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MONTE CARLO COLLISION OPERATOR FOR THE TEST PARTICLE TRACING IN FUSION NON MAXWELLIAN PLASMA

Oleg A. Shyshkin[1, 2], Boris Weyssow[1]

[1] *Université Libre de Bruxelles, Association EURATOM-Etat Belge, Campus Plaine, C.P. 231, B-1050 Bruxelles, Belgium*

[2] *Department of Physics and Technology, Kharkiv "V.N.Karazin" National University, Svobody sqr.4, Kharkov-77, 61077, Ukraine*

The expression for the discretized collision operator of a general Monte Carlo equivalent form in terms of expectation values and standard deviation for the non Maxwellian distribution function is derived for a magnetized plasma assuming distribution function isotropy. The case for δ - function distribution is studied in details. Additional numerical study displays significant modifications of collision frequencies for a wide kinetic energy range of a test particle interacting with non Maxwellian plasma.

GENERAL APPROACH

The Fokker-Planck collision operator acting on the distribution function $f_a(\mathbf{v})$ of an arbitrary particle species "a" under the assumption of isotropy for the distribution function $f_b(\mathbf{v}')$ of plasma species "b" could be rewritten as follows

$$\frac{df_a}{dt} = \nu_d(\mathrm{v})\,\mathrm{L_C} + \frac{1}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left[\mathrm{v}^3\left(\frac{m_a}{m_a+m_b}\nu_S(\mathrm{v}) f_a + \frac{1}{2}\nu_\parallel\,\mathrm{v}\frac{\partial f_a}{\partial \mathrm{v}}\right)\right], \qquad (1)$$

where $\mathrm{L_C} = \frac{1}{2}\frac{\partial}{\partial\lambda}\left[\left(1-\lambda^2\right)\frac{\partial f_a}{\partial\lambda}\right]$ is Lorentz collision operator, $\mathrm{L^{ab}} = \ln\Lambda\left(4\pi Z_a Z_b e^2/m_a\right)^2$ is function of Coulomb logarithm $\ln\Lambda$, charge numbers Z_a and Z_b, and the mass m_a. $\nu_S = \mathrm{L^{ab}}\left(1+m_a/m_b\right)\left(\partial\varphi_b/\partial\,\mathrm{v}\right)/\mathrm{v}$, $\nu_d = -2\mathrm{L^{ab}}\left(\partial\psi_b/\partial\,\mathrm{v}\right)/\mathrm{v}^3$ and $\nu_\parallel = -2\mathrm{L^{ab}}\left(\partial^2\psi_b/\partial\,\mathrm{v}^2\right)/\mathrm{v}^2$ are slowing down, deflection and parallel velocity diffusion frequencies respectively. The Rosenbluth potentials φ_b and ψ_b are functions of the relative velocity of particles $u = |\mathbf{v}-\mathbf{v}'|$ and distribution function f_b.

The Monte Carlo equivalent of the collision operator of the general form expressed in terms of time derivative of expectation values and the square of the standard deviation reads

$$F_n = F_o + \left(d\langle F\rangle/dt\right)\Delta\tau \pm \sqrt{\left(d\sigma_F^2/dt\right)\,\Delta\tau}\,, \qquad (2)$$

where $\Delta\tau$ is the integration time step and the function F could be replaced either by the kinetic energy $K = m\mathrm{v}^2/2$ or by the pitch angle $\lambda = \mathrm{v}_{\|}/\mathrm{v}$. The sign $\pm$ is to be chosen randomly but with the equal probability [1].

COLLISIONS WITH A "DELTA-FUNCTION" BACKGROUND

The collision operator for isotropic Maxwellian distribution can be derived in analytical form. It has been already described in details [1, 2, 3]. Since the non Maxwellian plasma is the point of our interest we assume the distribution function for plasma particles moving with the average velocity $\mathbf{V}'$ to be in the form of δ-function $f_b(\mathbf{v}') = n_b\,\delta(\mathbf{v}' - \mathbf{V}')$. This approach could be valid for electrons colliding with the plasma ions since the electron thermal velocity is much higher then of the ions. The other application could be the deceleration of fusion α-particles with high energy (3.52MeV) on the same plasma ions. Consistently with these applications we neglect $\mathbf{V}'$ and consider the distribution function depending only on the magnitude of $\mathbf{v}'$. In this case the collision operator becomes

$$\frac{df_a}{dt} = \mathrm{L}^{ab}\frac{n_b}{4\pi}\frac{1}{\mathrm{v}^2}\left(\frac{m_a}{m_b}\frac{\partial f_a}{\partial \mathrm{v}} + \frac{1}{\mathrm{v}}\mathrm{L_c}\right).$$

