The Interaction Between Tearing Modes and Transport in Plasmas

M. James[1,2], D.J. Applegate[2], J.W. Connor[2] and H. R. Wilson[3]
[1]University of Bristol, H H Wills Physics Laboratory, Royal Fort, Bristol BS8 1TL UK
[2]EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxon OX14 3DB UK
[3]Department of Physics, University of York, Heslington, York YO10 5DD UK

Abstract: A sheared slab model of the plasma is employed to explore the interaction between tearing modes and transport. Two processes are considered. In the first we calculate the effect of the island on the electron density profile as a first step towards a self-consistent calculation of the polarisation current. In the second, we employ a reduced model for these profile effects to calculate the impact of the island on the stability of the ion temperature gradient mode. We find that while a local theory predicts that the island has a strong stabilising influence, the more complete global WKB theory predicts the existence of new, localised, unstable modes that are not permitted in the absence of the island.

1. INTRODUCTION

Small scale magnetic islands are important to understand in tokamak plasmas for a number of reasons. One is the triggering of neoclassical tearing modes (NTMs), where (typically) another MHD event generates a small "seed" island which can then grow to large amplitude if its width exceeds a certain threshold. This threshold is thought to arise from a combination of the ion polarisation current and cross-field transport effects. The polarisation current itself will depend on the density profile across the island, so understanding the impact of the island on the transport processes is a crucial ingredient towards a model of the NTM threshold. A second issue is the recent experiments to suppress ELMs using resonant magnetic perturbations (RMPs) to degrade the confinement in the pedestal region. Understanding how the resulting small scale islands influence the transport is important for developing a model for the observed suppression of ELMs and hence whether or not the technique is viable for ITER. In this paper we explore two issues: how an island affects the density and potential profiles, and how these profile modifications influence the micro-instabilities responsible for turbulent transport: the ion temperature gradient, ITG, mode in particular.

2. PLASMA PROFILES NEAR A MAGNETIC ISLAND

Consider an island chain in a sheared slab of the plasma, where the magnetic field is given by:

$$B = B_0 \nabla z - \nabla \psi \times \nabla z \qquad\qquad \psi = -\frac{B_0 x^2}{2L_s} + \tilde{\psi}\cos K_y y \tag{1}$$

This provides an island chain along the *y*-direction, of half-width $w = \left(4L_s\tilde{\psi}/B_0\right)^{1/2}$ and length $2\pi/K_y$ at the flux surface where x=0. Density and temperature gradients (far from the island) are maintained in the *x*-direction with scale lengths L_n and L_T respectively. We consider long, thin islands satisfying $K_y w$<<1 and work in the rest frame of the island chain. A flux function $\chi=2x^2/w^2-\cos K_y y$ is defined, so that $\chi=1$ denotes the separatrix. We adopt the following simplified model equation for the electron distribution function:

$$v_{/\!/}\nabla_{/\!/}f_e = -D\frac{\partial^2 f_e}{\partial x^2} + \nu\left(f_e - nF_M\right) \tag{2}$$

where $v_{||}$ is the component of velocity parallel to the magnetic field lines, ν is the collision frequency, F_M is the Maxwellian velocity distribution, n is the electron density, and D is the cross-field diffusion coefficient.

Far from the magnetic island the cross-field diffusion is negligible, and the solution is of the form $f_e=n(\chi)F_M$. To determine $n(\chi)$ we introduce diffusion perturbatively. Defining angled brackets to represent the flux surface average operator that annihilates the parallel derivative, and integrating over velocity space we find that

$$\left\langle \frac{\partial^2 n}{\partial x^2}\right\rangle = 0 \Rightarrow n(\chi) = \frac{1}{2\sqrt{2}}\frac{w}{L_n}\int_1^{\chi}\frac{d\chi}{Q} + a \qquad Q = \frac{1}{2\pi}\oint\sqrt{\chi+\cos\xi}\,d\xi \tag{3}$$

