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Global stability analysis of ITG modes with parallel shear flow

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Abstract

A global analysis of the ITG mode stability in a magnetized plasma shows that unstable localized eigenfunctions can exist not only around the resonant radius, where $k_{\parallel} = 0$, but also close to the maximum value of the normalized ion temperature gradient. This new aspect of the ITG instability is investigated including the effect of a field-aligned shear flow.

Introduction

The apparent anomalous nature of the angular momentum diffusion, as observed in many experiments [1-4], calls for a unified modeling of plasma energy and toroidal rotation in the study of ITG mode stability. To this aim, we have developed a drift fluid model, in slab geometry, based on the ion continuity, parallel velocity and pressure equations, where the magnetic field curvature is taken into account through a suitable drift velocity [5]. The system of equations is solved as a boundary value problem, in the slab coordinate *x*, for the complex eigenvalue $\omega_k = \omega_k^R + i\gamma_k$. The spectra of the unstable eigenvalues are calculated together with the corresponding eigenfunctions. It is shown that, for a given **k**-vector, for not too steep ion temperature profiles, two branches of unstable eigenvalues appear, one localized where $k_{\parallel} = 0$, and another close to the maximum of $\left|R/L_{T_i}\right|$. The properties of the eigenvalues and of the eigenfunctions are studied for different profiles of ion temperature and parallel flow velocity.

The physical model

Let us consider a plasma slab in $-a < x < +a$. The magnetic field $\mathbf{B} = \mathbf{B}_p + \mathbf{B}_t = B_p\hat{\mathbf{e}}_p + B_t\hat{\mathbf{e}}_t$ (indices "p" and "t" refer to the "poloidal" and "toroidal" directions) depends on *x* as well as all the other relevant parameters, i.e. the ion density n_i, temperature T_i, pressure p_i, and parallel flow velocity $U_{\parallel}$. For perturbations with $|\omega_k| << \gamma_{ci}$, the transverse ion dynamics can be described in terms of a drift velocity, which is the sum of the electric, the diamagnetic, the polarization, and the magnetic curvature drifts. The projections of a given wave-vector $\mathbf{k} = (k_p, k_t)$ parallel and perpendicular to the local magnetic field, that is $k_{\parallel}(x)$ and $k_{\perp}(x)$, are functions of *x*. It is convenient to introduce a new reference frame rotated around the *x*-

axis by an angle $\vartheta(x)=\tan^{-1}\dfrac{B_p(x)}{B_t(x)}$, such that, $B_y=0$ and $B_z=|\mathbf{B}|$, for any *x*. The stability analysis proceeds by linearizing the relevant equations and considering a normal mode $\tilde{A}(x,y,z,t)\ =\ \tilde{A}_{k_y,k_z}(x)e^{-i(\omega_k t-k_y y-k_z z)}$. The equations for $\tilde{\varphi}_k=\dfrac{T_e}{en_i}\tilde{n}_k$, $\tilde{v}_{z,k}$, and $\tilde{p}_{i,k}$ write:

$$-i\overline{\omega}_k\frac{e\tilde{\varphi}_k}{T_e}+\frac{\tilde{v}_{x,k}}{L_n}+\frac{d\tilde{v}_{x,k}}{dx}+ik_y\tilde{v}_{y,k}+ik_z\tilde{v}_{z,k}=0, \qquad (1)$$

$$ik_zn_iZe\tilde{\varphi}_k+Mn_i\left(U'_z-U_y\vartheta'-k_yk_z\rho_{L,i}v_{ti}\right)\tilde{v}_{x,k}+i\frac{p_i}{\Omega_{ci}}\left[-\frac{k_z}{L_n}(\eta_i+1)+\frac{k_z}{L_B}+\frac{k_y\vartheta'}{2}-k'_z\right]\tilde{v}_{y,k}- \qquad (2)$$