In accordance to the general formula (2) the pitch-angle scattering operator in terms of deflections frequency $\nu_d = \mathrm{L}^{ab} n_b \left(4\pi\,\mathrm{v}^3\right)^{-1}$ is

$$\lambda_n = \lambda_o\left(1 - \nu_d\,\Delta\tau\right) \pm \sqrt{\nu_d\left(1 - \lambda_o^2\right)\,\Delta\tau}\ ,$$

and the energy slowing down operator in terms of collision frequency $\nu_K = \mathrm{L}^{ab}\dfrac{n_b}{4\pi}\dfrac{m_a}{m_b}\dfrac{1}{\mathrm{v}^3}$ is

$$K_n = K_o - 2K_0\,\nu_K\,\Delta\tau\ . \qquad (3)$$

Here the important point is the fact that the time derivative of square of standard deviation is $d\sigma_K^2/dt = 0$ excluding the broadening of

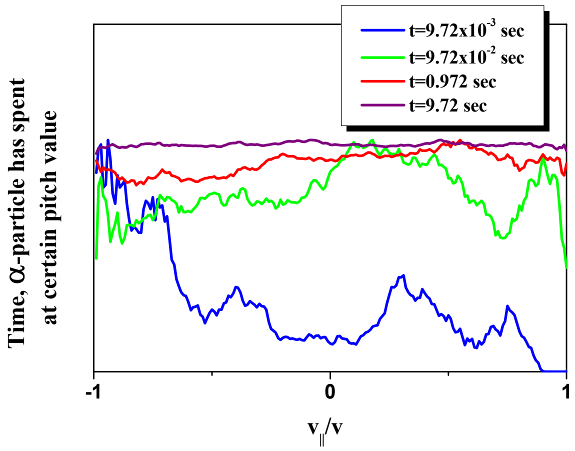

Fig 1. Relaxation in pitch space. After many collision time steps the particle pitch-angle is distributed uniformly among the values $-1 \le \lambda \le 1$.

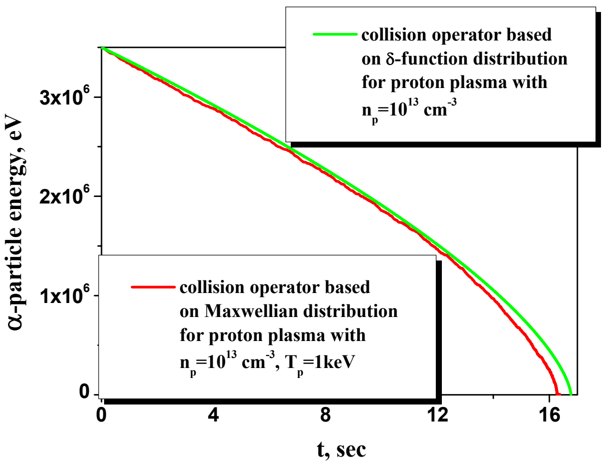

Fig 2. Deceleration of α-particle in uniform Maxwellian proton plasma with $n_p = 10^{13}\,cm^{-3}$ and $T_p = 1keV$ (red curve), and on protons distributed in accordance with δ–function (green curve).

distribution function. The relaxation in the pitch space represented by means of pitch-angle scattering operator for fusion α -particle colliding with protons distributed in accordance with the δ -function is displayed in figure 1. The slowing down of a fusion α -particle calculated by means of the operator (3) is compared to the same process calculated by the collision operator based on Maxwellian distribution (see figure 2). Good agreement of the results is observed. The advantage of the operator (3) is the simple analytical representation and as a consequence the significant CPU time reduction needed for the calculations.

COLLISION FREQUENCIES FOR AN ARBITRARY DISTRIBUTION FUNCTION

The derivation similar to the previous one under assumption of an arbitrary isotropic distribution function leads us to more general expression for the energy slowing down and scattering operator

$$K_n = K_0 - 2K_0\,\Delta\tau\left(\frac{m_a}{m_a+m_b}\nu_S - \frac{5}{2}\nu_\parallel - K_0\left.\frac{\partial\nu_\parallel}{\partial K}\right|_{K=K_0}\right) \pm 2K_0\sqrt{\nu_\parallel\,\Delta\tau}\,, \qquad (4)$$

while the expression for pitch-angle scattering stays unchanged as in the previous case. Now we have no the possibility to derive the analytical form for the collision operator basing on arbitrary distribution function for the background plasma particles. Nevertheless we can estimate numerically the collision frequencies for certain plasma distribution function. Assume that the proton plasma total distribution function has the following form $f(\mathrm{v}) = n_1/(\pi^{3/2}\mathrm{v}_1^3)e^{-(\mathrm{v}/\mathrm{v}_1)^2} + n_2/(\pi^{3/2}\mathrm{v}_2^3)e^{-(\mathrm{v}-\mathrm{V}/\mathrm{v}_2)^2}$ where $\mathrm{v}_i = \sqrt{2T_i/m_p}$ (see figure 3). On figures 4, 5 and 6 the numerical estimation for the modified collision frequencies due to presence of energetic particle fraction is displayed.

