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Generation of kinetic Alfvén waves by Non-conventional Global Alfvén Eigenmodes

O.P. Fesenyuk[1], A. Weller[2], A. Werner[2], Yu.V. Yakovenko[1], Ya.I. Kolesnichenko[1]

[1] *Institute for Nuclear Research, Kyiv 03680, Ukraine*

[2] *Max-Planck-Institut für Plasmaphysik, D-17491 Greifswald, Germany*

Introduction. Alfvén instabilities driven by energetic ions are often observed in tokamak and stellarator plasmas. They may deteriorate the confinement of the fast ion population that excites the instability and, in some cases, even that of the bulk plasma [1]. It was suggested [2] that the deterioration of the plasma energy confinement in W7-AS experiments [1] is caused by the oscillating electric field of kinetic Alfvén waves (KAW) emitted by an ideal Alfvén instability via tunnelling as described in [3]. The instability was identified as a Non-conventional Global Alfvén Eigenmode (NGAE) [2], i.e., an eigenmode with the frequency lying slightly above the frequency of an Alfvén continuum branch [in contrast to the Global Alfvén Eigenmode (GAE), which has the frequency slightly below a continuum branch]. The NGAE-modes in stellarators were studied in detail in a recent work [4]; they are partly similar to reversed-shear Alfvén eigenmodes (or Alfvén cascade eigenmodes) [5] observed in tokamaks.

In the present work, the generation of a KAW by an excited NGAE-mode is studied in detail. The obtained results are applied to a W7-AS shot #40173.

Model. Introducing the current due to finite Larmor radius effects [6] into the Alfvén wave equation [7], we obtain

$$\omega^2\nabla\cdot\left(\frac{1}{v_A^2}\nabla_\perp\Phi\right)+B\nabla_\parallel\left\{\frac{1}{B^2}\nabla\cdot\left[B^2\nabla_\perp\left(\frac{1}{B}\nabla_\parallel\Phi\right)\right]\right\}+8\pi\nabla\cdot\left\{\frac{\mathbf{B}\times\kappa}{B^4}\left[(\mathbf{B}\times\nabla p)\cdot\nabla\Phi\right]\right\}$$
$$-\frac{4\pi}{c}\nabla\cdot\left[\frac{1}{B}\mathbf{B}\times\nabla\left(\frac{j_\parallel}{B}\right)\nabla_\parallel\Phi\right]+\tau\nabla_\perp^4\Phi=0, \qquad (1)$$

where ω is the frequency; Φ is the scalar potential of the wave; v_A is the Alfvén velocity; $\nabla_\parallel=\mathbf{b}\cdot\nabla$; $\nabla_\perp=\nabla-\mathbf{b}\nabla_\parallel$; $\mathbf{b}=\mathbf{B}/B$; $\mathbf{B}$, p, κ and $j_\parallel$ are the magnetic field, the plasma pressure, the field line curvature, and the longitudinal current, respectively, in the equilibrium state; $\tau=\alpha\rho_i^2\omega^2/v_A^2$; $\alpha=3/4+T_e/T_i$; $\rho_i=c(M_iT_i)^{1/2}/(e_iB)$ is the ion gyroradius; M_i is the ion mass; e_i is the ion charge; T_i and T_e are the ion and electron temperatures, respectively. The last term of equation (1) describes nonideal effects. To simplify our consideration, we disregard the compressibility effects, which may be important for low-frequency GAE-modes [4]; however, such effects could be easily incorporated into our analysis.

Considering the case of low $\beta=8\pi p/B^2$ and high aspect ratio and keeping only one Fourier harmonic of the wave, $\Phi(r,\theta,\phi) = \Phi(r)\exp(im\theta - in\phi)$ with θ and ϕ the poloidal and toroidal angles, respectively, we write Eq. (1) as follows:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\omega^2-\omega_A^2}{\omega_{A0}^2}\rho r\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right) - \frac{m^2}{r^2}\frac{\omega^2-\omega_A^2}{\omega_{A0}^2}\rho\Phi - \frac{\tilde{k}}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\tilde{k}}{\mathrm{d}r}\right)\Phi - \frac{R^2}{\delta}\Xi(r)\Phi$$
$$+R^2\delta\tau\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} - \frac{m^2}{r^2}\right)^2\Phi = 0, \tag{2}$$

where $\omega_A = \tilde{k}\omega_{A0}/\rho^{1/2}$ is the continuum frequency, $\tilde{k} = m\iota - n$, $\omega_{A0} = \bar{v}_A/R$, $\bar{v}_A$ is average v_A at the magnetic axis, $R = L_a/(2\pi)$, L_a is the magnetic axis length, $\rho = n_i(r)/n_i(0)$, ι is the rotational transform, the dimensionless quantity $\delta(r)$ characterizes the cross section elongation.

