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GYROKINETIC LIMITATIONS AND IMPROVEMENTS

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Abstract: For a tokamak, we consider gyrokinetic quasineutrality limitations when evaluating the axisymmetric radial electric field; a gyrokinetic entropy production restriction on the ion temperature pedestal; and a hybrid gyrokinetic-fluid treatment valid on transport time scales.

Gyrokinetic quasineutrality limitation: Gyrokinetic quasineutrality is often used in the steady state, axisymmetric, long wavelength limit to determine a tokamak radial electric field [1] violating intrinsic ambipolarity [2]. In an axisymmetric tokamak, intrinsic ambipolarity [2] requires the heat and particle fluxes be independent of electrostatic potential to leading order in the ion gyroradius ρ_i. This property is most easily seen by considering the drift kinetic equation for the leading correction f_{1i} to lowest order Maxwellian ions f_{0i} given by

$$v_{\|}\vec{n}\cdot\nabla f_{1i} - C_{1ii}\{f_{1i}\} = -\vec{v}_{di}\cdot\nabla\psi\frac{\partial f_{0i}}{\partial\psi} = -v_{\|}\vec{n}\cdot\nabla\left(\frac{Iv_{\|}}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\right),$$

where $\vec{v}_{di}$ is the magnetic plus electric drifts, C_{1ii} the linearized ion-ion collision operator with $C_{1ii}\{v_{\|}f_0\} = 0$, and $\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi = B\vec{n}$. Letting $g_i = f_{1i} + (Iv_{\|}/\Omega_i)(\partial f_{0i}/\partial\psi)$ gives

$$v_{\|}\vec{n}\cdot\nabla g_i = C_{1ii}\{g_i - \frac{Iv_{\|}}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\} = C_{1ii}\{g_i - \frac{If_{0i}v_{\|}}{\Omega_i T}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial T_i}{\partial\psi}\},$$

showing the only drive for g_i is $\partial T_i/\partial\psi$, and giving a vanishing ion particle flux since $\langle n\vec{V}_i\cdot\nabla\psi\rangle_\psi = \langle\int d^3v f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = 0$, where $\langle...\rangle_\psi$ denotes flux surface average. A moment procedure for the electron particle flux using $C_{1e}\{f_{1e}\}$ = $C_{1ee}\{f_{1e}\}$ + $C_{ei}\{f_{1e}\}$ with C_{1ee} the electron-electron operator and $C_{ei}\{f_{1e}\} = L_{ei}\{f_{1e} - (m/T_e)V_{\|i}v_{\|}f_{0e}\}$ the unlike operator, gives the electron particle flux as $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi = (mcI/e)\langle B^{-1}\int d^3v v_{\|}C_{1e}\{f_{1e} - (m/T_e)V_{\|i}v_{\|}f_{oe}\}\rangle_\psi$, with L_{ei} the Lorentz operator. Since the electron drift kinetic equation can be written as $v_{\|}\vec{n}\cdot\nabla g_e$ = $C_{1e}\{g_e + (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\|i}v_{\|}f_{0e}\}$ with $g_e = f_{1e} - (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi)$, the $\partial\Phi/\partial\psi$ drives in the collision operator cancel, so g_e is independent of the radial electric field and we see $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi$ = $(mcI/e)\langle B^{-1}\int d^3v v_{\|}C_{1e}\{g_e + (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\|i}v_{\|}f_{oe}\}\rangle_\psi$ does not depend on the radial electric field to an order higher since C_{ii}/C_{ee} ~ $(m/M)^{1/2}$ ~ ρ_i/L is normally assumed, with L the radial scale length. In addition, a moment description can be used to further demonstrate that intrinsic ambipolarity must be satisfied to order ρ_i^2/L^2 since it is then the flux surface average of conservation of toroidal angular momentum that must give the radial electric field. To order ρ_i^2/L^2 the viscosity is diamagnetic (and so collisionless to lowest order) and may be written in terms of the ion gyroviscosity $\vec{\vec{\pi}}_{ig}$ as [3] $\langle R^2\nabla\zeta\cdot\vec{\vec{\pi}}_{ig}\cdot\nabla\psi\rangle_\psi = \langle (MI/B)\int d^3v v_{\|} f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = 0$. Inserting $f_{1i} = g_i - (Iv_{\|}/\Omega_i)(\partial f_{0i}/\partial\psi)$, using $\langle \int d^3v f_{0i}(v_{\|}/B)^2 v_{\|}\vec{n}\cdot\nabla(v_{\|}/B)\rangle_\psi = 0$, and recalling g_i depends only on $\partial T_i/\partial\psi$ gives a $\partial\Phi/\partial\psi$ independent result. Hence, the correct neoclassical radial electric field must be determined from toroidal angular momentum conservation to next order. By considering a steady-state θ pinch using a model collision operator, we have explicitly shown gyrokinetics cannot determine the axisymmetric, long wavelength electrostatic potential to order ρ_i^2/L^2 [1]. In standard gyrokinetic treatments intrinsic ambipolarity is violated when the ion distribution function is retained to order ρ_i/L in the guiding center density and to order ρ_i^2/L^2 in the finite orbit polarization term. However, when f_i is kept to order ρ_i^2/L^2 in both places the radial electric cannot be determined and no inconsistency arises, as illustrated by the θ pinch case.

