35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.**32D**, P-4.024 (2008)

Study of collisionless TE and ITG modes in an ITER-like equilibrium

M. Ansar Mahmood[1], J. Weiland[1], M. Persson[1], and T. Rafiq[2]

[1] *Chalmers University of Technology and Euratom-VR Association, S-41296 Gothenburg, Sweden*

[2] *Physics Department, Lehigh University, Bethlehem, PA 18015, USA*

Abstract The linear stability of collisionless trapped electron (TE) and ion temperature gradient (ITG) modes is studied in an ITER-like magnetic field configuration. An advanced fluid model for ITG-TE mode [1] is used and an eigenvalue equation along a field line is derived using the ballooning mode formalism. The eigenvalue problem is solved numerically using a standard shooting technique and applying WKB type boundary conditions. The growth rates and real frequencies of the most unstable modes and their eigenfunctions are calculated. Effects of the magnetic shear, trapped electrons fraction, wavevector, temperature ratios, and temperature gradients on mode stability are discussed.

The eigenvalue problem

The two main instabilities believed to be responsible for the anomalous transport in the core of tokamak plasmas are the toroidal ion temperature gradient (ITG) mode and the collisionless trapped electron (TE) mode. While anomalous transport is nonlinear it is essential to understand the underlying linear instabilities deriving the losses. The purpose of the present work is thus to study the linear stability of ITG and collisionless TE modes in an ITER-like geometry. A full magnetic field configuration that corresponds to ITER Scenario-4 [2] is computed using the variational moments equilibrium code VMEC and is then mapped to the Boozer coordinate system (s, θ, ζ) [2-4], where $s = 2\pi\psi/\psi_p$ is the normalized flux (radial) coordinate and θ, ζ are the generalized poloidal and toroidal angles, respectively. Here, $2\pi\psi$ is the poloidal magnetic flux bounded by the magnetic axis and ψ = constant surface, and $\psi_p = \pi B_0 \bar{a}^2/q$ is the total poloidal magnetic flux, where B_0 is the magnetic field at the magnetic axis, q is the safety factor and $\bar{a}$ is the average minor radius. Details of the magnetic field configuration can be found in Ref. [2,4].

The study is based on an advanced fluid model for the ITG-TE mode, which is derived in the collisionless electrostatic limit by using the continuity, energy and parallel momentum equations for the ion and trapped electron fluids. The free electrons are assumed to be Boltzmann distributed. The parallel velocity perturbation for the bounce averaged trapped electron fluid and effects of electron finite-Larmor-radius (FLR) and inertia are neglected. The closure relation used for the perturbed electrostatic potential is the quasi-neutrality, $\delta n_i/n_i = f_t \delta n_{et}/n_{et} + (1-f_t)\delta n_{ef}/n_{ef}$, where f_t is the fraction of the trapped electrons. The drift wave equation derived in a standard way is reduced to an ordinary differential equation along a field line by employing WKB assumptions in the coordinates (ψ, α, ζ) and by using the standard ballooning mode formalism [2, 4]. The resulting eigenvalue equation in the Boozer coordinates is written in the following form:

$$\frac{d^2\Phi}{d\zeta^2} + \left(\frac{2\chi JB}{\bar{a}\epsilon_n q\bar{R}\dot{\psi}}\right)^2 \frac{D_2(\Omega)}{D_1(\Omega)}\Phi = 0, \tag{1}$$

where

$$D_1 = \bar{N}_e\left[\Omega + \frac{5}{6}\tau\bar{a}\epsilon_n\Omega_d + (\eta_i - \frac{2}{3}\tau) + \frac{5}{3}\tau\left(\Omega + \frac{\bar{a}\epsilon_n}{2}\tau\Omega_d\right)\right] - \frac{5}{3}\tau f_t\left(\Omega + \frac{\bar{a}\epsilon_n}{2}\tau\Omega_d\right)F(\Omega),$$

$$D_2 = \bar{N}_e\left[\bar{N}_i - \Omega\left(1 - \frac{1}{2}\bar{a}\epsilon_n\Omega_d\right) + \frac{1}{2}\left(\eta_i - \frac{7}{3}\right)\bar{a}\epsilon_n\Omega_d + \frac{5}{3}\tau\left(\frac{1}{2}\bar{a}\epsilon_n\right)^2\Omega_d^2\right]$$
$$+\bar{N}_e\left(\tfrac{\chi B_0}{B}\right)^2\hat{k}_\perp^2\left(\Omega + \tau(1+\eta_i)\right)\left(\Omega + \tfrac{5}{6}\tau\bar{a}\epsilon_n\Omega_d\right) - f_t\bar{N}_i F(\Omega),$$