$$\Xi(r) = \frac{8\pi}{B^4}\frac{m^2}{r^2}\frac{\mathrm{d}p}{\mathrm{d}r}\left(4\pi\frac{\mathrm{d}p}{\mathrm{d}r} + B\frac{\mathrm{d}B}{\mathrm{d}r}\right) - \frac{1}{B^2r^2}\left[\frac{\mathrm{d}^2B_\theta}{\mathrm{d}r} + \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{4\pi B_\theta}{\bar{B}^2}\frac{\mathrm{d}p}{\mathrm{d}r}\right)\right]m\tilde{k}. \tag{3}$$

Here the first term represents the effect of the plasma pressure gradient (the expression in parentheses is, in fact, the field line curvature); the second term, of the longitudinal current gradient.

Introducing the wave function $\Phi_1 = r^{1/2}\rho^{1/2}\Phi$, we obtain an equation with the highest-order derivative term in the form $(\mathrm{d}/\mathrm{d}r)[(\omega^2-\omega_A^2)(\mathrm{d}\Phi_1/\mathrm{d}r)]$. We assume that the function $\omega_A(r)$ has an extremum at some radius $r = r_*$ and seek for a NGAE-mode localized at r_*. In this approximation we keep the radial dependence only in Φ and $\omega_A^2(r) \approx \omega_{A*}^2(1 - x^2/L^2)$, where the subscript $*$ refers to $r = r_*$, $x \equiv r - r_*$, $L \equiv (-2\omega_{A*}/\omega_{A*}'')^{1/2}$, prime denotes differentiation in r (for a NGAE-mode $\omega_{A*}'' < 0$). To reduce the order of equation Eq. (2), we use the transformation $\Phi_1(x) = (r_*/m)\int_{-\infty}^{\infty}(p^2+1)^{-1/2}\Psi(p)\exp[ip(m/r_*)x]\mathrm{d}p$. We obtain the Schrödinger equation

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}p^2} + [E - V(p)]\Psi = 0, \tag{4}$$

with $E = -L^2(m^2/r_*^2)(\omega^2 - \omega_{A*}^2)/\omega_{A*}^2$ and

$$V(p) = \frac{3}{(p^2+1)^2} - \frac{b}{p^2+1} - \eta\left(p^2+1\right), \tag{5}$$

where $\eta = (m^4/r_*^4)[R^2L^2\delta\tau\omega_{A0}^2/(\rho\omega_A^2)]|_{r=r_*}$,

$$b = 2 - \frac{L^2}{\omega_{A*}^2}\left\{\frac{\omega_{A0}^2}{\rho}\frac{\tilde{k}}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\tilde{k}}{\mathrm{d}r}\right) + \frac{R}{\delta\rho}\omega_{A0}^2\Xi + \frac{1}{2r^{1/2}\rho^{1/2}}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{\omega^2-\omega_A^2}{r^{1/2}\rho^{1/2}}\frac{\mathrm{d}(r\rho)}{\mathrm{d}r}\right]\right\}\Bigg|_{r=r_*}. \tag{6}$$

Let us first disregard non-ideal effects ($\eta = 0$). When $b > 0$, the potential energy $V(p)$ has a minimum (two minima for $0 < b < 6$), where bounded states (eigenmodes) may exist (conditions of their existence can be found in, e.g., Ref. [4]). These bounded states are characterized by $E < 0$, which corresponds to the frequency levels above the maximum of ω_A when $\omega_A'' < 0$.

Generation of a KAW. A sketch of $V(p)$ is shown in Fig. 1. Nonideality ($\eta \neq 0$) makes tunnelling of the wave to $p = \pm\infty$ possible. When the tunnelling is weak, it turns the bounded states into quasi-steady states with the wave leaking slowly through the barriers. This leakage means conversion of the mode into KAWs [3] (the dependence of V on p at large $|p|$ corresponds to the KAW dispersion). We assume that there are no KAWs incident to the barriers and tailor WKB solutions for the KAWs with the mode trapped in the potential well. We obtain that E has a non-zero imaginary part, which means that $\omega = \omega_R + i\gamma$, where ω_R is the frequency found in the ideal limit, and the radiation damping rate γ is given by

$$\frac{\gamma}{\omega_{A*}} = -\frac{r_*}{2\pi Lm}\left(\frac{\delta\omega}{\omega_{A*}}\right)^{1/2} \exp\left[-\pi\sqrt{\frac{\rho_*}{\alpha\delta_*}\frac{\delta\omega}{\omega_{A*}}\frac{L}{\rho_i}}\right] \quad (7)$$

with $\delta\omega \equiv \omega - \omega_{A*}$.