Entropy production constraint on ion temperature variation: Using canonical angular momentum, $\psi_* = \psi - (Mc/e)R^2\vec{v}\cdot\nabla\zeta = \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi - (Iv_{\|}/\Omega_i)$, as the radial variable allows strong gradients to be treated conveniently gyrokinetically [4]. Recall that the vanishing of the entropy production on a flux surface, $\langle\int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\}\rangle_\psi = 0$, requires the lowest order axisymmetric ion distribution function f_{0i} to be a local Maxwellian, with f_{0i} independent of poloidal angle in the banana regime. However, in the pedestal or an internal barrier (or on axis), drift departures from flux surfaces can become comparable to the local scale length ($\rho_{pi}\nabla\ell n n \sim 1$ with n the plasma density) and the entropy production argument has to be modified to account for the loss of locality due to finite poloidal ion gyroradius ρ_{pi} effects, requiring an equilibrium to be established over the entire pedestal (or barrier). Using the new gyrokinetic variables, we find that entropy production must vanish in the pedestal [4]:

$$\int_{\Delta V} d^3r \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\} = 0,$$

where ΔV is the volume of the pedestal (between the top of the pedestal where $\rho_{pi}\nabla\ell n n << 1$ and the separatrix) or the internal transport barrier (between inner and outer bounding flux surfaces having $\rho_{pi}\nabla\ell n n << 1$). As a result, f_{0i} must be drifting Maxwellian at most, giving $C_{1ii}\{f_{0i}\} = 0$. In the banana regime f_{0i} is independent of poloidal angle as well. Consequently, to make the Vlasov operator vanish $f_{0i} = f_{0i}(\psi_*, E, \mu)$, where $E = v^2/2 + e\Phi/M$ is the total energy and μ the magnetic moment. It is only possible to make a drifting Maxwellian out of these variables by ignoring the μ dependence and assuming the drift is nearly a rigid toroidal rotation of frequency ω_i with the ion temperature variation slow compared to the poloidal ion gyroradius ($\rho_{pi}\nabla \ell n T_i << 1$, $\rho_{pi}\nabla \ell n \omega_i << 1$) as for an isothermal Maxwellian [4,5]:

$f_{0i}(\psi_*,E) = n(M/2\pi T_i)^{3/2}\exp[-M(\vec{v} - \omega_i R^2\nabla\zeta)^2/2T_i] = \eta(M/2\pi T_i)^{3/2}\exp(-ME/T_i - e\omega_i\psi_*/cT_i)$,

where $\eta = n\exp[(e\Phi/T_i) + (e\omega_i\psi/cT_i) - (M\omega_i^2R^2/2T_i)]$ must also be nearly constant ($\rho_{pi}\nabla \ell n\eta << 1$). Thus, for a density pedestal having a scale length $L \sim -_{pi}$, the background ion temperature profile must have a much wider pedestal. In addition, for a density scale length of $-_{pi}$, lowest order perpendicular momentum balance gives $\omega_i = -c[d\Phi/d\psi + (en)^{-1}d(nT_i)/d\psi]$ with $cR(en)^{\omega l}d(nT_i)/d- \sim v_i$ = ion thermal speed. Consequently, in a subsonic pedestal in the banana regime it must be that to lowest order the ions are electrostatically confined [4] with $ed\Phi/d\psi \approx -(T_i/n)dn/d\psi$. Using total pressure balance we then see the electrons must be magnetically confined with a mean flow $\vec{V}_e$ comparable to the ion thermal speed ($\vec{V}_e \sim v_i$).

