*35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-4.038 (2008)*

Instabilities in toroidal geometry: a Water Bag approach

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Introduction

Low frequency instabilities are assumed to be responsible for the phenomenon of anomalous transport observed in magnetic confinement fusion experiments. Among these micro-instabilities ITG (Ion Temperature Gradient) instabilities may play an important role in explaining the anomalous transport. Indeed, core turbulence is usually interpreted as the non linear saturated state of the ITG driven modes.

Solving 3D fluid equations is the most convenient way to compute the plasma response to the perturbed electromagnetic field when wave-particle interactions are neglected. However, a fluid description can overestimate turbulent fluxes because the resonant wave-particle interactions can not be fully described with fluid equations.

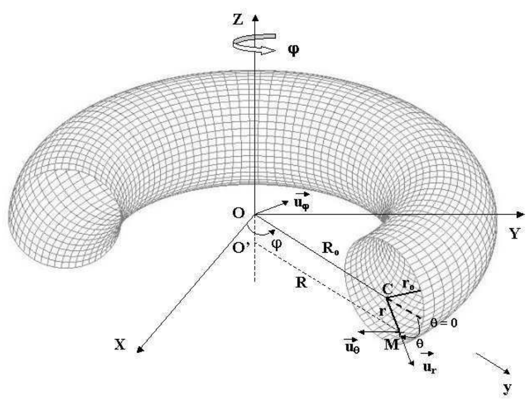

FIG. 1 – Toroidal coordinate system.

A drastic improvement is given by the gyrokinetic theory [1],[2] for strongly magnetized plasmas, which allows us to reducing the 3D velocity space into a 1D space ($v_{\|}$). However computing the resulting equations is still a non-trivial task. Recently, an alternative approach has been proposed to solve the Vlasov equation in a cylindrical geometry, based on a water bag representation of the distribution function which is not an approximation but rather a special class of initial conditions allowing to reduce the full kinetic Vlasov equation into a set of hydrodynamic equations while keeping its kinetic character [3], [4]. These results have been obtained in a simplified drift-kinetic model in a cylindrical geometry. The goal of this paper is to show how waterbag model can be pushed beyond the drift-kinetic approximation into the fully gyrokinetic model in a toroidal geometry.

Model description

Since the phase velocity of the instabilities is much lower than the electron thermal velocity, electronic kinetic effects are assumed to be negligible so that the electron distribution function is close to a maxwellian, which enables us to use a fluid model with an isothermal compression to close the system of equations. Moreover, the parallel electron motion is so fast that that the electrons can establish a Boltzmann equilibrium, so electrons are taken to follow the adiabatic law.

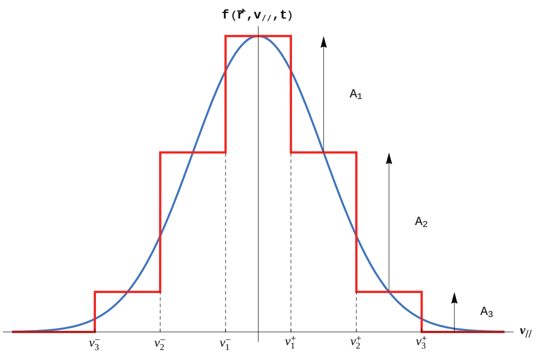

FIG. 2 – MWB function plotted against the parellel velocity, for M=3bags

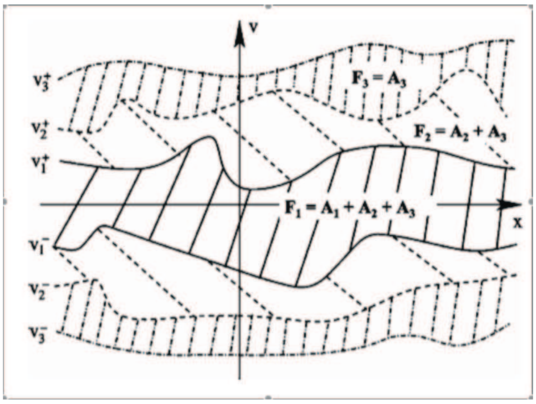

FIG. 3 – Bags contours in the phase space (x,v)

When the ion thermal velocity is close to the phase velocity, resonant interactions between waves and particles can play an important role in determining the instability growth rate. A basic kinetic model that describes the details of the distribution function is required. We consider low frequency perturbations ϕ with the following gyro-ordering in a small parameter ε :

$$\frac{\omega}{\Omega_{ci}} \sim \frac{k_{\parallel}}{k_{\perp}} \sim \frac{e\phi}{k_B T_e} \sim \frac{r_{Li}}{l_{\perp}} \sim \varepsilon$$

Then the Vlasov equation can be expanded in $r_{Li}/l_{\perp}$ and averaged over the cyclotron motion resulting in the drift kinetic equation. Moreover it is possible to take into account finite Larmor radius effects by adding gyro-averaging and polarization drift into the drift kinetic equation, leading to the gyrokinetic model.

