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Nondiffusive transport in plasma turbulence

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Introduction

Experimental evidence indicates that transport in magnetically confined fusion plasmas deviates from the standard diffusion paradigm. In particular, transport in fusion plasmas lacks of characteristic scales in space and time. Several theoretical approaches have been explored in the last years to explain these experimental findings. Among them, continuous-time random walk (CTRW) models with Lévy probability distribution functions (pdf) stand out since transport lacks any characteristic scale in these models [1]. In the fluid limit, these models lead to a description of transport in terms of fractional differential equations [2],

$$\frac{\partial^\beta n}{\partial x^\beta} = D_\alpha \frac{\partial^\alpha n}{\partial |x|^\alpha} \tag{1}$$

Here, we try to confirm the existence of this kind of distributions in numerical simulations of plasma turbulence. We have followed the evolution of fluctuations for two different fluid models: 1. pressure-gradient-driven turbulence in toroidal geometry (ballooning modes) [3]; 2. dissipative trapped electron mode in cylindrical geometry [4]. Both correspond to electrostatic turbulence in tokamaks.

To characterize the transport properties induced by the turbulence, we investigate the time evolution of pseudo-particle tracers. From the information on tracer orbits, we calculate the pdf of flights, and determine power-law tail exponents. From the information on the Lagrangian correlation of the velocity field along the particle tracer orbits, we determine the Hurst exponent. The results for both electrostatic modes show that the transport is superdiffusive and non-Markovian.

Pressure-gradient-driven turbulence

We study the pressure-gradient-driven turbulence in toroidal geometry by means of a reduced set of equations in the electrostatic limit. The $E \times B$ velocity is written in terms of the stream function, which is proportional to the electrostatic potential. The model consists of two equations, the perpendicular momentum equation for the stream function evolution, and the equation of state for the pressure evolution. Dissipative terms are included in both equations. Details of the equations and numerical methods can be found in Ref. [3]. We concentrate our studies on the near critical regime. It corresponds to a range of β values in which the time-averaged pressure profile during the steady state phase of the dynamical evolution is close to the marginally stable profile to resistive pressure-gradient-driven instabilities.

By changing the value of the dissipation terms, we change the spectrum of unstable modes. To check the dependence of the transport properties with the spectrum, we have made two simulations for the case $\beta_0 = 0.01$. In the first simulation, modes with $n > 35$ are stable, and the linear growth rate of modes with $n < 30$ is only weakly affected by the collisional dissipation. In the calculation, we have included 1700 Fourier components, with a maximum n value of 50, and the radial resolution is $\Delta\rho = 2 \times 10^{-3}$ times the minor radius [3]. In the second, the linear growth rate is weakly affected for modes with $n < 100$, we have included 8187 Fourier components, with a maximum n value of 120, and the radial resolution is 5×10^{-4}.

To study the transport properties, we investigate the time evolution of pseudo-particle tracers. An arbitrary parallel velocity is added to the turbulent flow to give the velocity of the tracers [3]. This parallel velocity acts as a randomizing term for the tracer particles. Using the information of flights, we try to evaluate the tail exponents of the waiting time (β) and step-size (α) pdfs. The easiest way of estimate these exponents is to calculate the pdf of particle displacements with respect to their initial position as a function of time. However, this method does not work in the case of pressure-gradient-driven turbulence because the averaged number of particle flights before leaving the plasma is seven, and there is not possible to identify the tail of the distribution [3].

