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ANALYTICAL THEORY OF RADIATIVE ABLATION FRONTS FOR DIRECT DRIVE ICF TARGETS

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Radiation transport energy around the ablation front play a significant role in many target materials (CH or CH doped, Berilium, etc) of interest in direct drive ICF [1]: Interpretation of experiments and simulations, Rayleigh-Taylor instability (RTI), double ablation fronts (DAF). The analytic stability analysis of the ablative RT instability is based on a single-temperature for the transport of the energy. The heat flux is proportional to the temperature gradient, and the thermal conductivity follows a power law of the temperature, $\kappa = \kappa_a (T/T_a)^\nu$, where κ_a, T_a are the thermal conductivity and temperature calculated at the peak density ρ_a, and ν is the power index [2]. If the radiated energy is negligible (low-Z materials such deuterium-tritium) the energy is transported mainly by electronic heat conduction. In this case, the power index $\nu = 2.5$ (as given by Spitzer) and $L_{Sp} = [(\gamma_h - 1)/\gamma_h]\kappa_a/(\rho_a V_a)$ is the characteristic thickness of the ablation front, where γ_h is the ratio of specific heats, and V_a is the ablation velocity, respectively [1]. However, if a significant amount of energy is present in the radiation field, then an accurate estimated of the energy transport requires the use of multigroup radiation transport models [3]. Because of the complexity of multigroup radiation transport models, the analytic theories carried out in the past were based on a single-group diffusive model (one temperature) for the energy transport, and determining the parameters ν, the Froude number $Fr = V_a^2/gL_0$, and $L_0 \equiv [(\gamma_h - 1)/\gamma_h]\kappa_a/(\rho_a V_a)$ of such a model, by fitting the 1-D analytic hydro-profiles with those obtained from 1-D simulations including multigroup radiation transport [1]. Notice that in this case, because $\nu \neq 5/2$, κ_a and hence L_0 are unknown. However, there is no guarantee that the two dimensional effects are correctly included in the one group model even though the one-dimensional hydro profiles are well reproduced. Moreover, an essential requirement for the stability analysis is to assure that the linearization is performed about the right equilibrium and obviously, a one-group diffusion model could not reproduce or explain the structure of double ablation fronts (DFA) hydro-profiles.

In the present work we are reconsidering firstly the analytical theory of 1-D subsonic steady ablation fronts extensively used in the past. We are taking into account radiation transport by means of a diffusive grey approximation. We use a nonequilibrium diffusion theory in which the radiation field can have an arbitrary spectral distribution and energy density. This two-group (radiation and matter temperatures) diffusive transport model seems to be a reasonable approximation, qualitatively and quantitatively [4], for material of low-medium ion charge number Z in ICF, as it is supported by simulations. First, an analytical 1D model having two temperatures (radiation temperature T_R and temperature of the matter T) is developed and solved as a boundary value problem. Supported by extensive simulations and opacity calculations in many material [4], the model use two expressions for the mean opacities ($\sim 1/T^{q_1}$ for $T < T_t$ and $\sim 1/T^{q_2}$ for $T > T_t$) with T_t being a transition temperature depending of the material, and typically $q_1 \sim 3$, $q_2 \sim 7$ and T_t is a little larger than the temperature at the ablation front T_a. The theory predicts the conditions for the apparition of DAF and it is used to derive the dispersion relation of the RTI in double ablation fronts. We use an isobaric model for the ablation region which is extended along the y direction. The basic hydrodynamic equations are the mass, momentum and energy conservation [$\nabla \cdot (\frac{\gamma_h}{\gamma_h - 1} \rho T \vec{v} + \vec{q}_e + \vec{S}_r) = 0$], where $\vec{q}_e$ and $\vec{S}_r$ are the electron heat and radiation energy flux respectively. For the radiation energy flux $\vec{S}_r$ we use a diffusion approximation in the frequency-averaged form [3], $\nabla \cdot \vec{S}_r = cK_P(U_p(T) - U)$ and $\nabla U = -3K_R \vec{S}_r / c$, where $U \propto T_R^4$, is the light energy density, $U_p \propto T^4$ is the equilibrium radiant energy density and K_P, K_R the Planck and Rosseland mean opacities. The one dimensional equilibrium solution can be obtained from the previous equations. Normalizing with the values at the ablation front ($\theta = T / T_a$, $\theta_R = T_R / T_a$, $\phi = S_r / \frac{\gamma_h}{\gamma_h - 1} \rho_a T_a V_a$) we get the equation governing the structure of the ablation front region