$$F(\Omega) = \left[\bar{N}_e - \Omega\left(1 - \frac{1}{2}\bar{a}\epsilon_n\Omega_{dt}\right) - \frac{1}{2}(\eta_e - \frac{7}{3})\bar{a}\epsilon_n\Omega_{dt} - \frac{5}{3}\left(\frac{1}{2}\bar{a}\epsilon_n\right)^2\Omega_{dt}^2\right],$$

$$\bar{N}_e(\Omega) = \Omega^2 - \frac{10}{6})\bar{a}\epsilon_n\Omega_{dt} + \frac{5}{3}\left(\frac{1}{2}\bar{a}\epsilon_n\right)^2\Omega_{dt}^2,$$

$$\bar{N}_e(\Omega) = \Omega^2 + \frac{10}{6})\bar{a}\epsilon_n\Omega_d + \frac{5}{3}\tau^2\left(\frac{1}{2}\bar{a}\epsilon_n\right)^2\Omega_d^2,$$

and

$$\Phi(\zeta) = \frac{e\phi}{T_e}, \qquad \Omega_d = \Omega_d(s,\alpha,\zeta) = \chi B_0\bar{R}\,(\frac{\mathbf{B}\times(\boldsymbol{\kappa} + \boldsymbol{\nabla}\ln B)}{B^2})\cdot\hat{\mathbf{k}}_\perp,$$

$$\Omega = \frac{\omega}{\omega_{*e}}, \qquad \chi = \epsilon^{-1}\frac{q\rho_{s0}}{\bar{a}}\frac{\partial S_e}{\partial\alpha}, \qquad \rho_{s0} = \frac{c_s}{eB_0/m_i}, \qquad \epsilon_n = L_n/\bar{R},$$

$$\hat{\mathbf{k}}_\perp = \frac{\bar{a}}{q}\left[\boldsymbol{\nabla}\zeta - q\boldsymbol{\nabla}\theta - \left(\frac{\zeta-\zeta_0}{q} - \theta_k\right)\dot{q}\boldsymbol{\nabla}s\right],\ b = (k_\perp\rho_{so})^2 = \chi^2\left(\hat{\mathbf{k}}_\perp\cdot\hat{\mathbf{k}}_\perp\right)|_{\zeta=\zeta_0}.$$

Results

The eigenvalue problem, equation (1), is solved numerically in an ITER-like magnetic field configuration [2]. Details of the boundary conditions and the numerical method used are given in Ref. [4]. Effect of trapped electrons on ITG mode threshold is displayed in figure 1 for two magnetic flux surfaces, which have global magnetic shear values approximately equal in magnitude but opposite in signs. It is found that the fraction of trapped electrons significantly reduces the ITG mode threshold. In both cases (with and without electron trapping), a higher threshold is found for negative shear magnetic surface than for the positive shear magnetic surface. The fraction of trapped electrons is also found destabilizing for both ITG and TEM modes, as shown in figure 2(a). From figure 2(b), we find that the maximum growth rate for the ITG mode shifts towards a larger wavelength (smaller values of b) when the fraction of trapped electrons is introduced, which tends to increase the transport by increasing the the correlation length in the plasma. Figure 2(c) shows the effects of temperature ratios on both ITG (ion-diamagnetic directed) and TE (electron-diamegnetic directed) modes. Large values of $\tau = T_i/T_e > 1$ are found to be stabilizing for the ITG modes and destabilizing for the TE modes.