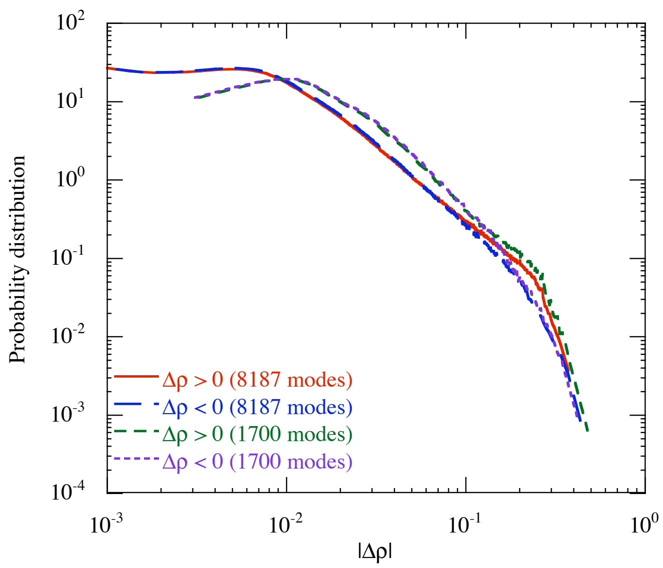

Figure 1: Convergence of the probability distribution function of the flights with the number of modes included in the calculation.

An alternative method to obtain the step-size pdf is through the pdf of flights. The problem is that the definition of flight is somewhat arbitrary. Here, we use a simple definition of flight. A tracer performs a flight while moves on a trajectory keeping the same sign in the radial velocity, with the exception of the case when the radial displacement is smaller than the radial resolution. In Fig. 1 we have plotted the pdf of flights for the cases when we include 1700 and 8187 Fourier components. The power law decay index is -2.1 ($\alpha = 1.1$) in the case with 1700 components and -1.9 ($\alpha = 0.9$) in the case with 8187 components, showing a very weak dependence with the number of modes. The power law region covers more than a decade for both cases, and is larger when we increase the number of modes and the radial resolution.

The second exponent, β, is computed by using the relation $\beta = \alpha \cdot H$, where H is the Hurst exponent [2]. The Hurst exponent is computed by applying the R/S statistics analysis technique to the radial Lagrangian velocity time series, averaged over many realizations [3]. We find that $H \simeq 0.63$, which corresponds to superdiffusive transport.

Dissipative trapped electron mode turbulence

For the dissipative trapped electron mode turbulence, we use a single-equation fluctuation model of plasma drift waves derived in the limit of long wavelengths for cylindrical geometry [5]. The description of the dynamics is reduced to a pair of coupled partial differential equations, which respectively describe the temporal evolution of the mean and fluctuating densities. The details of the equations and numerical methods are described in Ref. [4]. The equation for the mean density includes the nonlinear interactions between fluctuations, a source, and a diffusive term. The nature of the radial transport was observed to gradually become more diffusive-like as the intensity of the diffusive term was raised from zero [4].

As in the case of the pressure-gradient-driven turbulence, we follow tracers to characterize the transport properties. In this model, the fluctuating electrostatic potential is determined by the density fluctuations. The exponent α can be determined through the pdf of flights as we did before. However, for this case we will use a method described in Ref. [2]. The exponent α is determined by the pdf of the radial Lagrangian velocities averaged over many realizations. The Hurst exponent is computed by applying the R/S statistics analysis technique to the radial Lagrangian velocity time series, as before. The results are shown in Fig. 2 for four values of the diffusivity. Its values are such that most of the radial transport is carried by the turbulence. In the absence of the diffusion channel ($D_0 = 0$), $H \simeq 0.74$, $\alpha \simeq 0.98$ and $\beta \simeq 0.73$. That is, radial transport is supperdiffusive, but also strongly non-Gaussian and non-Markovian, as in the case of pressure-gradient-driven turbulence in the near critical regime. The Lagrangian method also quantifies the gradual transition towards Markovianity and Gaussianity of the dynamics, consisted with the results reported in Ref. [4]. As the value of D_0 is increased, α tends to 2,

