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PHYSICAL COLLISION FREQUENCY $\nu_{\mathrm{eff}}(T, \omega)$ FOR METALS AND WARM DENSE MATTER

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The collision frequency between electrons and ions is a central quantity for laser absorption and transport processes in dense plasma. Here we study the collision frequency in metal-like, partially degenerate, warm dense matter, where free electrons play the major role. In particular, the transition from cold metallic to hot plasma states through states near the Fermi temperature is of considerable practical interest. This regime at temperatures T = 0 – 100 eV is poorly understood, so far. The Spitzer formula applying to high temperature plasma breaks down in this regime, and typically ad hoc fit formulas are used [1, 2]. In addition, for absorption of VUV light at frequencies ω larger than the plasma frequency ω_p, the collision frequency $\nu_{\mathrm{eff}}(T, \omega)$ depends on both T and ω. VUV absorption has been the primary motivation for this work [3, 4].

Here we derive $\nu_{\mathrm{eff}}(T, \omega)$ from the bremsstrahlung cross section, using Sommerfeld's exact expression [6]. The essential new result [5, 8] is obtained for "slow electrons" ($u < Z\alpha c$, Z ion charge, α = 1/137), corresponding to the low temperature regime considered here. For slow electrons we find that the collision rate depends on electron velocity u like $1/u$, different from the usual $1/u^3$ scaling which applies to fast electrons and leads to the Coulomb logarithm with appropriate cut-offs after thermal averaging. For slow electrons, however, averaging $< 1/u >$ over a Fermi distribution leads to a finite analytical expression which represents the central result of this work. The new formula, containing no free parameters, covers the whole temperature regime of warm dense matter down to metals at room temperature and frequencies from $\omega = 0$ (DC) to optical and VUV photon energies. For $\omega < \omega_p$, $\nu_{\mathrm{eff}}(T, \omega)$ is replaced by $\nu_{\mathrm{eff}}(T, \omega_p)$ to account for Debye screening. The formula agrees surprisingly well with existing data. This will be shown in the talk.

1. Derivation of the rate of inverse bremsstrahlung absorption

Sommerfeld's cross-section for bremsstrahlung emission at $\hbar\omega = m(u^2 - u'^2)/2$ of an electron colliding with an ion of charge Z and changing velocity from u to u' [1] reads

$$\frac{d\sigma(u \to u', \omega)}{d\omega} = \frac{64\pi^2}{3} \frac{Z^2 \alpha_f^5 c^2}{(u-u')^2} \frac{u'}{u} \frac{-d\left|F(\xi)\right|^2 / d\xi}{(1-\exp(-2\pi\eta'))(\exp(-2\pi\eta)-1)} \frac{a_B^2}{\omega}$$

where $\alpha_f = e^2/\hbar c$, $\eta = Ze^2/\hbar u$, $\eta' = Ze^2/\hbar u'$, $a_B = \hbar^2/me^2$; $F(\xi) = F(i\eta', i\eta, \xi)$ is the hypergeomtric function with $\xi = -4uu'/(u-u')^2$. For ion density n_i and frequency interval $\Delta\omega$, the corresponding spontaneous emission rate is $R_e^{sp}(u \to u', \omega) = u n_i (d\sigma/d\omega)\Delta\omega$.

Adding stimulated emission, the total rate becomes $R_e^{tot} = R_e^{sp}(1+N_{ph})$, where the photon number per mode is $N_{ph} = \left(\pi^2 c^3/\omega^2\Delta\omega\right) n_{ph}$, and the photon density is $n_{ph} = E_0^2/(8\pi\hbar\omega)$ for a light wave with electric field E_0. We now make use of detailed balance [9] to obtain the rate of absorption by inverse bremsstrahlung $R_a(u,\omega\to u')$ from the rate of spontaneous bremsstrahlung emission. One finds

$$\frac{R_a(u,\omega\to u')}{R_e^{sp}(u'\to u,\omega)} = \frac{dZ(E')/dE'}{dZ(E)/dE\; N_{ph}^{-1}} = \frac{u'}{u} N_{ph}$$

where $dZ(E')/dE'$ and $dZ(E)/dE\; N_{ph}^{-1}$ denote final state densities. The total absorption rate is then obtained as R_a reduced by stimulated emission $R_e^{sp} N_{ph}$, leading to

$$w_t = R_a^{tot}(u,\omega\to u') = \frac{\pi^2 c^3}{\omega^2}\frac{E_0^2/8\pi}{\hbar\omega} n_i \left(u'\frac{d\sigma(u'\to u,\omega)}{d\omega} - u\frac{d\sigma(u\to u',\omega)}{d\omega}\right) \qquad (1.1)$$

Here the total photon absorption rate w_t is expressed in terms of Sommerfeld's cross-section. It is related to the effective electron-ion collision frequency ν_{eff} and the absorption coefficient μ by $\mu c_{gr} = \nu_{eff}\omega_p^2/\omega^2 = w_t n_e / n_{ph}$ with plasma frequency $\omega_p = (4\pi e^2 n_e/m)^{1/2}$, electron density n_e, photon density $n_{ph} = E_0^2/(8\pi\hbar\omega)$, and group velocity $c_{gr} = n_{ref} c$ of light in plasma.