$$-iMn_i\left[\overline{\omega}_k-k_yv_{ti}\frac{\rho_{Li}}{L_B}+k_yv_{ti}\frac{\rho_{Li}}{L_n}(\eta_i+1)\right]\tilde{v}_{z,k}-\frac{p_i\vartheta'}{2\Omega_{ci}}\frac{d\tilde{v}_{x,k}}{dx}-i\frac{p_ik_z}{\Omega_{ci}}\frac{d\tilde{v}_{y,k}}{dx}+i\left(k_z+\frac{k_yU'_z}{\Omega_{ci}}\right)\tilde{p}_i=0,$$

$$(\eta_i+1)\frac{p_i}{L_n}\tilde{v}_{x,k}+i\gamma p_ik_y\tilde{v}_{y,k}+i\gamma p_ik_z\tilde{v}_{z,k}+\gamma p_i\frac{d\tilde{v}_{x,k}}{dx}-i\overline{\omega}_k\tilde{p}_i=0, \qquad (3)$$

where the perpendicular components of the fluctuating velocity are

$$\tilde{v}_{x,k}=-iC\left\{\frac{Zek_y}{M\Omega_{ci}}\tilde{\varphi}_k+\frac{k_y}{n_iM\Omega_{ci}}\tilde{p}_k+\right.$$

$$\left.+\frac{\overline{\omega}_k}{\Omega_{ci}}\left[\frac{p'_i}{n_iM\Omega_{ci}}\frac{e\tilde{\varphi}_k}{T_e}-\frac{Ze}{M\Omega_{ci}}\frac{d\tilde{\varphi}_k}{dx}-\frac{1}{n_iM\Omega_{ci}}\frac{d\tilde{p}_k}{dx}-\frac{\rho_{Li}v_{ti}}{R}\left(\frac{e\tilde{\varphi}_k}{T_e}-\frac{\tilde{p}_k}{p_i}\right)\right]\right\}, \qquad (4)$$

$$\tilde{v}_{y,k}=-\left(U_y+\frac{ZeE_x}{M\Omega_{ci}}\right)\frac{e\tilde{\varphi}_k}{T_e}+\frac{Ze}{M\Omega_{ci}}\frac{d\tilde{\varphi}_k}{dx}+\frac{1}{n_iM\Omega_{ci}}\frac{d\tilde{p}_k}{dx}-\frac{k_y\overline{\omega}_k}{\Omega_{ci}^2}\left(Zc_s^2\frac{e\tilde{\varphi}_k}{T_e}+\frac{\tilde{p}_k}{n_iM}\right)-\frac{\rho_{Li}v_{ti}}{R}\frac{\tilde{p}_k}{p_i}. \qquad (5)$$

In Eqs.(1-5) $\overline{\omega}_k(x)=\omega_k-k_yU_y-k_zU_z$, $L_a=\dfrac{a}{da/dx}$, $\eta_i=\dfrac{d\ln T_i}{d\ln n_i}=\dfrac{L_n}{L_T}$, $R=R_o+x$, $C=1-\dfrac{U'_y}{\Omega_{ci}}-\dfrac{U_y\vartheta'}{\Omega_{ci}}$. Substituting Eqs.(4,5) into Eqs.(1-3), the system takes the form

$$\overleftrightarrow{\mathbf{A}}\ \tilde{\mathbf{y}}\ =\ \omega_k\overleftrightarrow{\mathbf{B}}\ \tilde{\mathbf{y}} \qquad (6)$$

where we have defined the vector $\tilde{\mathbf{y}}=\left(\tilde{\varphi}_k,\tilde{p}_k,\tilde{v}_{z,k}\right)$ and the two 2nd order differential operators $\overleftrightarrow{\mathbf{A}}$, $\overleftrightarrow{\mathbf{B}}$ which are independent of $\omega_k=\omega_k^R+i\gamma_k$. Eq.(6) is solved as a generalized eigenvalue problem, in the interval $x\in(-a,+a)$, with boundary conditions $\tilde{\mathbf{y}}(\pm a)=0$.