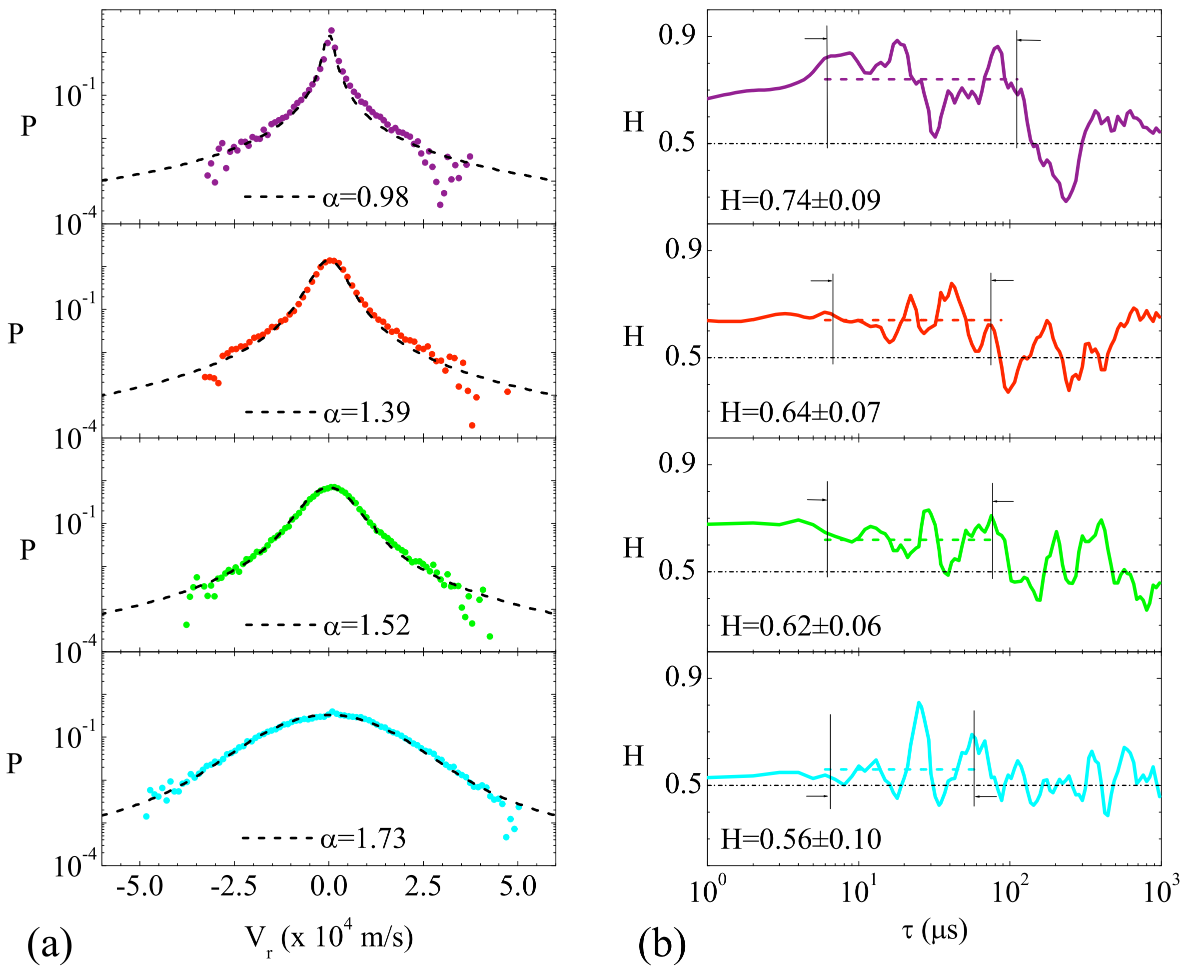

Figure 2: Probability density functions of Lagrangian velocities (Panel a) and Hurst exponent of Lagrangian velocity series versus elapsed time (Panel b) for various diffusivities [Top: $D_0 = 0$; Second: $D_0 = 0.07\,\mathrm{m}^2/\mathrm{s}$; Third: $D_0 = 0.18\,\mathrm{m}^2/\mathrm{s}$; Bottom: $D_0 = 1.19\,\mathrm{m}^2/\mathrm{s}$]

the Gaussian value; H tends to $1/2$, the diffusive value. And, consequently, β tends to 1, the Markovian value.

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