$$L_{Sp} \theta^{5/2} \frac{d\theta}{dy} = \theta + \phi - 1, \quad l_P \frac{d\phi}{dy} = Bo\, \delta_t (\theta^4 - \theta_R^4), \quad l_R \frac{d\theta_R^4}{dy} = -\frac{3}{Bo} \delta_t \phi, \qquad (1)$$

where l_P, l_R are the Planck and Rosseland photon mean free paths at the peak density, $\delta_t \sim \theta^{-q_1}$ (for $\theta < \theta_t \equiv T_t / T_a$) or $\delta_t \sim \theta^{-q_2}$ ($\theta > \theta_t$) [4], and $Bo \equiv 4\sigma T_a^4 / \frac{\gamma_h}{\gamma_h - 1} \rho_a T_a V_a$ is the Boltzmann number. The system (1) is an eigenvalue problem that is solved with boundary conditions in the coldest region and in the far away plasma where $S_r(+\infty) \approx cU/2$. It is found that the ratio of the radiation energy flux to the total energy flux ($b_1 \equiv S_r/(S_r+q_e)$) at the ablation front depends critically of the Boltzmann number, L_{Sp}/l_P and l_R/l_P. A second parameter characterizing the radiative ablation front is $a_1 \equiv dT_R/dT$ at the peak density, which is a measured about how much the radiation temperature turns off the temperature of the matter. The resolution of the eigenvalue problem (1) gives us a_1 and b_1 in terms of the relevant parameters of the problem:

$$\frac{3L_{Sp}}{l_R Bo} = \frac{4a_1(1-b_1)}{b_1}, \quad \frac{L_{Sp}Bo}{l_P} = \frac{b_1(1-b_1)}{4(a_1-1)}, \tag{2}$$

It can be shown that the characteristic length at the front, defined as (see eq. (1)) $L_0 = L_{Sp} + 4a_1 l_R Bo/3$, is finally given (see eq.(2)) by $L_0 = L_{Sp}/(1-b_1)$. In figure 1, we are giving the parameter $0 < b_1 < 1$ as a function of L_{Sp}/l_P for several values of the Boltzmann number Bo, and $l_R/l_P = 10$. In the figure 2 we are plotting the normalized temperatures versus normalized coordinate y/L_{Sp}. The parameter $a_1 > 1$ is varying through 3.5 (a) , 4 (b) , 5 (c), 10 (d), and 50 (e), while b_1 is fixed to 0.5; it was assumed $q_1 = 3,\ q_2 = 7$, $l_R/l_P = 10$ and $T_t/T_a = 4$. It is noticeable the emerging of the second ablation front (electronic) in cases a, b and c. The maximum density gradient scale length at the radiative ablation front, L_m/L_0 is about 11 in cases a, b, c, and d, while is about 6 in the case e.

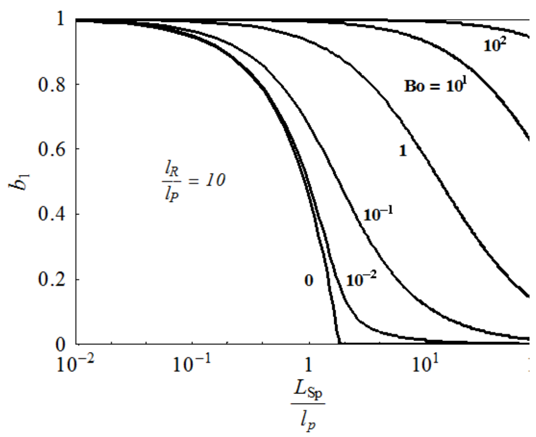

FIG.1. The function b_1 versus L_{Sp}/l_P for several values of the Boltzmann numer Bo.