The numerical results

Eq.(6) is solved by assuming different spatial distributions of the ion density, $n_i(x)=(n_0-n_a)\left(1-\xi^{\alpha_1}\right)^{\alpha}+n_a$, of the ion (electron) temperature,

$T_{i(e)}(x) = \left(T_{i(e)0} - T_{i(e)a}\right)\left(1 - \xi^{\beta_{i(e)1}}\right)^{\beta_{i(e)}} + T_{i(e)a}$, and of the ion parallel velocity, $U_z(x) = \left(U_0 - U_a\right)\left(1 - \xi^{\gamma_1}\right)^{\gamma} + U_a$, where $\xi = x/a$ and suitable values of the exponents are considered in order to test the stability of the system against more peaked or broader spatial profiles of the relevant physical quantities. The model is applied to a typical AUG discharge [6]: B_0=2T, $n_0=6\times10^{13}cm^{-3}$, $n_a=3\times10^{13}cm^{-3}$,$T_{e0}$=4.2keV, T_{ea}=1.4keV, T_{i0}=6.7keV, T_{ia}=2.2keV. In the cases considered here, $k_y\rho_{Li} \approx 0.4 - 0.6$ on the low field side (LFS) of the slab.

Case I (Fig.1) – "Round" density profile (α_1=2, α=1); U_z=0; three T_i profiles: β_{i1}=2, β_i=2.7 (plots *a,d*), β_{i1}=2, β_i=2 (*b,e*), β_{i1}=2, β_i=1 (*c,f*). In plots *a-c*, the most unstable eigenvalues are shown in the plane ω_k^R, γ_k going from a more peaked (*a*) to a flatter (*c*) T_i profile. Correspondingly, in plots *d-f*, $|\tilde{\varphi}_k|^2$of the unstable eigenmodes are shown in the LFS half-slab, 0<ξ<1. A transition from a single-branch to a double-branch regime of instability occurs by broadening the T_i profile in such a way that, in addition to the set of eigenfunctions close to the position ξ_0 (=0.4) where $k_{||}$=0 (full circles), new eigenmodes appear, around ξ_M (≈0.6-0.9) where $\left|R/L_{T_i}\right|$ is maximum (empty circles). The HFS remains stable due to the favorable curvature effect. Notice that, from *a* to *c*, the maximum value of γ_k around ξ_0 decreases, while the instability remains dominated by the modes at ξ_M.

Case II (Fig.2) – "Round" density profile (α_1=2, α=1); several $U_z(x)$ profiles (the marks used throughout Fig.2 correspond to the profiles in *d*) with U_0=300 km/s and U_a=30 km/s; β_{i1}=2, β_i=1.75. In plot *a* the most unstable eigenvalues are shown in the plane ω_k^R, γ_k; in *b* the corresponding values of $\left| -{}_{k}^{R}\ \omega k_z U_z \right|$ are shown *vs* ξ (the full line refers to $\sqrt{2}k_z v_{ti}$). In *c* the growth rates γ_k are plotted as functions of the normalized velocity gradient $\left|U_z'/\gamma_{ci}\right|$, calculated at the peak of the corresponding eigenfunction.

In both cases it is seen that the most unstable eigenfunctions are localized around ξ_M, which is closer to plasma periphery than ξ_0. Moreover the instability covers a broad radial region.

Conclusions

Solving the system of linearized fluid equations as a generalized boundary value problem in a plasma slab, physical conditions for the onset of a double-branch regime of unstable eigenvalues are found. Under such conditions a wide portion of the plasma slab can be made unstable by a single wave-vector, with important impact on the radial anomalous transport.