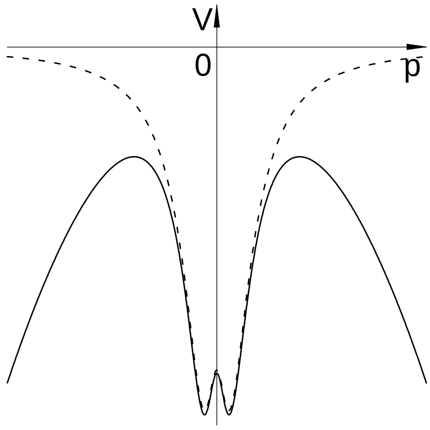

Figure 1: Sketch of $V(p)$ for $\eta = 0$ (dashed line) and $\eta \neq 0$ (solid line).

The amplitude of the emitted KAWs for $x^2 \gg (r_*^2/m^2)\, b$ and $b > 6$ [i.e., when $V(p)$ has a single minimum] is given by the following expression:

$$\left|\frac{\Phi(x)}{\Phi_A}\right|^2 \approx \frac{1}{4} b^{3/4}(2l+1)^{1/2}|\eta|^{1/2}\left|\frac{r_*}{mx}\right|^3 \exp\left[-\pi\sqrt{\frac{\rho_*}{\alpha\delta_*}\frac{\delta\omega}{\omega_{A*}}\frac{L}{\rho_i}}\right] \quad (8)$$

where Φ_A is the wave amplitude at $x^2 \ll (r_*^2/m^2)\, b^{1/2}$, where the ideal NGAE-mode prevails, l is the radial mode number.

W7-AS experiment. Two instabilities with almost steady-state amplitudes and the frequencies of 16 kHz and 28 kHz were observed in the W7-AS discharge No. 40173 about $t = 0.35$ s. The instabilities had the same poloidal mode number $m = 3$. The corresponding eigenmodes were identified in Ref. [8] as GAE- and NGAE-modes, respectively, with $(m,n) = (3,1)$. In Ref. [8], the ι-profile was modified so that the maximum and the minimum of the Alfvén continuum branch $(3,1)$ fit the observed frequencies of the instabilities (see the Alfvén continuum in Fig. 2). The continuum in Fig. 2 is calculated with the plasma compressibility taken into account [the frequency of the branch $(0,0)$ is the geodesic acoustic mode (GAM)]. Since

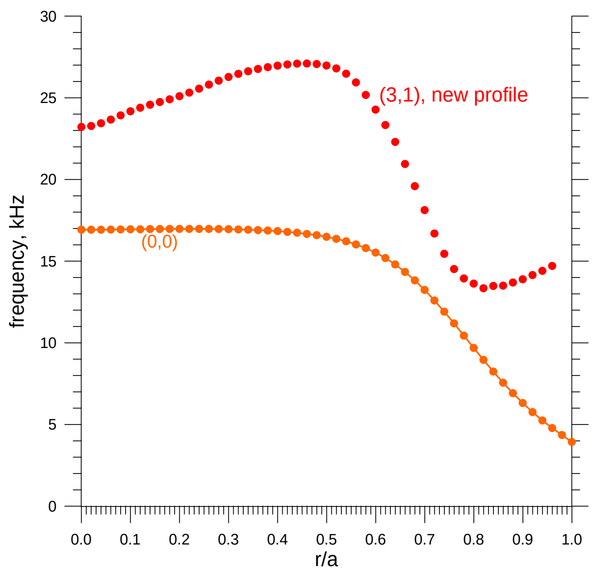

Figure 2: Alfvén continuum branches labelled by the numbers (m,n) for discharge No. 40173 of W7-AS.

the frequency of the 28-kHz mode is almost twice higher than the GAM frequency, we neglect the effect of compressibility for this mode. Analysing the $(3,1)$-branch of the Alfvén continuum, we find relevant parameters: $r_* = 0.51a$, $L = 0.27a$, $b = 4.0$, $\rho_i = 0.12\,\text{cm}$, $\eta = 1.1\times10^{-2}$. Assuming that $l = 0$, we find: $\delta\omega/\omega = 0.15$, $\gamma/\omega = 1.3\times10^{-5}$. Thus, the radiative damping of this mode is rather weak, much less than the line width observed in experiment, which means that this mechanism is not dominant for this particular mode.

Conclusions. The rate of the radiative damping of NGAE-modes and the amplitudes of the KAWs radiated by these modes have been found. They are in agreement (up to coefficients of order of unity) with estimates given in Ref. [2]. Analysis of an NGAE mode observed in the W7-AS discharge No. 40173 shows that the radiative damping of that specific mode is rather weak.

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