Radio Frequency Effects on the Charging of Dust Grains

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Introduction

Dusty plasmas are attracting increasing interest due to their applications both in industry and in basic physics research. One of the fundamental properties of a dust grain which is central to every model of its dynamic behaviour in a plasma environment is its charge. The most common approach to determining this property is through the OML (Orbital Motion Limited) approach. Many dusty plasma systems involve time varying phenomena. These include radar surveys of the mesopause [1] and experiments in RF discharges. In the latter case, the solid particles need not be confined in the sheath region but can reside instead in the bulk plasma, as in microgravity [2] and applied temperature gradient experiments [3]. Previous work [4, 5] considered the sheath region and adopted a different approach. Fluid and kinetic simulations [6, 7] predict a small RF electric field in the bulk plasma. Also, theory and experiments [8] provide indication of a combination of a stationary plus a drifting (SPD) Maxwellian distribution function for the electrons carrying the current in the bulk plasma of an RF discharge. Our main aim is to study how each of these two mechanisms affects the charging of dust grains.

Orbit motion limited (OML) approach

In OML, the unperturbed ion and electron distribution functions far from the dust grain, are used in conjunction with the trajectories of particles in the vicinity of the solid particle to compute the currents onto it. Conservation of energy and angular momentum is assumed to calculate the cross-section, $\sigma_j$, for charging collisions. The current is calculated by integrating over the distribution of the plasma particles.

If a Maxwellian distribution is assumed for the ions and electrons the currents take the form

\begin{align}
I_{i,\text{OML}} &= \pi r_d^2 \rho_i \left( \frac{8k_B T_i}{\pi m_i} \right)^{1/2} \left( 1 - \frac{e \Phi_d}{k_B T_i} \right) \\
I_{e,\text{OML}} &= -\pi r_d^2 \rho_e \left( \frac{8k_B T_e}{\pi m_e} \right)^{1/2} \exp \left( \frac{e \Phi_d}{k_B T_e} \right)
\end{align}

(1)

The floating potential of the dust grain is found using the condition that at steady state the ion and electron currents balance. OML has been shown, for the drift-free case, to be sufficiently accurate for grains much smaller than $\lambda_{de}$. 
Figure 1: (Left) Solution of the differential equation for the charge evolution for 6.78, 13.56 and 27.12 MHz. (Right) The normalised dust grain potential as a function of the normalised electric field for 6.78, 13.56 and 27.12 MHz. The electric field is normalised to $k_B T_e/e$.

Dust grain charging in a time varying electric field

Assumptions

The model assumes that we have an isolated dust grain within a uniform and neutral plasma with the electrons and the ions having initially a Maxwellian distribution. The applied electric field, $E_1 = E_{RF} \cos(\omega_{RF} t)$, is in the x direction and it is only a function of time. The frequency is, $\omega_{pi} << \omega_{RF} << \omega_{pe}$. Hence, the ions are not affected by the applied electric field. On the other hand, the electrons respond to the applied electric field instantaneously. It is also assumed that the electric field of the dust grain is small compared to the time dependent one. This assumption is valid for the whole volume of the plasma, except a small sphere around the dust grain where its electric field is dominant. Finally, it is assumed that the time for the electrons to travel through this region is much smaller than the period of the RF field.

Model

The applied electric field is experienced only by the electrons and acts as a perturbation to their distribution. The linearised Vlasov equation is solved and an expression for the perturbed electron distribution function is obtained.

$$\frac{\partial f_1}{\partial t} - \frac{eE_1}{m_e} \cdot \frac{\partial f_0}{\partial u} = 0$$  \hspace{1cm} (3)

$$f_1 = \frac{eE_{RF} \sin(\omega_{RF} t)}{m_e \omega_{RF}} \frac{\partial f_0}{\partial u_x}$$  \hspace{1cm} (4)

By following the OML formalism the current of electrons on the dust grain due to the per-
Figure 2: (Left) Comparison of the solution of the differential equation for the charge evolution for 13.56MHz with the charge given by a DC current with the amplitude of the current $I_0$ and the $I_{RMS} = I_0/\sqrt{2}$. (Right) The normalised dust grain potential as a function of the factor $\beta$ for 13.56MHz. The drift velocity is normalised to the electron thermal speed.

turbed distribution $f_1$ can be calculated.

$$I_{e,RF} = \frac{2\sqrt{\pi}eE_{RF} \sin(\omega_{RF}t)}{m_e\omega_{RF}} \left[ \frac{3}{4} A \exp \left( -A^2 \right) + \frac{\sqrt{\pi}}{2} \left( \frac{3}{4} - A^2 \right) \text{erfc}(A) \right]$$

where $A = u_{e,min}/u_T$, with $u_{e,min} = \left( -\frac{2e\Phi_d}{m_e} \right)^{1/2}$ being the minimum speed an electron needs to reach a negatively charged dust grain and $u_T$ the electron thermal speed. The rate of change of the charge of the dust grain is equal to the sum of the currents hitting it.

$$\frac{dQ}{dt} = I_{i,OML} + I_{e,OML} + I_{e,RF}$$

By solving this differential equation for the charge of the dust grain and taking the vacuum capacitance approximation we can find the floating potential of the grain, some results of this approach can be seen in Figure 1.

**Charging from SPD-Maxwellian distributions**

We consider the case of a SPD-Maxwellian distribution comprised of a large stationary population, $f_1$, and a smaller drifting population, $f_2$, with a drift velocity $u_d$ which is a sinusoidal function of time, with angular frequency $\omega_{RF}$. We call $\beta$ the ratio of the number density of the small population to the overall density, $\beta = n_2/n_0$. For a given current density we calculate the corresponding $u_d = \frac{j_e}{e_n \beta}$. The electron currents take the form, see [9].

$$I_e = (1 - \beta) I_{e,OML} + \pi r_d^2 e_n \beta \left( \frac{8k_BT_e}{\pi m_e} \right) [G_1(\chi_e) + G_2(\chi_e)\Phi_d]$$

(7)
where $\chi_e \equiv \left( \frac{m_e u_e^2}{2k_B T_e} \right)^{1/2}$ and $G_1(\chi_e), G_2(\chi_e)$ are defined in [9]. The same formalism is used to calculate the floating potential. We study for a given current the effect of the variation of the $\beta$ term. Results are given in Figure 2.

**Conclusions**

We have studied two models for the charging of dust grains in the bulk plasma of an RF discharge. The first model, considers the charging of the particle in an externally applied spatially homogeneous electric field. In this case, it is shown that there is an increase in the resulting average potential of the dust grain. The grain’s instantaneous potential fluctuates around an average constant value, see Figure 1 (left). This increase increases with increasing electric field amplitude and decreasing RF frequency, see Figure 1 (right). In the second model, we assumed that the alternating currents in the bulk plasma of an RF discharge are described by an SPD-Maxwellian distribution. It was shown that in this case the dust grain acquires an increased average potential compared with OML, calculated for a Maxwellian distribution of the same temperature, while the grain’s instantaneous potential fluctuates around an average constant value, see Figure 2 (left). The increased potential of the dust grain is a function not only of the amplitude of the current but also of the part of the distribution carrying the current, see Figure 2 (right).

**References**