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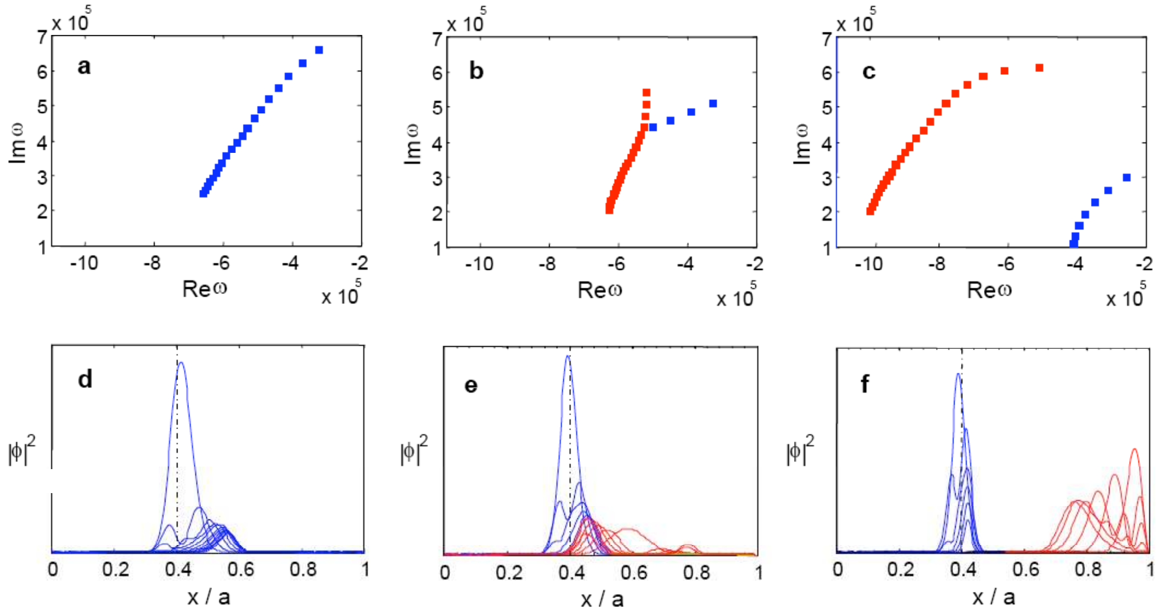

Fig.1

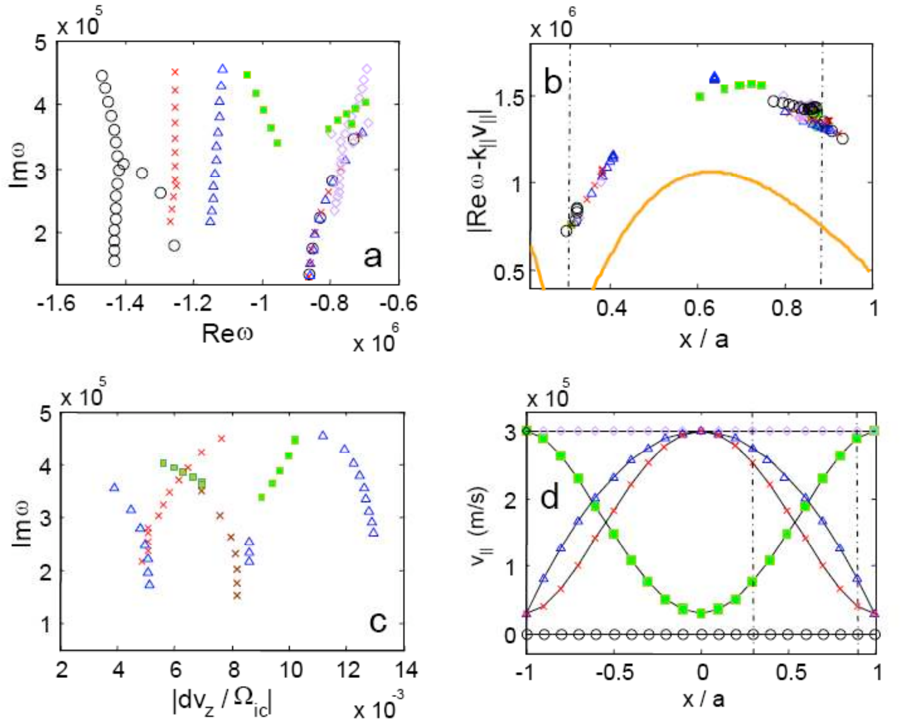

Fig.2