Hybrid gyrokinetic-fluid description: Simulating tokamaks on transport time scales requires evolving drift turbulence with axisymmetric neoclassical and zonal flow radial electric field effects allowed to the same order. Full electric field effects are more difficult to retain than density and temperature evolution effects since the need to satisfy intrinsic ambipolarity in the axisymmetric, long wavelength limit requires evaluating the ion distribution function to higher order than standard gyrokinetics. An electrostatic hybrid gyrokinetic-fluid treatment using moments of the full Fokker-Planck equation removes the need to go to higher order. This hybrid description evolves electrostatic potential, plasma density, ion and electron temperatures, and ion and electron flows using conservation of charge, number, ion and electron energy, and total and electron momentum, respectively [7]. All electrostatic effects with wavelengths much longer than an electron gyroradius are retained so that ion temperature gradient (ITG) and trapped electron mode (TEM) turbulence and the associated zonal flow as well as all neoclassical behavior are treated. Closure for the electrons is obtained by solving the electron drift kinetic equation to find the leading order correction to the Maxwellian electrons f_{0e} needed to evaluate the parallel electron viscosity (or pressure anisotropy) as well as the momentum and energy exchange terms with the ions. In addition, the $\vec{v}v^2/2$ moment of the exact electron Fokker-Planck equation is used, along with this first order correction to f_{0e}, to evaluate the electron heat flux (collisional plus diamagnetic), thereby achieving closure for the electrons. Ion closure is achieved similarly by solving the ion gyrokinetic equation to leading order in ρ_i/L. However, ion closure is somewhat more complicated because the $\vec{v}\vec{v}$ as well as the $\vec{v}v^2/2$ moment of the ion Fokker-Planck equation must be used to evaluate the ion gyroviscosity and perpendicular viscosity, along with the ion heat flux. Moreover, to recover the correct results in the axisymmetric, long wavelength limit, the gyrokinetic variables must be determined to one order higher than normal [1]. Once this is done complete closure is obtained and a description valid on transport time scales is obtained that properly evolves the electrostatic potential and flows, as well as density and temperatures. In this hybrid description distribution functions are only used to evaluate moments needed for closure and collisional exchange [7]. Moment equations evolve all other quantities such as density, temperatures, flows, and potential. As a result, either PIC or continuum gyrokinetic, and drift kinetic results, may be employed, and the kinetic equations need not be solved in conservative form. In addition, the flux surface average of conservation of toroidal angular momentum contains axisymmetric radial electric field terms from both the Reynolds stress and the collisional perpendicular ion viscosity whose respective coefficients compare as

$$\left\langle \frac{\tilde{n}}{n} \frac{\partial}{\partial \zeta} \left(\frac{e\tilde{\Phi}}{T_i} \right) \right\rangle_{\psi} \quad \text{vs} \quad \frac{q^2 R}{L_\perp} \frac{\nu_{ii}}{\Omega_i}$$

with tildes denoting fluctuating quantities, $L_\perp$ the local perpendicular scale length, R the major radius, and q the safety factor. For $\tilde{n}/n \sim e\tilde{\Phi}/T_i \sim 10^{-2}$ with 0.1 de-phasing, both quantities are of order 10^{-5}, for $L_\Phi \sim q^2 -_{pi}$ and ITER like numbers of B = 5.3 T, T_i = 8 keV, n = 10^{19} m^{-3}, and R = 6 m. Consequently, even though momentum relaxation is expected to be anomalous, the axisymmetric steady state radial electric field may be determined by a competition between the turbulent Reynolds stress and collisional perpendicular ion viscosity.

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References

[1] F. I. Parra and P. J. Catto, Plasma Phys. Control. Fusion **50**, 065014 (2008)

[2] L. M. Kovrizhnikh, Zh. Eksp. Teor. Fiz. **56**, 877 (1969) [Sov. Phys. JETP **29**, 475 (1969)]; and P. H. Rutherford, Phys. Fluids **13**, 482 (1970)

[3] R. D. Hazeltine, Plasma Phys. **15**, 77 (1973); and P. J. Catto and A. N. Simakov, Phys. Plasmas **12**, 012501 (2005)

[4] G. Kagan and P. J. Catto, to appear in Plasma Phys. Control. Fusion **50**, (2008)

[5] P. J. Catto and R. D. Hazeltine, Phys. Plasmas **13**, 122508 (2006)

[6] A. N. Simakov and P. J. Catto, Plasma Phys. Control. Fusion **49**, 729 (2007)

[7] P. J. Catto, A. N. Simakov, F. I. Parra and G. Kagan, submitted to Plasma Phys.Control. Fusion