If at initial time we impose the distribution function is equal to $f = A$ where A is a constant between two curves $v^+(r)$ and $v^-(r)$, f remains constant, according to phase space conservation property of the Vlasov equation in the case of collisionless plasmas. The two curves are called the contours of the bag. Now let us consider $2M$ contours in phase space labelled $v_j^+(\mathbf{r})$ and $v_j^-(\mathbf{r})$, where $j = 1, \ldots, M$ (see Fig. 2 and 3). Water-Bags can be obtained by projecting a continuous distribution function on a step function. We get a M bag system where the distribution function takes on M different constant values. The properties of the system are completely described by the knowledge of the contours.

We get a set of hydrodynamic equations, the system behaves as fluids coupled by the electromagnetic fields [3]. There is no differential operator associated with the parallel velocity but we keep the kinetic aspect of the problem with the same complexity as a multi-fluid model. It is called a Multi-Water-Bag model. One interesting property of the MWB model is to convert analytical problems related to the $v_{\parallel}$ variable into algebraic ones, wich is very useful in order to obtain linear dispersion relations.

$$\frac{\partial v_j^{\pm}}{\partial t} + \dot{\mathbf{r}}.\nabla v_j^{\pm} - \dot{v}_j^{\pm} = 0 \tag{1}$$

Model equations

Consequently, in toroidal geometry (Fig.1) the resulting equations take the following form :

$$\frac{\partial v_j^{\pm}}{\partial t} + \dot{\mathbf{r}}.\nabla v_j^{\pm} - \dot{v}_j^{\pm} = 0 \tag{2}$$

$$\dot{\mathbf{r}} = v_{\parallel}\mathbf{b} - \frac{\nabla < \phi > \times \mathbf{B}}{B^2} + \left(\frac{\mu}{q_i} + \frac{v_{\parallel}^2}{\Omega_{ci}}\right)\nabla \times \mathbf{b} \tag{3}$$

$$\dot{v}_{\parallel} = -\left(\frac{q_i}{m_i}\nabla < \phi > + \frac{\mu_i}{m_i}\nabla B\right).\left(\mathbf{b} + \frac{v_{\parallel}}{\Omega_{ci}}\nabla \times \mathbf{b}\right) \tag{4}$$

$$n_e = Z_i\left[< n_i > + \nabla_{\perp}.\left(\frac{n_i}{\Omega_{ci}B}\nabla_{\perp}\phi\right)\right] \tag{5}$$

with : $n_i = \sum_{j=1}^{M} A_j \left(v_j^+ - v_j^-\right)$. The $\mathbf{b}$ vector is along the magnetic field lines, and <.> is the gyroaverage operator.

Equations 3 and 4 describe the motion of a charged particle in an inhomogeneous magnetic field [5]. Equation 5 is the quasi-neutrality equation which includes polarization drift and gyroaverage effects.

Linear analysis

The physical features of the ITG instabilities can be obtained by the linear analysis of the model. The equilibrium function and other quantities are separated from their perturbations by keeping only the perturbations at first order.

These perturbations are projected on a Fourier basis in φ direction as :

$$\phi(\mathbf{r},t) = \delta\phi g(\theta)\exp[-i(\omega t - n\varphi)]$$

and

$$v_j^{\pm}(\mathbf{r},t) = \pm a_j(r,\theta,t) + \delta v_j^{\pm} g(\theta)\exp[-i(\omega t - n\varphi)]$$

System equilibrium is given by keeping only the zero-order terms in (2)-(5). We assume that there is no electric field at the equilibrium.

Then we get : $a_j(r,\theta) \simeq a_j(r,0)\sqrt{1+\frac{2\mu B_o}{m_i a_j^2(r,0)}\frac{r}{R_o}(\cos\theta - 1)}$.

This equilibrium definition allows us to describe the trapped particles population. In the multiple waterbag description, trapped particles correspond to close contours in phase space.

When linearized the system (2)-(5) takes the form :

$$A_{nj}^{\pm}(r,\theta)\delta v_j^{\pm} + B_{nj}^{\pm}(r,\theta)\partial_\theta \delta v_j^{\pm} + C_{nj}^{\pm}(r,\theta)\delta\phi + D_{nj}^{\pm}(r,\theta)\partial_\theta \delta\phi = 0 \tag{6}$$

$$E_{nj}^{\pm}(r,\theta)\delta\phi = \sum_{j=1}^{M} A_j \left(\delta v_j^{+} - \delta v_j^{-}\right) \tag{7}$$

This system holds a coupling between different perturbed contours of the Water-Bag distribution. Although the resolution of (6)-(7) is not straightforward, this system formally provides the basis to study the linear properties of the plasmas for any arbitrars equilibrium distributions. As compared to the full gyro-Vlasov equation, the complex integral operators with respect to the $v_{\|}$ variable are replaced by a discrete sum over the bag (see Eq.7), which is a much more simpler task from a mathematical point of view. Resolution of this system is a work in progress.

Références

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