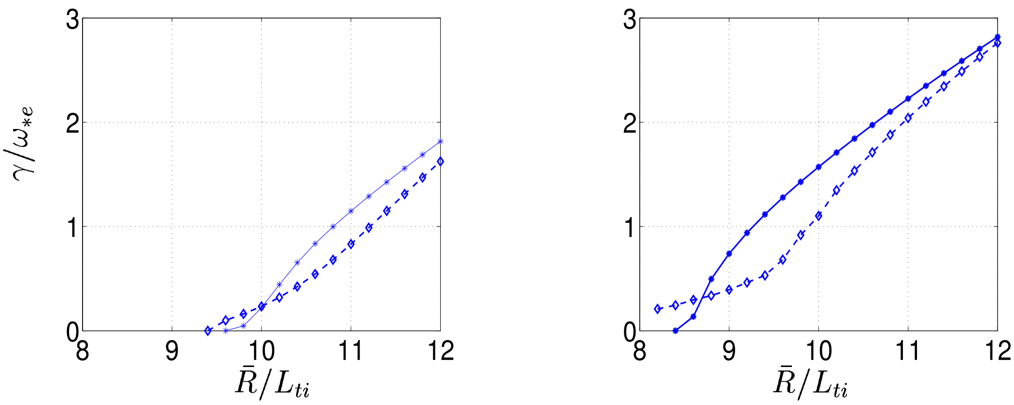

Figure 1: *Normalized growth rate, ω/ω_{*e}, as a function of $\bar{R}/L_{ti}$ for $f_t = 0$ (left) and $f_t = 0.2$. The *-solid curves for $s = 0.4$ (negative shear magnetic surface) and diamond-dashed curves for $s = 0.61$ (positive shear magnetic surface). The other parameters are $\bar{R}/L_{te} = 6$, $\tau = 1$, and $\theta_k = 0$.*

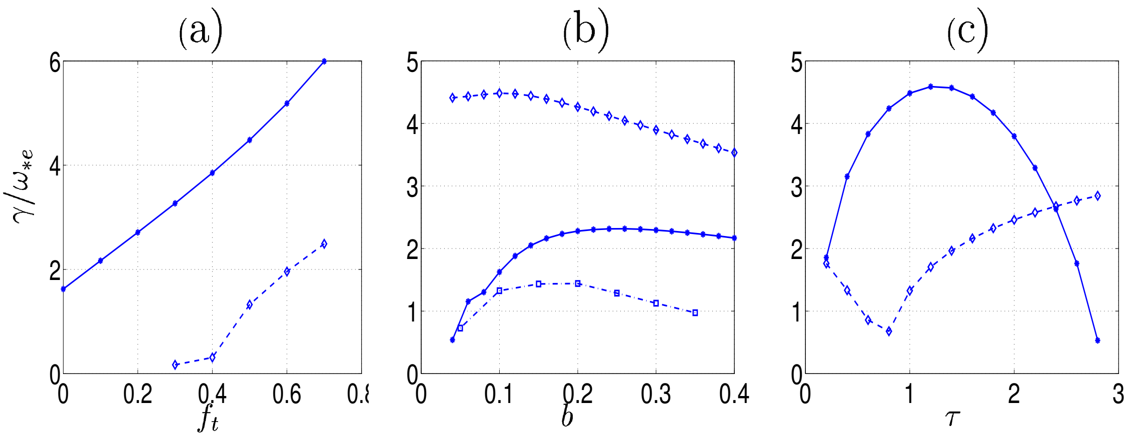

Figure 2: (a) *Normalized growth rate as a function of* f_t *for modes in the ion (*∗*-solid) and in the the electron (diamond-dashed) diamagnetic direction. The parameters are:* $s = 0.61$, $\bar{R}/L_n = 1$, $\bar{R}/L_{ti} = \bar{R}/L_{te} = 12$, $\tau = 1$ *and* $\theta_k = 0$. (b) *Normalized growth rate as a function of* b*, where *-solid curve is for* $f_t = 0$*, diamond-dashed for modes in the ion diamagnetic direction at* $f_t = 0.5$ *and square-dashed-dotted for modes in the electron diamagnetic direction at* $f_t = 0.5$*. The other parameters are same as in (a).* (c) *Normalized growth rate as a function of* $\tau = T_i/T_e$ *with* $f_t = 0.5$ *for modes in the ion (*∗*-solid) and in the electron (diamond-dashed) diamagnetic direction. The parameters are the same as in (a).*

Acknowledgment

One of the authors, T. Rafiq, would like to acknowledge the support of the U.S. DoE under Grant No. DE-FG02-92ER54141.

1. J. Weiland, *Collective Modes in Inhomogeneous Plasma* (Bristol and Philodelphia: Institute of Physics Publishing) p 118.

2. M. Ansar Mahmood, T Rafiq, and M Persson *Plasma Phys. Control. Fusion* **48**, 1019 (2006).

3. A. H. Boozer, Phys. Fluids **25**, 520 (1982).

4. T. Rafiq, J. Andersson, M. Nadeem, and M. Persson, *Plasma Phys. Control. Fusion* **43**, 1363 (2001).