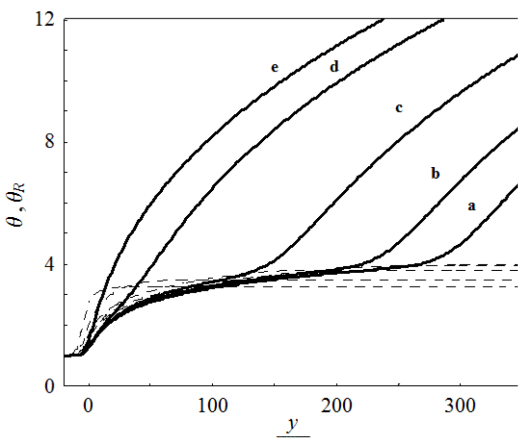

FIG.2. Normalized temperatures θ and radiation temperature θ_R (dashed lines) versus y/L_{Sp} .

Although a complete stability analysis will be done in the next future, here we present some preliminary results of the RT instability in a DAF structure. The model is simplified by taking for the total heat flux the expression: $\vec{q} \propto -l_R T^{3+q_1} \nabla T$, for $T < T_*$, and $\vec{q} \propto -T^{5/2} \nabla T$ for $T > T_*$

, where T_* is some characteristic temperature where the second (electronic) ablation front is located. The model is crude but still very relevant for ICF [4]. This description of the DAF structure would be qualitatively and quantitatively accurate for $L_{Sp}/(l_R Bo)<1$. The analysis is carried out in the standard way expanding variables as $f = f_0 + f_1(y)e^{ikx+\gamma t}$, where f_0 is the equilibrium solution, k the wave number of perturbation and γ the linear growth rate. Until now, only numerical results of the γ-eigenvalue problem we are presenting, reserving for the immediate future the analytical study of the problem.

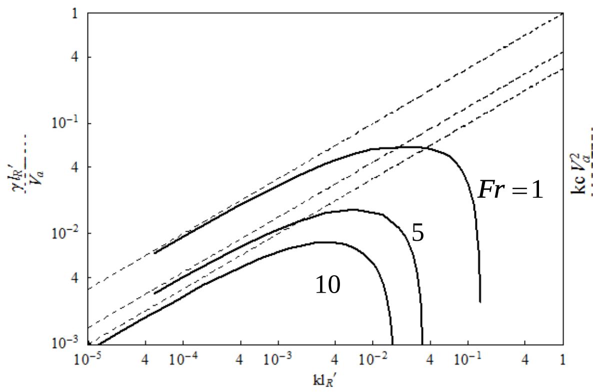

FIG.3. Normalized linear growth rate versus normalized wave number for several values of the Froude number and $L_{Sp}/l'_R = 1$. The dashed straight line correspond to the classical RT, $\gamma = \sqrt{kg}$.

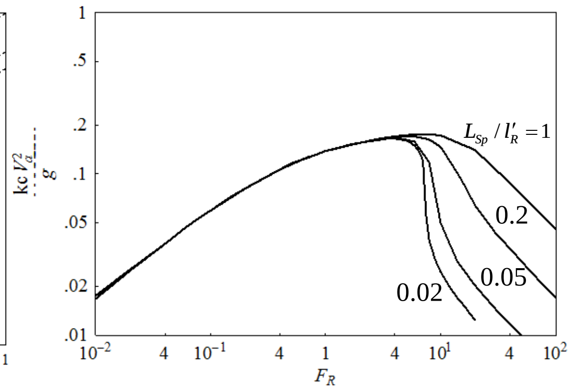

FIG.4. Normalized cutoff wave number versus Froude number for several values of L_{Sp}/l'_R.

In the figure 3 we are plotting the normalized growth rate versus normalized wave number. Here, $l'_R = l_R Bo$, and the Froude number is defined as $Fr = V_a^2/gl'_R$. In figure 4 we plot the normalized cutoff wave number versus Froude number for several values of L_{Sp}/l'_R. As it is noticeable in this figure, for very small values of L_{Sp}/l'_R almost every perturbation with Froude number larger than 10 is stable.

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