2. Evaluation in different parameter regions

The absorption rate (1.1) depends on the Coulomb parameter $Ze^2/\hbar u$ and the photon-to-electron energy ratio $\hbar\omega/mu^2$. Different asymptotic regions are marked in Fig. 1. In region (I), referring to fast electrons and low frequency, the Born approximation is valid giving the classical collision frequency $\nu_{eff} = (4\pi Z^2 n_i \hbar m/3e^2)/u^3$. It scales $\sim 1/u^3$ with velocity. Averaging over a Maxwell distribution leads to Spitzer's result

$$\text{region (I):} \quad \nu_{eff} = \frac{4\sqrt{2\pi}}{3}\frac{Z^2 e^4 n_i}{m^2 u_{th}^3}\ln\frac{mu_{th}^3}{Ze^2\omega_p}, \qquad (1.2)$$

where $mu_{th}^2 = k_B T_e$ defines thermal velocity at temperature T_e. Here the Coulomb logarithm corresponds to cut-offs at the Debye length $\lambda_D = u_{th}/\omega_p$ and at the classical closest approach $b_{\min} = Ze^2/mu_{th}^2$ as maximum and minimum impact parameters. Region (II) has been analysed in [8]. One obtains the same expression as before, except that ω_p is replaced by ω:

$$\text{region (II):} \quad \nu_{eff} = \frac{4\sqrt{2\pi}}{3}\frac{Z^2 e^4 n_i}{m^2 u_{th}^3}\ln\frac{mu_{th}^3}{Ze^2\omega} \qquad (1.3)$$

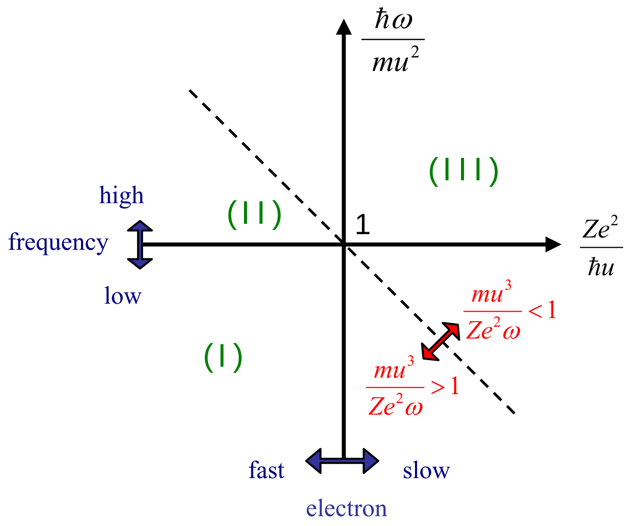

Fig. 1 Different regions for radiative Coulomb collisions in the plane spanned by the Coulomb parameter $Ze^2/\hbar u$ and the photon-to-electron energy ratio $\hbar\omega/mu^2$.

It is clear that these logarithmic results break down in the region $mu^3/Ze^2\omega < 1$ above the diagonal in Fig. 1. Careful evaluation of eq.(1.1) showed [8] that $w_t \sim 1/u$ (instead of $\sim 1/u^3$) in region (III). Thermal averaging then leads to finite results without any logarithmic cut-offs [5]. The central new result is

$$\text{region (III):} \quad \nu_{eff} = \frac{4\pi^2 2^{2/3}\Gamma(1/3)}{15\cdot 3^{5/6}\Gamma(2/3)} \frac{Z^2 e^4 n_i}{m^2 u_{th}^3} \left(\frac{mu_{th}^3}{Ze^2\omega}\right)^{2/3} \left\langle \frac{u_{th}}{u} \right\rangle . \qquad (1.4)$$

It applies to VUV absorption in warm dense matter, where $Ze^2/\hbar u > 1$ and $\hbar\omega/mu^2 > 1$. In this region, electrons are partially degenerate, and averaging has to be performed over a Fermi distribution $f(\varepsilon) = (1+\exp[(\varepsilon-\mu)/k_B T])^{-1}$. The integral including Pauli blocking leads to

$$\left\langle \frac{u_{th}}{u} \right\rangle = \frac{3u_{th}}{mu_F^3} \int_0^\infty d\varepsilon f(\varepsilon)(1-f(\varepsilon+\hbar\omega)) = 3\left(\frac{u_{th}}{u_F}\right)^3 \ln\left(\frac{1+e^y}{1+e^{y-z}}\right) \frac{1}{(1-e^{-z})} ,$$