where $\xi=K_y y$. We have employed the boundary condition that, far from the island, $(1/n)dn/dx=-1/L_n$ and a is a constant of integration, to be determined. If the cross-field diffusion can be neglected everywhere, then equation (3) will hold right up to the island separatrix. The density is a constant inside the island, $n(\chi)=n_0$. Matching at the separatrix determines $a=n_0$. This profile is shown as the dashed line in Fig 1. Notice that the density gradient is discontinuous at the separatrix, so that the second derivative is infinite there. This demonstrates that there is a narrow layer around the separatrix where diffusion cannot be neglected, no matter how small D is. This region was first identified in [1] and then treated rigorously in the kinetic limit in [2]. We use the formalism presented in [2] in which Eq (2) is solved in a narrow layer close to the separatrix, treating diffusion and parallel streaming on an equal footing. Collisions can be neglected here. As one moves away from the separatrix, where $\chi=1+\delta$, (δ<<1) the distribution function takes a linear form [2]:

$$f_e = c_1\left[\left(\frac{\pi w^3 u}{L_s a D}\right)^{1/2}\frac{\delta}{8} + 0.855\right] \tag{4}$$

where $u=|v_{||}|$ and c_1 is to be determined by matching to the outer region $f_e=n(\chi)F_M$ in the limit $\chi\to 1$. Note that

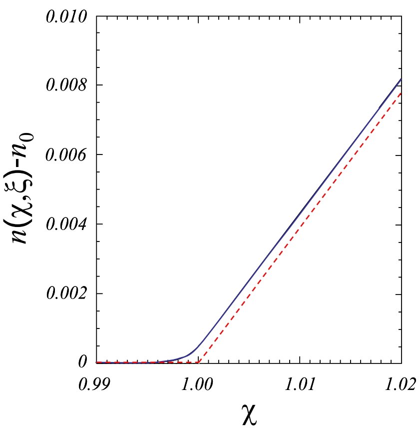

Figure 1: Density profile across the island O-point in vicinity of the separatrix with (full) and without (dashed) diffusion effects.

this matching cannot be performed directly as the solution in Eq (4) is not a Maxwellian. We therefore introduce an intermediate matching region, writing the solution there as

$$f_e = n(\chi)F_M + A(v)g(\chi) \quad \text{where} \quad \frac{d}{d\chi}\left(Q\frac{dg}{d\chi}\right) = 2\frac{dQ}{d\chi}\frac{w^2 v}{8D}g \tag{5}$$

We solve for g numerically with the boundary condition that $g \to 0$ far from the island. Equations (4) and (5) can be used to match the forms for f_e and its derivatives with respect to χ, to provide two equations for the 3 unknown quantities a, c_1 and $A(v)$. The final constraint results from the condition that the integral of A over velocity space must be zero (as $n(\chi) = \int f_e d^3v$). The resulting density distribution is shown as the full curve in Fig 1. Note that the density is now smoothed in the vicinity of the separatrix. In addition, it is no longer a flux surface quantity in that region, nor is it Maxwellian in velocity space. All of these will influence the polarisation current, which is work in progress.

3. MICROSTABILITY NEAR A MAGNETIC ISLAND

Our earlier work described how the modifications to the density and flow profiles in the vicinity of a magnetic island affect ITG mode stability [3]. We developed a local (in y) theory to show that the island has a strong stabilising influence, primarily because of the density gradient flattening inside the island. Indeed, the flow shear from the electrostatic potential outside the island was found to be slightly destabilising. In that work, we described a WKB theory to develop the full 2-D mode structure, writing the perturbed potential in the form $\tilde{\varphi}(x,y) = F(x,\varepsilon y)e^{i\int k_y dy}$ where the small parameter $\varepsilon=(K_y/k_y)^{1/2}$, and k_y represents the characteristic wavenumber of the fluctuations in the y-direction. To leading order in ε, we then derived the following eigenmode equation:

$$\frac{\rho_s^2}{\tau}\frac{\partial^2\tilde{\varphi}}{\partial x^2} + \left[\frac{L_n^2\omega_{*e}^2}{L_s^2\tau\rho_s^2(\Omega_0-\alpha_d S)^2}x^2 - \left[\frac{(\Omega_0-\alpha_d S)-(\omega_{*e}-\alpha_n S/\omega_E)}{(\Omega_0-\alpha_d S)\tau+(1+\eta_i)(\omega_{*e}-\alpha_n S/\omega_E)} + b\tau^{-1}\right]\right]\tilde{\varphi} = 0 \tag{6}$$

where the slow y-dependent profile modifications are described through the function $S(x,y)$ (see [3] for this and other definitions). Solutions of this equation are described in [3], where the higher order WKB theory is also outlined to derive the mode-structure in the y-direction. Specifically, at O(ε), we find that k_y must be chosen so that $\partial\Omega_0(k_y, y)/\partial k_y=0$ and this generally requires k_y to be complex. Indeed, the Cauchy-Riemann conditions mean that this constraint is equivalent to identifying the point where the growth rate is stationary in the plane of complex k_y. Figure 2 shows such a minimum, corresponding to a strong growth rate $\sim 0.5 v_{thi}/L_n$, comparable to the growth rate with no island and much greater than the prediction of the local theory. Such a solution (ie with $\mathrm{Im}(k_y)\neq 0$) would not be permitted in the absence of an island

