

Amplitude bifurcation in the peeling-relaxation ELM model

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1. Introduction

Edge Localised Modes (ELMs) are Tokamak instabilities which occur at the edge of the plasma and are characterized by repeated losses of both particles and energy to the surrounding vacuum [1]. ELMs are only encountered in the H mode of operation as they result from the large current and pressure gradients that arise when the H mode pedestal is formed after the transition from L mode. The plasma ejected by these instabilities can deposit large amounts of energy on plasma facing components such as the divertor, thus leading to reductions in divertor lifetime. The tolerances for ELM size on ITER are very limited, with an upper limit on fractional energy losses from the edge pedestal ($W_{\text{ELM}}/W_{\text{Ped}}$) of less than 5% [2]. Current ELM studies on existing experiments have indicated that $W_{\text{ELM}}/W_{\text{Ped}}$ can range up to as much as 25%. Therefore understanding ELMs is of key importance.

In this paper we will build on an existing ELM theory [3] by introducing the bootstrap current which is associated with the edge pedestal region. We will predict both ELM widths and $W_{\text{ELM}}/W_{\text{Ped}}$ for a cylindrical plasma and show that these quantities are highly dependent on the bootstrap current. A full account of the Taylor relaxation theory of ELMs can be found in [3]; however, the underlying premise is that the plasma is initially unstable to peeling mode instabilities and this unstable plasma then Taylor relaxes [4] radially inwards from the edge, until a stable point (r_e) is reached. By examining the radial distance relaxed in a cylindrical model, it is possible to calculate the ELM width.

The bootstrap current [5] arises spontaneously as a result of the confinement of the plasma in a Tokamak. It is caused by density and temperature gradients and the interactions of trapped particles with passing (free) ions and electrons and can form a significant part of the toroidal current in a Tokamak. The bootstrap current is important in the study of ELMs as there is a significant bootstrap current in the edge pedestal which will affect the plasma current and safety profiles in this region, and therefore the stability of the plasma to peeling modes.

2. Incorporating the Bootstrap Current into the Relaxation Model

In previous work [3] the initial plasma state was modelled to have a parabolic safety profile; we now consider a more complex parameterisation of the q profile which allows for a localized increase in the current near the edge due to bootstrap. A \tanh -like function added to the parabolic profile can conveniently represent the effect of adding the bootstrap current, because the step function added to the q profile corresponds to a peak in the current density profile $I = (1/r)d/dr(r^2/q)$; where I is the non-dimensional current density, q is the safety profile and r is the radial distance. The profile

$$q_i = q_0 + (q_a - q_0)r^2 - d_{iq}r^2(1 + \text{Tanh}(\frac{r - r_b}{d_{ia}})) \quad 1$$

was chosen, where d_{iq} is a measure of the size of the bootstrap, d_{ia} is measure of both the magnitude and width of the bootstrap term, and r_b is its radial position. This form allows the results in [3] to be replicated by setting d_{iq} to zero, thus removing the bootstrap term.

Following [3], the final (post ELM) state is taken to be a Taylor relaxed state for $r_e \leq r \leq a$, whose profile is determined by conservation of flux and helicity, with current sheets at $r = r_e$ and $r = a$. With the modified profile (1), this requires numerical integration. The position of r_e is determined to be the largest value which gives a stable profile to all modes.

The dependence of maximum predicted ELM width ($d_{e \max}$) value on the edge q for three different magnitudes of bootstrap current is shown in Figure 1. The three curves correspond to no bootstrap current (blue), a moderate bootstrap current (green) and a large bootstrap current (red). It can be seen that for the moderate and large bootstrap currents, the ELM widths bifurcate into two distinct bands. One band has large ELM widths (up to 0.8 in normalised units for the large bootstrap current) while the other band has ELM widths of comparable size to the non-bootstrap case. It is notable that there are no intermediate ELM widths; the bands are distinct with clear gaps between them. As this banding is not present in the non-bootstrap case, it must be concluded that this banding of results is a direct consequence of the inclusion of this current. Finally, it is noted that the ELM widths and the gap between the bands both increase in size as the bootstrap magnitude increases. Note that the ELM widths generated by the large bootstrap current are included for illustrative purposes as they are unrealistically large. In order to explain these results we examine the stability graphs (Figure 2) for two points with similar q_a but with vastly differing ELM widths. Note that $d_e = \frac{(a - r_e)}{a}$ is determined to be the lowest value where $-\delta W$ becomes negative (i.e. is stable).

