2}{2\tau T_i}\right],$$

where $v_1 \equiv n_N/n_\infty$. By taking moments of Eq. (4) and using the dimensionless parameters such as $L \equiv C_s/W(z)$, $y \equiv \int[W(z)/C_s]dz$, or $y \equiv zW/C_s$, $M \equiv V_z/C_s$, $n \equiv n_i/n_\infty$, one can get the following dimensionless equations:

$$M\frac{dn}{dy} + n\frac{dM}{dy} = 1 - n + k_i n - k_r n^2,\ \frac{dn}{dy} + nM\frac{dM}{dy} = (M_\infty - M)[1-(1-\alpha)n] - k_i nM, \quad (5)$$

where k_i, k_m, k_e and k_r are the normalized ratios of ionization ($\equiv (K_I Z n_N a/C_s)(L/a)$), MAR ($\equiv (K_M Z n_M a/C_s)(L/a)$), EIR ($\equiv (K_E Z n_\infty a/C_s)(L/a)$) and total recombination ($\equiv k_m + k_e$) with respect to the cross-field transport contribution, respectively, and n_M is the density of relevant molecular ions. Here C_s is the ion acoustic speed ($\equiv \sqrt{(T_e + ZT_i)/m_i}$), n_N is the atomic neutral density and quasi-neutrality ($en_e = Zen_i$) is used. After some arrangements, one can get the following equations for dn/dy and dM/dy:

$$\frac{dn}{dy} = \frac{M_\infty - 2M - (M_\infty - M)(1-\alpha)n + (1+k_r n)nM}{1-M^2}, \quad (6)$$

$$\frac{dM}{dy} = \frac{1 - n - M(M_\infty - M)[1-(1-\alpha)n] + k_i n(1-M^2) - k_r n^2}{n(1-M^2)}. \quad (7)$$

Dividing Eq. (6) by Eq. (7) leads to

$$\frac{1}{n}\frac{dn}{dM} = \frac{M_\infty - 2M - (M_\infty - M)(1-\alpha)n + (1+k_r n)nM}{1 - n - M(M_\infty - M)[1-(1-\alpha)n] + k_i n(1-M^2) - k_r n^2}. \quad (8)$$

One can recover the same form of non-viscous Stangeby model[10] by putting $\alpha = 0,\ k_i = k_r = 0$, and that of strong viscous Hutchinson model[11] by putting $\alpha = 1,\ k_i = k_r = 0$, which confirms the validity of the fluid equation (Eq. (8)). General solutions of Eqs. (6) and (7) should be given numerically instead of solving Eq. (8) to avoid the possible problem of singularity [**?**] Figure 1 shows normalized density (n) profiles in terms of normalized fluid velocity (M) for $\alpha = 1$ and $M_\infty = 0.4$ with different ionization contribution ($k_i = 0.001$, 0.1, 1.0) with respect to the cross-field transport contribution. Both results by solving Eqs. (6) and (7), and Eq. (8) are exactly matched for this case. Figure 2 shows the ratio of ion saturation currents ($R \equiv J_{up}/J_{dn}$) in terms of ionization and recombination as an application of our model. For the small contribution, say less than 1% for ionization and much less than 0.1 % for recombination, there is no visible change in R. Larger ionization contribution ($K_i \geq 0.01$) makes R be decreased with the same Mach number (M_∞) indicating the decrease of the calibration factor ($K = \ln[R]/M_\infty$), while R increases with recombination, although we cannot handle the case somewhat larger contribution of recombination, say, larger than ($k_r \geq 10^{-3}$) due to singularity of our fluid model or due to un-known numerical instability in our numerical code. Calibration factor for deducing Mach number's decreases with ionization, leading to the higher Mach numbers for the same ratio of ion saturation currents, while it increases with recombination. With high neutral pressure and high electron temperature or larger connection length of the flux tube (L) of existing and future toroidal divertor-type machines, contributions of ionization and recombination would become larger due to strong magnetic field and neutral pressure (strong e-n and i-n collisions).

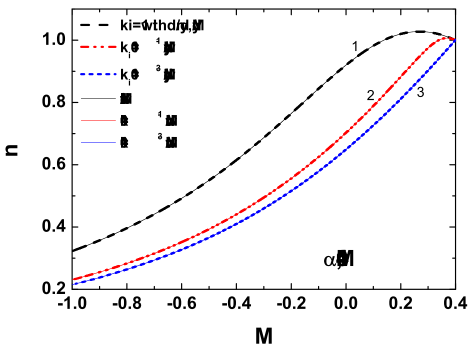

Figure 1: Normalized density (n) profiles in terms of normalized fluid velocity (M) for $\alpha = 1$ and $M_\infty = 0.4$ with different normalized ionization contribution (k_i = 0.001(line 3), 0.1 (line 2), 1.0 (line 1)) with respect to the cross-field transport contribution.

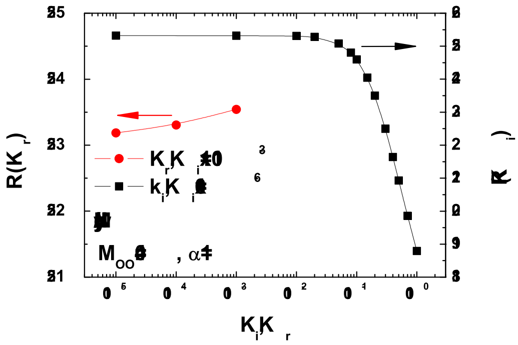

Figure 2: Ratio of ion saturation currents ($R \equiv J_{up}/J_{dn}$) in terms of normalized ionization rate (k_i) and recombination (k_r).

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