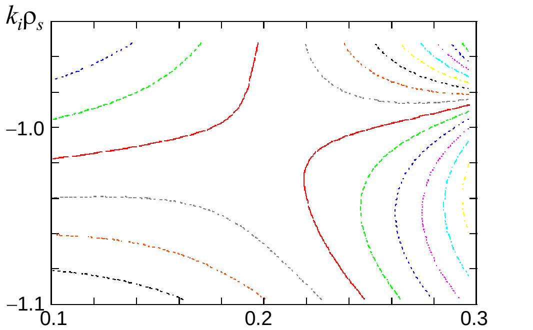

Fig 2: Contours of constant growth rate as a function of real and imaginary $k_y=k_r+ik_i$. The full contour corresponds to a growth rate $0.5v_{thi}/L_n$

(where the amplitude F is independent of y) as it would violate the periodicity in y. However, here we find that F is localised about a position in y of maximum or minimum instability (ie the X or O-points of the island). Specifically, it is a Gaussian, of width $\sigma\sim[(\partial^2\Omega_0/\partial k_y^2)/(\partial^2\Omega_0/\partial y^2)]^{1/4}\sim\varepsilon/K_y$ (ie intermediate between the island length and the wavelength of the fluctuations). Then we find that the imaginary part of k_y corresponds to a shift in the peak of the Gaussian, so that we can write the potential in the form:

$$\tilde{\varphi} = exp\left[-\left(y+\sigma^2 k_i/2\right)^2/\sigma^2\right]e^{ik_r y}\hat{F}(x,\varepsilon^2 y) \qquad (7)$$

where we have written $k_y=k_r+ik_i$. This shift is a significant fraction of the length of the island, so that the mode is not localised near y=0, where it is locally most unstable (ie the X-point). The result is rather unexpected: while the slab-like (ie local) ITG modes are strongly stabilised [3], the island permits a new type of mode structure, which is unstable. Specifically, it is highly localised in the y-direction, and this might therefore be expected to result in a reduced level of transport. A numerical solution of the full 2-D equations (without exploiting the WKB theory) reproduces these results. Fig 3 shows an example, confirming that the mode is localised in the y-direction at a point that is shifted relative to the X-point. Note that we have retained some gradient in density and temperature within the island for numerical convenience (ie $\alpha_n=\alpha_d=0.9$), so that although the gradients (and therefore drive) are much reduced compared to the situation without an island, there does remain a finite drive for the instability inside the island. Future work will aim to explore the consequences of further suppressing the gradients inside the island.

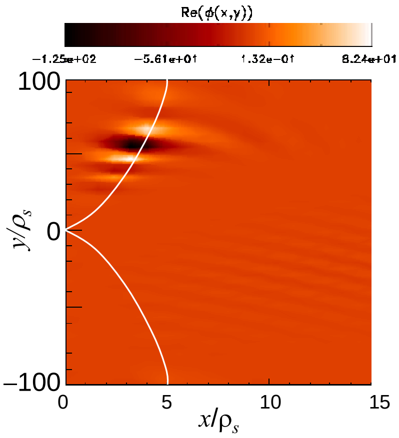

Fig 3: 2-D eigenmode structure. The full curve shows the position of the island separatrix.

REFERENCES

[1] R. Fitzpatrick, Phys Plasmas 2 (1995) 825
[2] R.D. Hazeltine, et al Phys Plasmas 4 (1997) 2920
[3] H.R. Wilson, et al, Proc EPS Conference (Warsaw) 2007

Acknowledgements: Helpful discussions with Per Helander are gratefully acknowledged. This work was funded by the United Kingdom Engineering and Physical Sciences Research Council and in part by the European Communities under the contract of Association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.