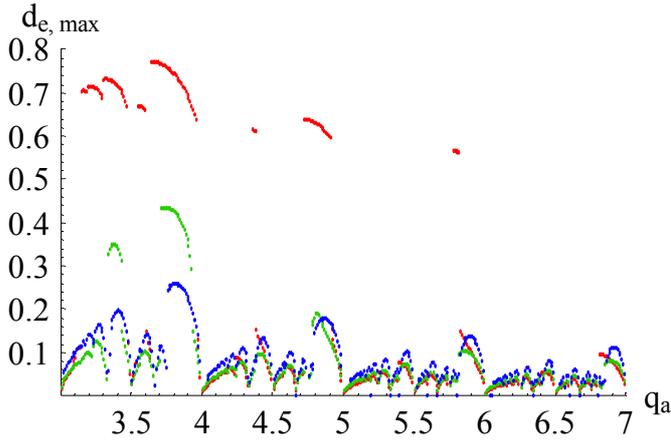


Figure 1: Plot of maximum predicted ELM width $d_{e,max}$ against edge q for no bootstrap (blue), moderate bootstrap (green), and a large bootstrap current (red)

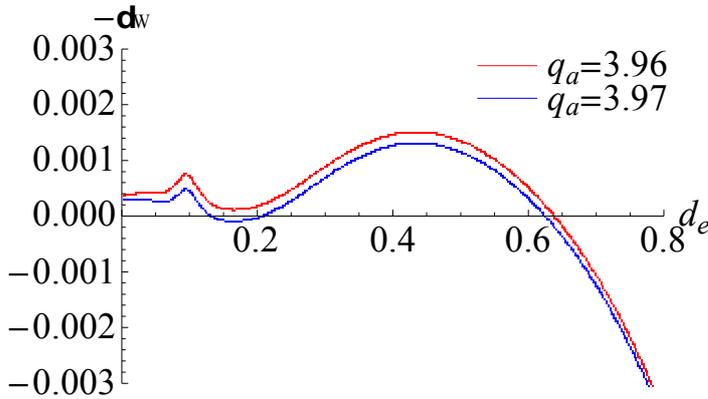


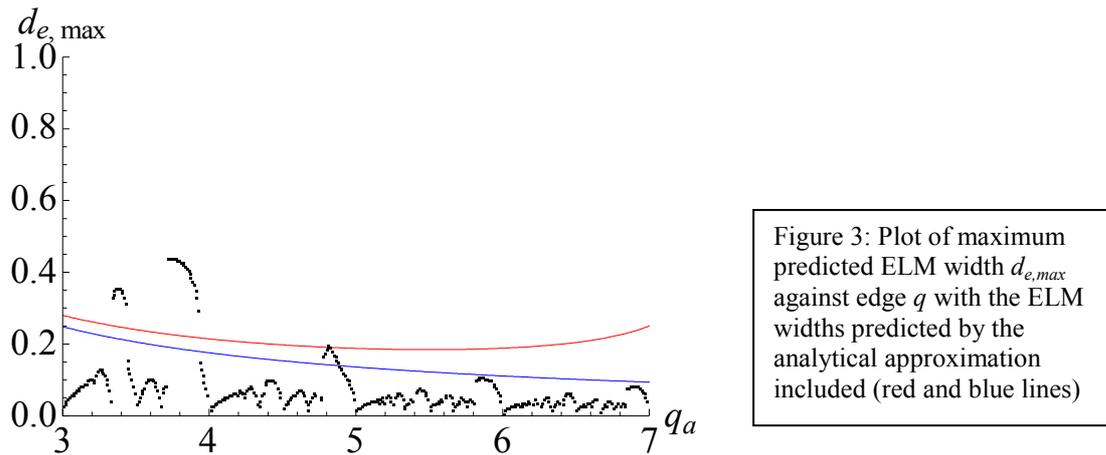
Figure 2: Plot of $-\delta W$ against width of relaxed region d_e for two edge q values.

It can be seen that the shape of the energy curve can lead to large jumps in the radial distance at which the plasma becomes stable, for very small changes in q_a .

We now proceed to use a perturbation approach to find an analytic approximation for the ELM width with a bootstrap current present. The initial current J_Z is Taylor expanded inwards from the plasma vacuum interface and the bootstrap is approximated by a delta function at r_b . The following expression for ELM width, \mathcal{E}_{max} , can be derived:

$$\mathcal{E}_{max}^2 = \begin{cases} r_e > r_B \rightarrow \frac{-3I_a^2}{4n(al'_a)} \\ r_e < r_B \rightarrow \frac{-I_a^2}{16n(\frac{I_B r_b}{a} + \frac{al'_a}{12})} \end{cases} \quad 2$$

where I_a is the toroidal current density at $r = a$, I_B is the bootstrap current density, and n is the toroidal mode number of the peeling instability. This result shows that there will be a split in ELM widths depending on whether the bootstrap current peak is reached by the relaxation process. This is very similar to the banding of results seen in Figure 1. The results, with parameter I_B calculated to fit the numerical profiles graphs, are shown in Figure 3 superimposed onto the results of numerical calculations for the same conditions.



It can also be shown that altering the other variables in (2) such as r_b and d_{ta} the degree of bifurcation and the gap between bands are both altered but the general characteristics of Figure 1 remain the same.

3. Conclusions

It is found that inclusion of the bootstrap current in the relaxation theory of ELM size [3] has a definite and dramatic effect on ELM width. In the analytical approximations (with a highly localized but small bootstrap), whether or not the bootstrap location is reached by the relaxation process is the major factor in causing the ELM widths to be split. In the numerical results, the larger widths of the current distributions studied result in the splitting occurring at less well defined points. We speculate that the two distinct ELM sizes found here could be a possible explanation for the differences between Type I and Type III ELMs. It is also hoped to link the results from this paper with present studies into ELM mitigation.

References

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