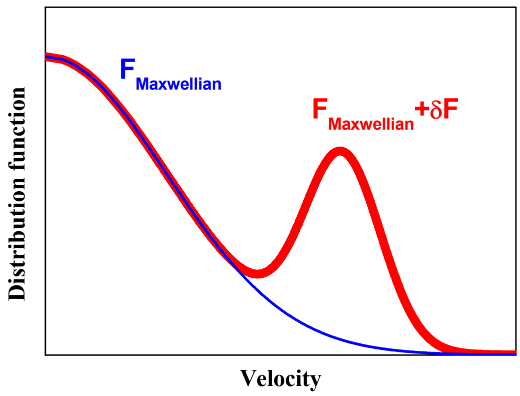

Fig 3. Superposition of Maxwellian proton plasma distribution function with the parameters $n_1 = 10^{14}\,cm^{-3}$, $T_1 = 1keV$ and energetic fraction of the same particle species with the average velocity V related to the energy of $3keV$, $n_2 = 2\times10^{12}\,cm^{-3}$ and $T_2 = 100eV$ (red curve).

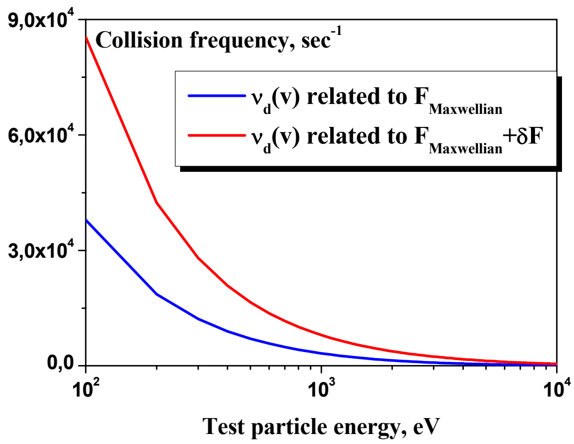

Fig 4. Deflection frequency versus the test particle energy for both Maxwellian proton plasma (blue curve) and plasma with the energetic particle fraction (red curve).
All parameters are the same as on figure 3.

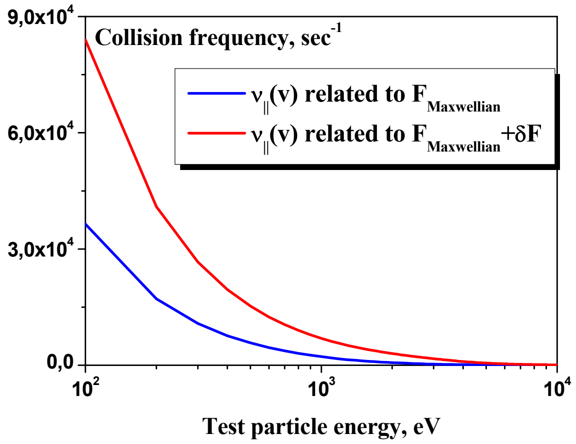

Fig 5. Parallel velocity diffusion frequency versus the test particle energy. All parameters are the same as on figure 3.

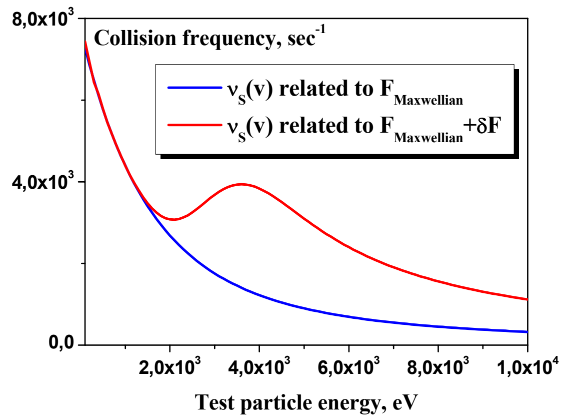

Fig 6. Slowing down frequency versus the test particle energy. All parameters are the same as on figure 3.

As one can see the modification of the plasma particles distribution function causes significant changes in collision frequencies.

CONCLUSIONS

The Monte Carlo equivalent of slowing down and scattering operator basing on plasma particles distribution in the form of δ - function is derived (3) and studied in details. Its validity to calculate energy slowing down for energetic particles like ion beams and fusion α - particles is proved. Significant variation in collision frequencies due to modification of the distribution function is shown by numerical integration of Rosebluth potentials. As soon as collision frequencies determine the transport coefficients for magnetized plasma the changing of transport properties is expected as well. The numerical integration of the collision operator (4) gives us possibility to study comprehensive picture of particle transport for different magnetic field configurations [4] and confinement scenarios characterized by non Maxvellian distribution of plasma particles.

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[1] A.H. Boozer, G. Kuo-Petravic, *Phys. Fluids* **24**, 851 (1981)

[2] W.D. D'haeseleer, C.D. Beidler, *Comput. Phys. Comm.* **76**, 1 (1993)

[3] O.A. Shyshkin, R. Schneider and C.D. Beidler, *Nucl. Fusion* **47**, 1634 (2007)

[4] A.A. Shishkin, *Nucl. Fusion* **42**, 344 (2002)