where Fermi velocity is defined by $mu_F = \hbar(3\pi^2 n_e)^{1/3}$, and $y = \mu/k_B T$, $z = \hbar\omega/k_B T$. A convenient interpolation between the different regimes [5] is

$$\nu_{eff}(T_e,\omega) \simeq 2\sqrt{2\pi} \frac{Ze^4 n_e}{m^{1/2}(k_B T_e)^{3/2}} \ln\left[1 + \frac{1.32}{\sqrt{2\pi}} \frac{k_B T_e}{(m^{1/2} Ze^2 \tilde{\omega})^{2/3}}\right] F(T_e,\hbar\omega) \qquad (1.5)$$

where the Fermi factor is $F(T_e,\hbar\omega) = \sqrt{\frac{\pi}{2}} \left\langle \frac{u_{th}}{u} \right\rangle \Rightarrow \frac{3}{4}\sqrt{\frac{\pi}{2}} \frac{u_{th}}{u_F} \min(\hbar\omega/\varepsilon_F, 1)$ for $\mathrm{T} \to 0$ and 1 for $T \to \infty$. Furthermore, $\varepsilon_F = mu_F^2/2$ is the Fermi energy and $\tilde{\omega} = \max(\omega, \omega_p)$. More details are found in [5]. This result is plotted for solid-density Al targets in Fig. 2. The most pronounced effect in the warm dense matter is due to electron degeneracy. Pauli blocking reduces the number of electrons taking part in the absorption process for $\hbar\omega < \varepsilon_F$ and $\mathrm{T} \to 0$.

4. Comparison with experiments and conclusions

First data for VUV absorption in heated solid-density Al from FLASH/DESY [5] agree reasonably with the present theory; at the same time, optical light reflection from room temperature Al is reproduced. It is tempting to also explore the DC limit ($\omega \to 0$) of eq.(1.5) which gives $\nu_{eff}^{DC}(T) = 0.144 \cdot (me^2/\hbar^3) k_B T / n_i^{1/3}$. It is in surprising agreement with low temperature data [10] from DC conductivity measurements. It is concluded that the first-principle model developed here provides a unified description of radiative absorption in warm dense matter and cold metals over a wide-range of temperatures and frequencies.

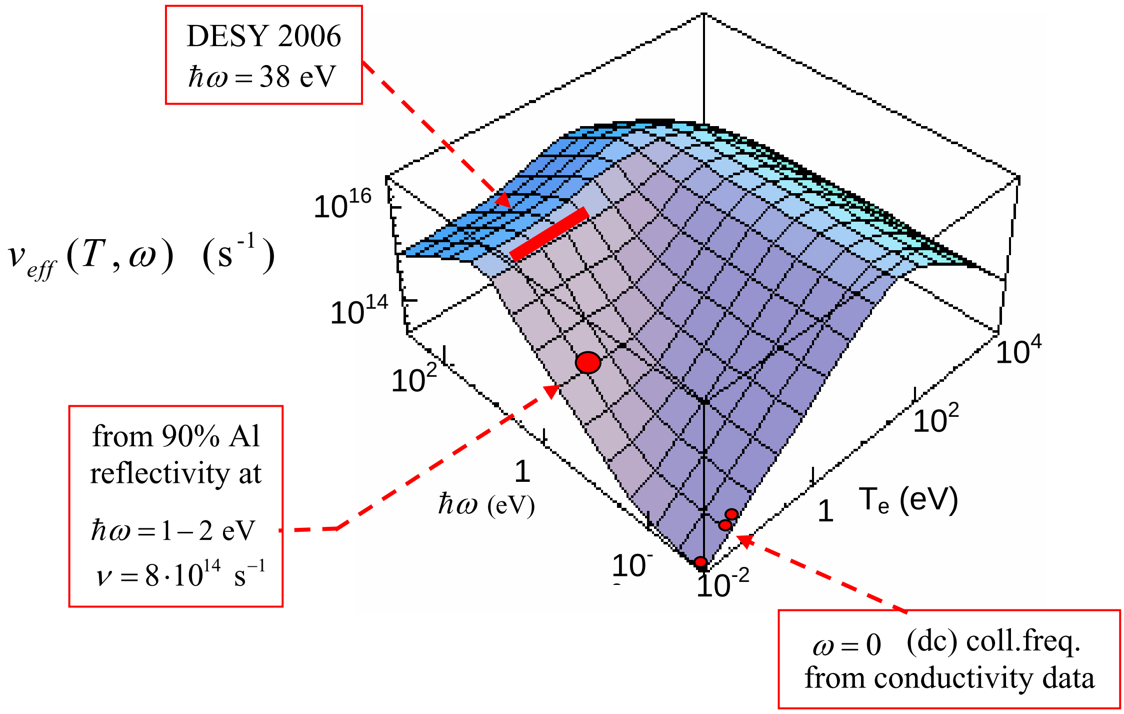

Fig. 2 Radiative collision frequency as function of T_e and ω according to eq. (1.5) and compared to the few existing data.

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